

# Optimizing of Discrete Time-cost Trade-off Problem in Construction Projects Using Advanced Jaya Algorithm

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## Abstract

This paper proposes a novel algorithm to optimize the discrete time-cost trade-off problem (DTCTP) in construction projects. As DTCTP is assumed NP-hard, the new metaheuristic models are investigated to contribute for decision-making of project managers. DTCTP can be modelled as multi-mode to represent real-life problems more practical. According to the model, the project activities have at least two or more durations and cost alternative modes. For solving this problem effectively, a novel optimization metaheuristic method named Advanced Jaya Algorithm (A-JA) is proposed, which is generated from Jaya Algorithm (JA). The benchmark function tests are applied to verify the model with other well-known metaheuristic methods. The key weakness of the base algorithm JA is that, it has unstable solution accuracy and low likelihood of escaping local optimums. According to the results, A-JA considerably improves these areas. Two case studies of DTCTP are carried out after verification to demonstrate the effectiveness and efficiency of the proposed algorithm in comparison with JA and three well-known methods. The results of A-JA are found to be more powerful than the base algorithm JA and the benchmark algorithms. The proposed method achieves the Pareto fronts to help decision makers to make trade-off between the objectives and choose the optimum solution considering on their preference.

## Keywords

construction management, Jaya algorithm, metaheuristic method, optimization, multi-objective modelling

## 1 Introduction

The level of competition in the construction industry is rising as more businesses enter the market and as already existing businesses increase their potential. Construction business strives to reduce cost and duration by reducing idle machinery and labor time in order to obtain a competitive edge over the rival. For this reason, the companies and their project managers need remarkable planning and arrangement of construction projects. Certainly, cost and time are the most significant objectives in planning of the projects. These concepts depend on each other intricately. Reducing both project cost and time is highly complex and requires selection of appropriate construction method for each project activity. The different application methods called modes have different time and cost alternatives. Accordingly, the project manager who desires to compress the total completion time has to accelerate some activities. In the view of conventional practice, the project manager reduces project time by hiring more workers, supplying extra resources or investing more efficient

equipment. Subsequently, these methods raise the direct cost of the project as inevitable. Shortening the duration and adjusting the constraints of a construction project are challenging tasks. Crashing project activity durations is an effective method which is used to reduce the overall project completion time. The goal of crashing is to achieve the largest amount of shortening while incurring the least amount of additional cost possible [1]. In the time-cost trade-off problem (TCTP), the project's cost and completion time are traded off. The project managers frequently use the TCTP methodology to complete projects on schedule while incurring the fewest additional costs [2].

In the construction projects, the lower cost solution usually takes longer while the crashed solution usually costs higher. In order to balance project completion time and overall project cost, TCTP seeks to identify the best possible combination of activity durations and activity costs [3]. The different types of functions can be used to simulate the time-cost connection, but the most well-known one is the

discrete time-cost trade-off problem (DTCTP), in which each project activity can only have one of a set of discrete time-cost points. DTCTP is useful for modeling general time-cost relationship and is commonly used in practice.

DTCTP makes the assumption that all project activities can be completed in the various cost and time modes. The goal of DTCTP is to choose the method of each activity that meets the deadline for the least amount of cost. Due to the objectives' strong incompatibility, DTCTP is an NP-hard (Non-deterministic Polynomial-time hard) problem [4]. It should be noted that there is not a single best solution to the issue since, when two competing objective functions are taken into account, it is impossible to improve one without degrading the other. The decision-makers frequently lack sufficient knowledge of how these competing purposes behave, making it impossible for them to express their preferences for the order in which these objectives should be pursued. Therefore, an appropriate tactic for avoiding these trade-offs is to produce non-dominated solutions on the Pareto front. Since DTCTP involves two objectives, when there is no other shorter time under a particular budget or no lower cost under a predetermined project deadline, the solution is considered to be Pareto optimum. The Pareto front is the minimal time-cost curve, which spans the range of minimum and maximum duration and has a negatively sloped convex cost curve.

The goal of this work is to demonstrate the applicability of an alternate metaheuristic optimization method for solving DTCTP, when discrete time-cost combinations are available for a project's activities. This study differs from earlier research in that it suggests a brand-new, enhanced strategy based on the Jaya Algorithm (JA), which has not yet been investigated enough in project planning. The method is applied using numerical experiments that simulate real-world undertakings. Additionally, showing the Pareto front allows decision-makers the flexibility to choose the best option in light of the project priorities.

The rest of this paper is structured as follows. Section 2 provides a quick overview of the DTCTP literature that has already been published. The details of the JA and A-JA algorithms are discussed in Section 3, and the suggested model is validated using benchmark analysis. The decision variables, the objective function, and problem constraints are built upon in Section 4 which is included presentation of the problem formulation and notations. The numerical experiments are then investigated. Additionally, the results are summarized, and Section 5 compares the effectiveness of the strategy. The report concludes in Section 6, which also presents the contributions of the study.

## 2 Literature review

About six decades ago, the significance of the time-cost trade-off was realized along with the emergence of project planning methodologies. Kelley and Walker [5] used parametric linear programming to offer the original time-cost trade-off solution procedures. Primarily, the researchers developed the solution procedures that consider trade-offs among time and cost with linear functions deterministically. Since TCTP has formed as discrete in later studies such as [6] and [7], the problem became NP-hard type. All versions of DTCTP are NP-hard, in the strict sense, as demonstrated in [8], which is the most important complexity result of the project planning literature. Therefore, the use of heuristic and metaheuristic methods has become necessary to solve the problem. In many years DTCTP has been extensively studied in the literature. Only a small number of research, nevertheless, have focused on employing multi-objective models to solve the problem [9].

