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Reliability Analysis of the Slope Stability of Homogeneous Earth Dam under Seismic Loading

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Abstract

Earth dams are widespread throughout the world and their safety is the overriding concern of civil engineers, especially in regions with high seismic activity. This contribution presents a deterministic and reliability-based approach of the slope stability analysis of homogeneous earth dam under seismic loading. The hydrostatic loading as well as the saturation line in the dam body is considered for different design situations to investigate the effects of water level fluctuations on earth dam slope stability. Seismic loading is introduced by horizontal and vertical pseudo-static forces, while the considered failure mode is circular slip, analyzed with the classical method of slices. Given the nature of the soils, uncertainties about their properties must be considered in the assessment of the slope stability. These uncertainties can be quantified by a reliability analysis. The proposed reliability method is approached by the failure probability assessment of the slope stability of an earth dam based on Monte-Carlo simulation. The used random variables are the mechanical shear properties of the embankment body as well as the acceleration coefficient of the seismic zone. For the failure probability evaluation, the log-normal distribution law was used to generate the random variables. A computer program was developed using Matlab®. The developed approach was applied, for a practical example, to analyze the slope stability of an earthen dam located in Médéa (Algeria), whose dimensions are realistic. The results obtained with the developed analytical model were compared with those obtained with the Flac2D© software. The confrontation turned out to be satisfying.

Keywords

slope stability, earthen dam, pseudo-static forces, probability of failure, Monte Carlo simulation, reliability

1 Introduction

The stability of an earth dam depends on the stability of its upstream and downstream embankments on its foundation. It is determined by its ability to resist, along the potential sliding surfaces, to shear forces deriving from the application of hydrostatic loads, accidental loads (earthquake) and dead loads (weight of the dike embankment). There is no global method to calculate the slope stability of such a system without making assumptions. It has been observed so far that the circular-shaped failure surface is the most critical slip shape, especially for a homogeneous earth dam. This hypothesis is generally accepted, and one generally takes a cylindrical surface with a horizontal axis, which appears as a slip circle in the dike vertical cross-section. Based on this hypothesis, the unstable mass is cut into vertical slices of small juxtaposed thicknesses in order to study the overall balance. It is important to remember that this so-called method of slices is used by most numerical calculation programs because it adapts very well to the complex geometries of the dike slopes, the variable conditions of the embankment, the external stresses, as well as the water pressure conditions. Johari and Rahmati [1] used the method of slices in a reliability analysis based on FORM and verified with Monte Carlo simulation by considering the soil parameters as random. Many deterministic calculation methods for the analysis of the sliding stability of earth dams have been proposed, depending on the nature of the assumptions made on the interactions between the slices and the pore pressure.

The Ordinary Method of Slices is one of the earliest methods for analyzing slope stability. It assumes that the inter-slice forces resultant for all slices is tilted at an angle parallel to the slice base. The main flaw of this method is its simplifying assumption, which does not satisfy the inter-slice equilibrium where adjacent slices have different base tilts.

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Bishop's simplified method [2] no longer neglects horizontal inter-slice forces and assumes that all shear forces between slices are nil. This method satisfies the vertical forces equilibrium of each slice as well as the moment equilibrium regarding to the failure circle center. Malkawi et al. [3] used these two previous methods in a reliability analysis, based on FORM and verified with Monte Carlo simulation by considering also the uncertainties of soil parameters.

The simplified method of Janbu assumes that the inter-slice forces are horizontal, which underestimates the factor of safety (FS). Bolton et al. [4] used and compared Janbu's simplified method with Spencer's method, without considering the seismic effect. The Lowe and Karafiath method [5] assumes that the inter-slice forces are tilted at an angle equal to the average of the embankment surface and the slices base angles.

Spencer's method rigorously satisfies static equilibrium by assuming that the inter-slice forces resultant has a constant but unknown tilt. Agam et al. [6] conducted a deterministic analysis based on Spencer's method to study the influence of the water table location on factor of safety, without considering the seismic effect.

Morgenstern and Price proposed a method similar to that of Spencer, except that the tilt of the inter-slice resultant force is considered to vary as a function of some part of an arbitrary function. Zhu et al. [7] conducted a deterministic analysis based on Morgenstern and Price, where only the horizontal component of the seismic loading is considered. The generalized method of Janbu suggests that the actual location of the thrust line is an additional unknown, and therefore, the equilibrium can be rigorously satisfied on the condition that the hypothesis selects the thrust line correctly. Sarma [8] uses the method of slices to calculate the magnitude of a horizontal seismic coefficient, needed to bring the unstable mass back to a limit equilibrium state. This allows, to the approach, to develop a relationship between the seismic coefficient and the presumed safety coefficient. The static safety coefficient will thereby correspond to the case of a zero-value seismic coefficient. The author uses a distribution function for the inter-slice forces and the value of the seismic coefficient can be calculated directly for the assumed factor of safety. All equilibrium conditions are always satisfied by this method.

Some interesting and recent studies of this last decade deserve to be cited in the same theme.

Khajehzadeh et al. [9] applied the Morgenstern and Price method to calculate the factor of safety for general shape of slip surface. In this deterministic study, the pseudo-static method is adopted considering only the horizontal component of the seismic loading.

Kádár and Nagy [10] carried out calculations with GeoStudio 7.10 GeoSlope software based on Bishop's slope stability method. The factor of safety values were validated in Plaxis 2D. A reliability analysis was applied with Monte Carlo method, where soil parameters are considered as random variables. The seismic effect is not considered.

In their research, Zeng et al. [11] formulated the factor of safety using the Morgenstern and Price approach that is implemented in the reliability calculation model. The soil parameters are considered random. We notice also, that in this study, only the horizontal component of the seismic action is considered as a deterministic variable.

In practice, the slope stability of earth dams is commonly evaluated using factors of safety, and dam slopes are considered stable if these factors are greater than 1. It is accepted that a factor of safety which approaches unity (one) can denotes an unsafe slope. In Ontario (Canada), the factor of safety to be considered against instability under seismic conditions is generally 1.1. However, given the uncertainties brought by the materials properties on the slope stability analysis, the calculated factor of safety should be greater than the admissible factors of safety (determined based on engineering feedback) so that a safety margin on the dam slope can be guaranteed.

Interestingly, the admissible factors of safety specified in Chinese, Korean and Japanese design codes for rockfill dams are between 1.30 and 1.50. Whereas the Spanish standard and the Maghrebian guide for the execution of studies and works on small dams [12] require earth dams to have a minimum factor of safety of 1.40 for any slip surface. The admissible factor of safety for the earth dam slope in United States Army Corps of Engineers (USACE) [13] and British Dam Society (BDS) [14] design guidelines is set at 1.50.

In this contribution, a stability analysis of a homogeneous earth dam is presented, using a deterministic and a reliability approach, at the ultimate limit state of a slope, with respect to the sliding, based on the ordinary method of slices and taking into account seismic loading. This study is based on the recommendations issued by the Maghrebian guide [12]; highlighting the influence of the embankment parameters, the seismic loading and finally the saturation line in the dike body directly linked to the water level in the dam