Although different exact approaches are available for small size experiments, DTCTP cannot be solved with deterministic methods effectively due to the NP-hard structure of the problem. As a result, various metaheuristics have been suggested for resolving problems involving multi-objective optimization. The metaheuristic algorithms work with mathematical imprecision in reaching the ideal solution, but they concentrate on finding good solutions in a fair amount of calculation time. These methods are more practical for dealing with real-world problems than exact procedures, because they allow for fast decision-making.

The Nondominated Sorting Genetic Algorithm (NSGA), Particle Swarm Optimization (PSO), and hybrid algorithms are just a few of the metaheuristic algorithms that have been presented for DTCTP over the past ten years. Some studies related DTCTP are presented in Table 1 [9–28].

The earlier research mentioned below demonstrated the efficiency of the metaheuristic methods on DTCTP. But the majority of them, the researchers looked into more hypothetical cases. As a result, the applications for construction projects' trade-offs have remained superficial. The practical applicability of the proposed method to the actual projects is the most significant contribution of this study.

### 2.1 Discrete time-cost trade-off problem (DTCTP) model

Regarding DTCTP is NP-hard, metaheuristic models have been developed to provide results that can help project managers make decisions. DTCTP can be considered as a multi-mode problem to represent the challenges more accurately faced in practice. The multi-modal modelling

**Table 1** Previous DTCTP studies

Publication	Procedure	Key Contributions
Afshar et al. [10]	Nondominated Archiving Multicolony Ant Algorithm	To assess the effectiveness of the suggested technique, a case study involving 18-activity was examined. The findings demonstrated that the suggested method outperformed well-known weighted method to produce non-dominated solutions in a combinatorial optimization problem.
Hazır et al. [11]	Robust Optimization Models	The models that were created wherein interval uncertainty was taken into account for the uncertain cost factors.
Son et al. [12]	Hybrid Optimization	The two distinct scenarios were mathematically merged using the novel formulation technique for solving DTCTP.
Ghoddousi et al. [9]	NSGA	The impacts of resource levelling on project time and cost were investigated.
Kaveh et al. [13]	CBO and CSS Algorithms	The application of Charged System Search (CSS) and Colliding Body Optimization (CBO) were introduced to solve well-established scheduling problems including DTCTP. The outcomes of the case study showed that CBO model obtained better solutions in a faster process compared to the CSS model.
Said and Haouari [14]	Two-Stage Solution Strategy	The uncertainties of the crashing options were considered in the model.
Aminbakhsh and Sonmez [15]	PSO	The model was developed to provide an efficient method for the large-scale types of DTCTP.
Bettemir and Birgönül [16]	Network Analysis Algorithm	The suggested algorithm's capacity to locate the global optimum and its pace of convergence were evaluated. According to test results, the algorithm reaches optimal or almost optimal answers quite quickly.
He et al. [17]	Variable Neighborhood Search and Tabu Search	The investigation has significance for project scheduling research due to the addition of a new objective as well as practical consequences for contractors to adjust their cash flows.
Li et al. [18]	Bi-Objective Hybrid Genetic Algorithm	The approach produced effective solutions by solving the DTCTP iteratively under various deadline constraints.
Leyman et al. [19]	Iterated Local Search	The model was created with the intention of taking activity progress into account when deciding in DTCTP when and how much to pay the contractor.
Albayrak [20]	Novel Hybrid Algorithm	On the DTCTP application, the method which was created by fusing PSO and GA was contrasted with traditional PSO.
Sonmez et al. [21]	Activity Uncrashing Heuristic	The study looked into a novel heuristic technique that can successfully produce solutions for large-scale type of DTCTP.
Panwar and Jha [22]	NSGA-III	The study offers a different planning approach that helps project managers choose the best trade-offs while building a facility.
EIMenshawy and Marzouk [23]	NSGA-II	The suggested model has the ability to choose the ideal scenario for DTCTP.
Huynh et al. [24]	Multiple Objective Social Group Optimization	A novel optimization method was suggested and then verified on case studies based on two construction projects.
Çakır et al. [25]	Exact methods	A new explicit integer linear programming model and constraint programming model were presented and compared.
Van Eynde and Vanhoucke [26]	Reduction Tree Method	The exact algorithm was proposed to obtain the complete curve of non-dominated time-cost alternatives for the project. The computational experiments show that the use of the reduction tree provides significant speedups.
Son and Nguyen Dang [27]	Hybrid MultiVerse Optimizer Model	The proposed method can accomplish high-quality solutions for medium and large-scale DTCTP and can be used to optimize the cost-time problems for real-life projects.
Yılmaz and Dede [28]	Rao Algorithms	Non-dominant sorting based Rao-1 and Rao-2 algorithms were applied to multi-objective DTCTP. The findings indicate, this approach can be considered a promising alternative to other metaheuristic algorithms.

allows the project manager to exercise greater flexibility in project execution by selecting possible compromises. Each activity with distinct application modes has varied activity time and cost options, according to that modelling. The DTCTP mathematical model is explained below.

Consider a project with  $N$  activities in which project activity utility data is represented as discrete points. There are  $m_i$  distinct points in each action  $i$  where  $m_i$ . Each discrete point represents a distinct method of performing the activity. We assume that  $d_i$  and  $c_i$  are variables to indicate

the time and cost of activity  $i$ . Also,  $d_{i1}$  and  $c_{i1}$  represent the normal point, while  $d_{imi}$  and  $c_{imi}$  correspond to the crash point. Normal and crash points overlap for activities with just one discrete point, and therefore  $m_{i1}$ . Fig. 1 shows the attributes of discrete activity in the time-cost relationship.

In order to obtain the best solution, an effective DTCTP model is created in this study. The model is developed in the binary linear programming that minimizes the total cost and duration. Overlapping across the activities is allowed, and discrete activity time-cost relationships are taken into account. The model includes zero-one (binary) variable constraints and logical constraints based on the network. A zero-one variable ( $x$ ) is required for every discrete point one per activity. In order to guarantee that only one discrete point is chosen for each action, binary variables are included. Eqs. (1) and (2) allow for the expression of the project's overall duration and cost (2). The binary variable  $x_{ij}$  is a component of discrete point  $j$  of activity  $i$  in the equations.

$$\text{Total dur.}(d_i) = d_{i1}x_{i1} + d_{i2}x_{i2} + \dots + d_{imi}x_{imi} = \sum_{j=1}^{m_i} d_{ij}x_{ij} \quad (1)$$

$$\text{Total cost}(c_i) = c_{i1}x_{i1} + c_{i2}x_{i2} + \dots + c_{imi}x_{imi} = \sum_{j=1}^{m_i} c_{ij}x_{ij} \quad (2)$$

### 3 Method

#### 3.1 Jaya algorithm (JA)

The Jaya Algorithm (JA), which has been suggested by Rao [29], is a new optimization algorithm that lack of specific parameters. Other significant characteristics of JA include easy and flexible implementations, as the solutions are updated using a single equation, in addition to specific parameter-less control. Additionally, it has been shown that JA can solve optimization issues that are constrained and unconstrained [30]. JA is a simple yet powerful metaheuristic optimization algorithm, which has been widely used to solve various types of optimization problems [31]. The findings demonstrate that JA outperforms other well-known optimization algorithms, such as GA, PSO, DE, ABC, and TLBO [29].

Initial solutions (P) are produced at random in the JA while adhering to the top and lower limits of the process variables. After that, Eq. (3) is used to stochastically update each variable in each solution.

$$O_{p+1,q,r} = O_{p,q,r} + \alpha_{p,q,1} \left( O_{p,q,best} - abs(O_{p,q,r}) \right) - \alpha_{p,q,2} \left( O_{p,q,worst} - abs(O_{p,q,r}) \right) \quad (3)$$

The value with maximum fitness, or the best value of the objective function, is the best solution, and the value with minimum fitness is the worst option (i.e., worst value

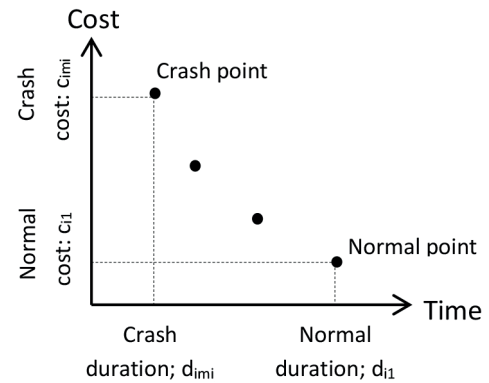


Fig. 1 Duration and cost relation

of the objective function). The terms best and worst in this context refer to the population's best and worst solutions, respectively. The indexes for variables, potential solutions, and iterations are  $p$ ,  $q$ , and  $r$ .  $O_{p,q,r}$  refers to the  $r$ -th candidate solution's  $q$ -th variable in the  $p$ -th iteration.  $\alpha_{p,q,1}$  and  $\alpha_{p,q,2}$  are numbers produced at random between  $[0, 1]$ . An initial population with an NP number of solutions is produced at random at the beginning. The non-dominant idea is then used to sort and rank this initial population. The best answer is chosen as the one with the highest rank (rank = 1). The worst answer is that with the lowest rating. The solution with the highest crowding distance is chosen as the best answer and if there are multiple solutions with the same rank. By doing this, the optimal solution will be chosen from the sparse area of the search space. This selection strategy is used so that the search process can be guided by solutions in less populated areas of the objective space. The updated solutions are based on the fundamental JA equation once the best and worst solutions are determined. A group of 2P solutions (where P is the size of the starting population) is created once all the updated solutions are mixed with the initial population. The crowding distance value for each solution is calculated when these solutions are ranked again. The suitable solutions are selected based on the new ranking and crowding distance value. JA's flowchart is shown in Fig. 2.

#### 3.2 Advanced Jaya algorithm (A-JA)

The Jaya Algorithm is a metaheuristic algorithm that is both simple and effective in terms of its population-based approach. In addition to its simplicity, it does not rely on any specific parameters associated with algorithms. Although it has these advantages, the JA suffers from some shortcomings including unwanted premature convergence and the possibility of being trapped in local minima due to insufficient population diversity [32]. When the objective

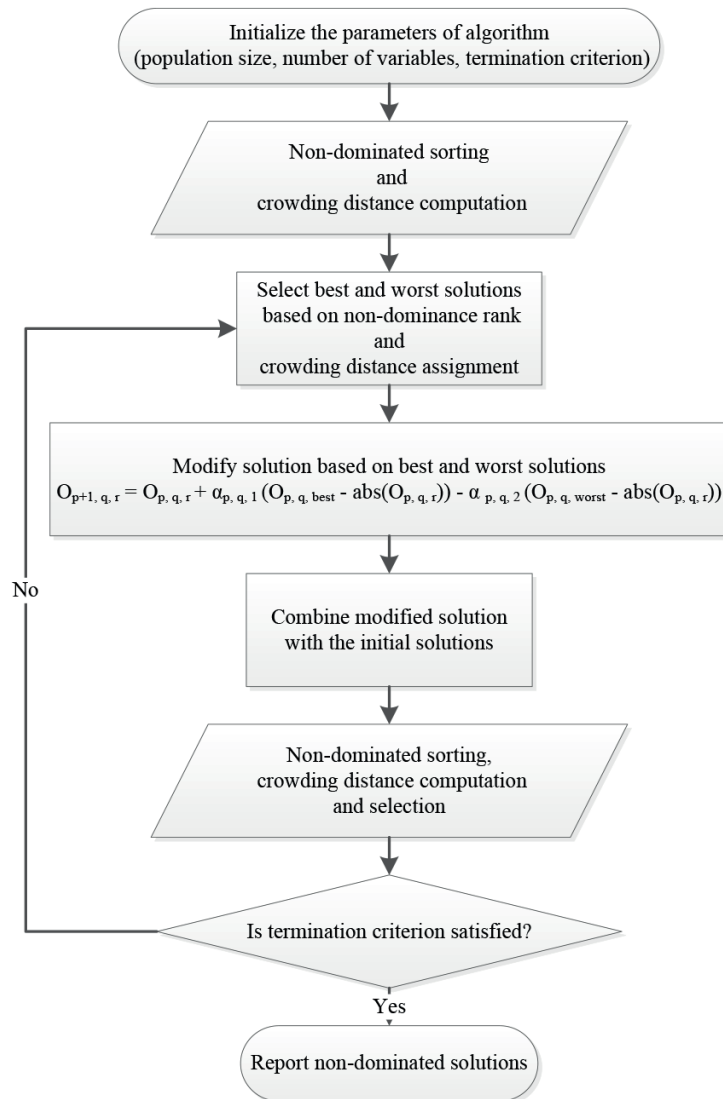


Fig. 2 Flowchart of JA

function converges to a local optimum, the population of the basic JA suffers from a loss of variety and early convergence that may take place. Therefore, the population's diversity needs to be enhanced in order to overcome the shortcomings of the basic JA. An effective optimization technique must also strike a balance between exploitation and exploration [33]. The former concept refers to a population's capacity to arrive at optimal answers as quickly as feasible, whilst the latter can be characterized as a search algorithm's capacity to explore various areas of a search space. While excessive exploration results in a random search, excessive exploitation only leads to a local search. The adjustment to JA that is suggested below will enhance both its global and local search capabilities, which will help to solve the difficulties with search, balancing, and convergence. The expression is given in Eq. (4).

$$O_{p+1,q,r} = O_{p,q,r} + \alpha_{p,q,1} \left( O_{p,q,best(RI)} - abs(O_{p,q,r}) \right) - \alpha_{p,q,2} \left( O_{p,q,worst} - abs(O_{p,q,r}) \right), \quad (4)$$

where  $RI$  is a random integer 1 or 2. Using Eq. (5), where  $RI$  is not a parameter of JA, the value of  $RI$  is determined arbitrarily with equal probability.

$$RI = round \left[ 1 + rand(0,1) \{1-2\} \right] \quad (5)$$

The algorithm does not receive the value of  $RI$  as an input, and instead chooses its value at random using Eq. (5). It has been found after numerous trials on numerous benchmark functions that the technique works best when the value of  $RI$  is between 1 and 2. However, it is discovered that the algorithm performs significantly better if the value of  $RI$  is either 1 or 2. As a result, in order

to make the algorithm simpler, it is advised that *RI* take either 1 or 2 based on the rounding up requirement provided by Eq. (5). Best (1) and best (2) are solution candidates with the lowest and the second values of function within the population. These randomized values ensure exploration by serving as scaling factors. The absolute value of the variable (instead of a signed value) also ensures exploration. The iterations' acceptable function values are all saved and utilized as input for the subsequent iteration. The solution obtained for a given problem in the suggested method moves toward the optimal solution while avoiding the worst option. The random number *RI* ensures that the search space is thoroughly explored. The proposed algorithmic flow is given as follows:

- Step 1: Enter the predetermined parameters, such as the population size, the termination criterion, the number of jobs, the number of machines that are available, the number of operations, and the corresponding processing durations.
- Step 2: Produce the initial population and assess the values of its objective functions. Decide which candidate solution has the highest and lowest function value, and then designate it as the best (or the worst) option.
- Step 3: Determine whether the termination criteria has been met; if so, move on to Step 9; otherwise, move on to Step 4.
- Step 4. Change the responses of the remaining candidates (with the best and worst solution using the proposed JA updating mechanism to generate new solutions)
- Step 5. If the acceptance requirement is not met, update the existing solution to the changed solution for each solution.
- Step 6. Update the existing solution to the changed solution if the acceptance requirement is met; else, keep the current solution.
- Step 7. Create a new population by putting the suggested local search to use on each potential answer. Find the new best solution (and the worst solution respectively).
- Step 8: Go back to Step 3 and repeat the process until the termination requirement is met.
- Step 9: Stop and present the best answer.

A family of functions known as ZDT was chosen for this study because it is a comprehensive and well-liked collection of test functions for evaluating the effectiveness of multi-objective Pareto optimization techniques [34]. Each of these test functions has a specific characteristic that is analogous to a real-world optimization issue that can make convergence to the Pareto front challenging. The performance of the well-known optimization techniques (Genetic

Algorithm-GA, Particle Swarm Optimization-PSO and Multiobjective Evolutionary Algorithm-MOEA) on these test problems is studied. Iteration number and population size are found to be 500 and 25, respectively, after test experiments. The investigated algorithms and their parameters are given in Table 2.

The most typical application of Pareto optimization, particularly in project planning applications, is two targets, which are present in all ZDT functions. Five runs of each method were made for every ZDT function in this paper. This was done in order to determine how reliable each algorithm's outcomes were and to make sure they weren't influenced by the initial conditions. The obtained results are shown in Table 3.

It is important to note that the simplicity and lack of additional control parameters required by the original JA were unaffected by the transition to A-JA. Five benchmark functions were utilized as case studies to assess how well the suggested A-JA performed in terms of convergence speed and solution precision. Benchmark function comparisons with other metaheuristic algorithms reveal that the suggested A-JA greatly improves the original JA's performance. Additionally, of all algorithms, it has the quickest global convergence, the greatest solution quality, and is the most reliable for practically all test functions. In contrast to single objective functions, multi-objective benchmark functions feature a large number of local optimal points. Therefore, they are appropriate for testing the exploration of a specific method. The problem also gets more challenging to solve as the number of local minima for multiple objective functions rises. The results show that A-JA considerably offers good solution accuracy and a higher probability of avoiding local optimums. As a result, it comes in first place among all compared algorithms. As a result, A-JA can be viewed as a very effective method for solving challenging optimization issues.

**Table 2** The parameters of the optimization techniques

Algorithm	Parameter	Value
GA	Crossover probability	1
	nGrid	30
PSO	Inertia weight	0.5
	c1	1
	c2	2
	Probability	1
MOEA	Differential weight	0.5
	Mutation distribution index	20
	Neighbourhood size	30

**Table 3** The benchmark results according to ZDT functions

Functions	Metric	A-JA	JA	GA	PSO	MOEA
ZDT1	Mean	2.03E-04	5.21E-03	7.46E-02	1.51E-02	1.28E-02
	Std.	2.45E-04	3.22E-03	3.57E-02	2.04E-03	3.41E-03
ZDT2	Mean	1.22E-03	5.29E-03	3.23E-03	3.63E-02	3.45E-02
	Std.	2.01E-03	7.89E-04	2.04E-03	2.37E-02	1.98E-02
ZDT3	Mean	3.657E-03	2.79E-03	1.85E-03	5.77E-03	3.31E-03
	Std.	4.04E-04	3.55E-03	1.54E-03	7.65E-03	4.01E-03
ZDT4	Mean	1.52E-03	6.67E-03	2.38E-03	6.03E-02	5.44E-03
	Std.	1.03E-03	5.08E-03	5.72E-03	5.47E-02	2.67E-03
ZDT6	Mean	1.88E-03	1.45E-03	1.32E-03	3.99E-02	4.08E-02
	Std.	3.43E-03	1.17E-03	1.59E-03	5.36E-02	6.02E-02

The learning factor and acceleration coefficient are employed for the initialization of other optimization approaches because they call for a scaling factor and crossover elements. So far as ignoring the effort of altering constraints and shortening the time needed for the optimization process are concerned, the JA and A-JA computation has a crucial advantage [35]. The goal of this work is to improve the algorithm for solving DTCTP. It is motivated by the effectiveness and possible applications of the JA and A-JA. The next section presents the case study of DTCTP using the proposed A-JA and JA comparatively.

#### 4 Application of discrete time-cost trade-off

##### 4.1 Mathematical modelling

The optimization implementations, which are performed on the example to confirm the comparative effectiveness of the A-JA and JA in regards of DTCTP, are discussed in detail. In order to achieve this, the data of the construction project with 18 activities and 63 activities are first explained, followed by a description of the mathematical model of DTCTP. The JA and A-JA algorithms were coded in MATLAB R2021a for this application. The time complexity is adopted to evaluate the runtime of the proposed method in terms of the number of the activities. The testing of the total runtime was done in a laptop with configuration of Windows 10 OS, 1.80 GHz CPU, 8GB RAM. The total runtime of the A-JA was about 30 sec and 150 sec for the projects with 18 and 63 activities, respectively. All of the experiments were performed 20 times in order to reduce statistical mistakes.

Equations (6), (7), (8), and (9) serve as constraints in the mathematical model of the DTCTP shown below, whereas Eq. (10) and Eq. (11) work as objective functions. In equations,  $c_t$  total cost of the project,  $t_t$  the project time,  $c_{ij}$  cost of the  $j$ th mode for  $i$ th activity,  $x_{ij}$  assignment of the  $j$ th

mode for  $i$ th activity,  $T_n$  initializing time of the  $n$ th activity,  $m_n$  mode alternatives,  $n$  total number of activity,  $T_{ij}$  duration of  $j$ th mode of  $i$ th activity and  $T_{max}$  represents maximum termination time. Equation (6) states that day 0 is where the algorithm begins. According to the Eq. (8), the project's maximum completion time should be equal to or less than the total of the initiation time of the  $n$ th activity, which is the last process, and the duration of the same activity in the  $j$ th mode. In accordance with Eq. (8), the starting time of the successor activity should be equal to or less than the total of the initial time of the predecessor activity and the duration of the  $j$ th mode. The final constraint, called as Eq. (9), states that only one mode, from  $j$  to  $m$ , can be chosen for all activities, from  $i$  to  $n$ . In light of this, it is obvious that  $x_{ij}$  has a binary variable value.

Constraint functions:

$$T_1 = 0, \tag{6}$$

$$T_n + \sum_{j=1}^{m_n} T_{nj} \cdot x_{nj} \leq T_{max}, \tag{7}$$

$$T_a + \sum_{j=1}^{m_a} T_{aj} \cdot x_{aj} \leq T_b; \text{ for all predecessors} \tag{8}$$

$a = 1, \dots, n; b = 1, \dots, n,$

$$\sum_{i=1}^n \sum_{j=1}^{m_i} x_{ij} = 1. \tag{9}$$

Objective functions:

$$\text{Min } c_t = \sum_{i=1}^n \sum_{j=1}^{m_i} c_{ij} \cdot x_{ij}, \tag{10}$$

$$\text{Min } t_t = \left[ T_n + \sum_{j=1}^{m_n} (T_{ij} \cdot x_{ij}) \right]. \tag{11}$$

The goal of the model's formulation is to reduce both project duration and overall cost. It incorporates multiple objective functions, depending on the optimization mode, as shown in Eq. (10) and Eq. (11). The first objective function in Eq. (10) aims to decrease overall project costs, whereas the second objective function in Eq. (11) seeks to reduce overall project duration.

#### 4.2 Case studies

In project planning literature, the activity list which holds all activity numbers in a sequentially order is a typical representation for metaheuristic optimization [19].

In this paper, the predecessors, different modes, timings, and cost amounts for each activity for 18-activity project are listed in Table 4 Feng et al. [36] provided the first case study issue, an 18-activity project, and Hegazy [37] identified time-cost modes. The table shows that the relationship between time and cost is the discrete model since some tasks can be accomplished more quickly at the expense of higher costs. Following to 18-activity project, larger scale project is discussed considering the same options to generate solutions for minimum time and minimum cost to illustrate the capability of the presented model better. The second case study large-scale project with 63-activities is taken from Sonmez and Bettemir [38]. Table 5 contains all the required data for 63-activity to be executed in the project.

The case studies have different modes. These modes relate to techniques used in accelerating construction projects. In real life, all project activities cannot have the same number of time and cost alternatives. Some project activities can have only one option. There may not be a cheaper, more expensive, longer or shorter alternatives than this option. So, the activity is single-modal. That is, the activity is without alternative. Sometimes the activity may have different time and cost alternatives, which is called multi-modal. To increase the complexity of the optimization, the presented examples are multimodal project planning problems.

#### 5 Results and comparison

Following test trials, the case study's iteration counts for the first and second cases, respectively, is 200 and 500. Also, the population sizes are determined to be 25 for both cases. Both case studies are solved using five metaheuristic methods. These methods are Genetic Algorithm, Particle Swarm Optimization, Multiobjective Evolutionary Algorithm, Jaya Algorithm and Advanced Jaya Algorithm. The proposed model is employed in this study to simultaneously reduce project time and expense. Also, both the logical relationship and mathematical constraints are satisfied in all algorithms. The termination criteria are determined as reaching of the maximum number of iterations.

**Table 4** 18-activity project information for the first case study of DTCTP

Act.	Pred.	Mode (1)		Mode (2)		Mode (3)		Mode (4)		Mode (5)	
		Dur.	Cost	Dur.	Cost	Dur.	Cost	Dur.	Cost	Dur.	Cost
1	-	14	2,400	15	2,150	16	1,900	21	1,500	24	1,200
2	-	15	3,000	18	2,400	20	1,800	23	1,500	25	1,000
3	-	15	4,500	22	4,000	33	3,200	-	-	-	-
4	-	12	45,000	16	35,000	20	30,000	-	-	-	-
5	1	22	20,000	24	17,500	28	15,000	30	10,000	-	-
6	1	14	40,000	18	32,000	24	18,000	-	-	-	-
7	5	9	30,000	15	24,000	18	22,000	-	-	-	-
8	6	14	220	15	215	16	200	21	208	24	120
9	6	15	300	18	240	20	180	23	150	25	100
10	2, 6	15	450	22	400	33	320	-	-	-	-
11	7, 8	12	450	16	350	20	300	-	-	-	-
12	5, 9, 10	22	2,000	24	1,750	28	1,500	30	1,000	-	-
13	3	14	4,000	18	3,200	24	1,800	-	-	-	-
14	4, 10	9	3,000	15	2,400	18	2,200	-	-	-	-
15	12	12	4,500	16	3,500	-	-	-	-	-	-
16	13, 14	20	3,000	22	2,000	24	1,750	28	1,500	30	1,000
17	11, 14, 15	14	4,000	18	3,200	24	1,800	-	-	-	-
18	16, 17	9	3,000	15	2,400	18	2,200	-	-	-	-

Note: Act. - Activity, Pred. - Predecessor, Dur. - Duration (days), Daily indirect cost: \$1500



**Table 5** 63-activity project information for the second case study of DTCTP

Act.	Pred.	Mode (1)		Mode (2)		Mode (3)		Mode (4)		Mode (5)	
		Dur.	Cost	Dur.	Cost	Dur.	Cost	Dur.	Cost	Dur.	Cost
1	-	14	3,750	12	4,250	10	5,400	9	6,250	-	-
2	-	21	11,250	18	14,800	17	16,200	15	19,650	-	-
3	-	24	22,450	22	24,900	19	27,950	17	31,650	-	-
4	-	19	17,800	17	19,400	15	21,600	-	-	-	-
5	-	28	31,180	26	34,200	23	38,250	21	41,400	-	-
6	1	44	54,260	42	58,450	38	63,225	35	68,150	-	-
7	1	39	47,600	36	50,750	33	54,800	30	59,750	-	-
8	2	52	62,140	47	69,700	44	72,600	39	81,750	-	-
9	3	63	72,750	59	79,450	55	86,250	51	91,500	49	99,500
10	4	57	66,500	53	70,250	50	75,800	46	80,750	41	86,450
11	5	63	83,100	59	89,450	55	97,800	50	104,250	45	112,400
12	6	68	75,500	62	82,000	58	87,500	53	91,800	49	96,550
13	7	40	34,250	37	38,500	33	43,950	31	48,750	-	-
14	1, 8	33	52,750	30	58,450	27	63,400	25	66,250	-	-
15	9	47	38,140	40	41,500	35	47,650	32	54,100	-	-
16	9, 10	75	94,600	70	101,250	66	112,750	61	124,500	57	132,850
17	10	60	78,450	55	84,500	49	91,250	47	94,640	-	-
18	10, 11	81	127,150	73	143,250	66	154,600	61	161,900	-	-
19	11	36	82,500	34	94,800	30	101,700	-	-	-	-
20	12	41	48,350	37	53,250	34	59,450	32	66,800	-	-
21	13	64	85,250	60	92,600	57	99,800	53	107,500	49	113,750
22	14	58	74,250	53	79,100	50	86,700	47	91,500	42	97,400
23	15	43	66,450	41	69,800	37	75,800	33	81,400	30	88,450
24	16	66	72,500	62	78,500	58	83,700	53	89,350	49	96,400
25	17	54	66,650	50	70,100	47	74,800	43	79,500	40	86,800
26	18	84	93,500	79	102,500	73	111,250	68	119,750	62	128,500
27	20	67	78,500	60	86,450	57	89,100	56	91,500	53	94,750
28	21	66	85,000	63	89,750	60	92,500	58	96,800	54	100,500
29	22	76	92,700	71	98,500	67	104,600	64	109,900	60	115,600
30	23	34	27,500	32	29,800	29	31,750	27	33,800	26	36,200
31	19, 25	96	145,000	89	154,800	83	168,650	77	179,500	72	189,100
32	26	43	43,150	40	48,300	37	51,450	35	54,600	33	61,450
33	26	52	61,250	49	64,350	44	68,750	41	74,500	38	79,500
34	28, 30	74	89,250	71	93,800	66	99,750	62	105,100	57	114,250
35	24, 27, 29	138	183,000	126	201,500	115	238,000	103	283,750	98	297,500
36	24	54	47,500	49	50,750	42	56,800	38	62,750	33	68,250
37	31	34	22,500	32	24,100	29	26,750	27	29,800	24	31,600
38	32	51	61,250	47	65,800	44	71,250	41	76,500	38	80,400
39	33	67	81,150	61	87,600	57	92,100	52	97,450	49	102,800
40	34	41	45,250	39	48,400	36	51,200	33	54,700	31	58,200
41	35	37	17,500	31	21,200	27	26,850	23	32,300	-	-
42	36	44	36,400	41	39,750	38	42,800	32	48,300	30	50,250
43	36	75	66,800	69	71,200	63	76,400	59	81,300	54	86,200
44	37	82	102,750	76	109,500	70	127,000	66	136,800	63	146,000
45	39	59	84,750	55	91,400	51	101,300	47	126,500	43	142,750

Continuation of **Table 5**

Act.	Pred.	Mode (1)		Mode (2)		Mode (3)		Mode (4)		Mode (5)	
		Dur.	Cost	Dur.	Cost	Dur.	Cost	Dur.	Cost	Dur.	Cost
46	39	66	94,250	63	99,500	59	108,250	55	118,500	50	136,000
47	40	54	73,500	51	78,500	47	83,600	44	88,700	41	93,400
48	42	41	36,750	39	39,800	37	43,800	34	48,500	31	53,950
49	38, 41, 44	173	267,500	159	289,700	147	312,000	138	352,500	121	397,750
50	45	101	47,800	74	61,300	63	76,800	49	91,500	-	-
51	46	83	84,600	77	93,650	72	98,500	65	104,600	61	113,200
52	47	31	23,150	28	27,600	26	29,800	24	32,750	21	35,200
53	43, 48	39	31,500	36	34,250	33	37,800	29	41,250	26	44,600
54	49	23	16,500	22	17,800	21	19,750	20	21,200	18	24,300
55	52, 53	29	23,400	27	25,250	26	26,900	24	29,400	22	32,500
56	50, 53	38	41,250	35	44,650	33	47,800	31	51,400	29	55,450
57	51, 54	41	37,800	38	41,250	35	45,600	32	49,750	30	53,400
58	52	24	12,500	22	13,600	20	15,250	18	16,800	16	19,450
59	55	27	34,600	24	37,500	22	41,250	19	46,750	17	50,750
60	56	31	28,500	29	30,500	27	33,250	25	38,000	21	43,800
61	56, 57	29	22,500	27	24,750	25	27,250	22	29,800	20	33,500
62	60	25	38,750	23	41,200	21	44,750	19	49,800	17	51,100
63	61	27	9,500	26	9,700	25	10,100	24	10,800	22	12,700

Note: Act. - Activity, Pred. - Predecessor, Dur. - Duration (days), Daily indirect cost: \$2300

Firstly, 18-activity project was solved using GA, PSO and MOEA. These algorithms were able to determine the optimal solutions of \$162390, 161270 and 161270, respectively. Secondly 63-activity project was solved and the same algorithms gave the following results: \$5334600, 5282450 and 5201750. Hence, when compared to GA, PSO and MOEA are more successful for converging the projects. Especially, in 63-activity project, MOEA outperformed to GA and PSO. The results obtained from these different methods can be shown in Table 6.

In first case study, according to JA results, the project can be terminated in 106 days with \$153240 cost. Following the JA, the case study is solved with A-JA. DTCTP results of A-JA have the values of 100 days and \$150270 cost. In second case study, according to JA results, solutions are 618 days for time and \$4990500 for cost. For the same case, A-JA have the values of 616 days and \$4911250 cost.

The Pareto optimal solutions of 18-activity project and 63-activity project obtained by A-JA are graphically presented respectively in Fig. 3.

In addition, according to the results, both JA and A-JA outperformed the other three well-known algorithms in terms of duration and cost. In all 20 runs, the A-JA was able to find the best solution to 18-activity problem. Therefore, further comparisons are made for these two algorithms. In A-JA, which is limited to 200 iterations for 18-activity project, The Pareto front values were noticed in the first 60 iterations. For 63-activity project, A-JA is terminated after 500 iterations and the Pareto front was concluded in 150 iterations. The graphics of number of iterations-fitness value for both cases are given in the Fig. 4. The figures lead to the conclusion that the parameters used and the number of iterations is enough.

**Table 6** DTCTP results of the case studies

Project	Objective	Methods				
		GA	PSO	MOEA	JA	A-JA
18-activity	Duration	113	110	110	106	100
	Cost	162390	161270	161270	153240	150270
63-activity	Duration	624	623	621	618	616
	Cost	5334600	5282450	5201750	4990500	4911250

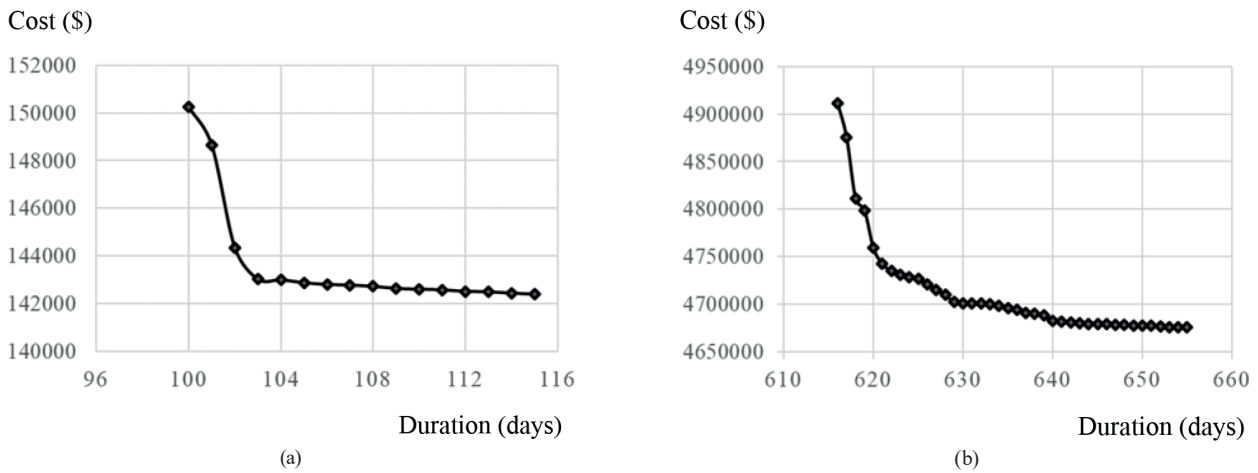


Fig. 3 The Pareto front solutions obtained by A-JA; (a) 18-activity project, (b) 63-activity project

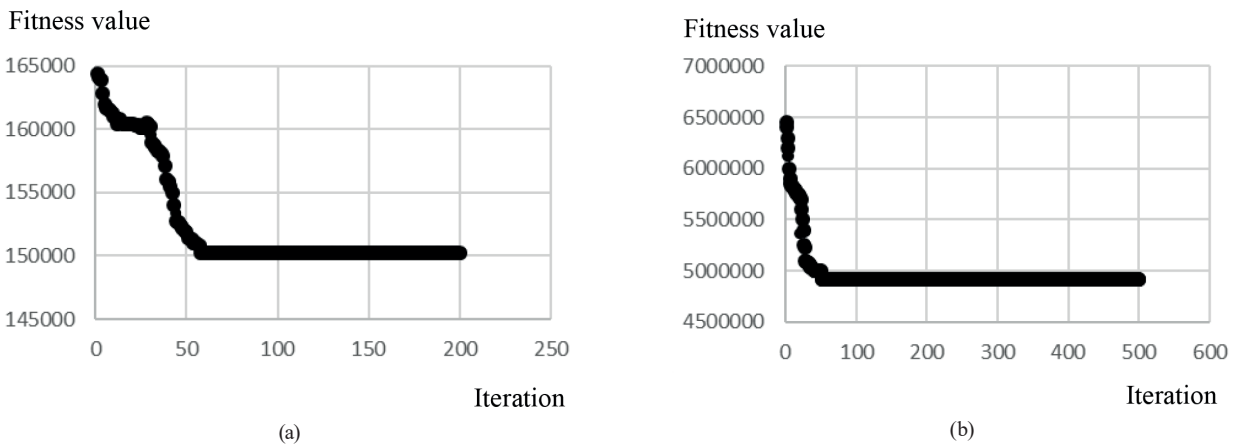


Fig. 4 The number of iterations-fitness value graphic of A-JA; (a) 18-activity project, (b) 63-activity project

Using 18-activity and 63-activity projects, the performances of JA and A-JA are compared. For each project, the metaheuristic methods are used 20 times. The mean deviation of 20 trials from the best solution is used to calculate convergence. To assess the metaheuristics' performance, the mean deviations of JA and A-JA are given and compared in Table 7.

For both the 18-activity and 63-activity implementations, JA and A-JA were capable of arriving at the optimal solution. However, JA had a mean deviation value between 1.62 and 2.11 for the same problems. The mean deviation values of A-JA were 0.96 and 1.14, respectively. The mean deviation of JA for 20 trials is 1.87 and mean deviation of

A-JA is 1.05. In all of the applications, A-JA performed better than the JA. As a result, A-JA presents a sufficient alternative for the DTCTP.

### 6 Conclusions

For the first time, the DTCTP is solved in this paper using an efficient metaheuristic multi-objective optimization algorithm based on the Jaya Algorithm (JA). The new metaheuristic method called A-JA was developed and proposed for DTCTP. In order to evaluate and compare the effectiveness and applicability of the suggested approach, various benchmark problems were simulated. After then, the case studies were developed and analyzed with JA, A-JA and other well-known algorithms GA, PSO, MOEA. Both JA and A-JA outperformed the other three algorithms. The modified algorithm A-JA performed relatively better than the basic algorithm JA. In comparison to the findings of JA, the proposed A-JA demonstrated increased efficacy and efficiency. This research contributes to the construction management body of knowledge in the following terms:

Table 7 The mean deviations (%) from the optimal solution

No. of runs	Project type	Mean deviation (%)	
		JA	A-JA
20	18-activity	1.62	0.96
20	63-activity	2.11	1.14

- Providing a comprehensive review for the construction project planning optimization literature, upon which researchers can rely to investigate the DTCTP in previous literature.
- Developing a novel optimization model that considers multi-mode alternatives for each activity in addition to all project optimization capabilities.
- The model successfully implements a metaheuristic method to optimize construction projects. It can be used in multi-mode optimization to simultaneously minimize duration and cost. It provides help decision makers to make trade-off among these objectives and choose the optimum solution based on their preference.

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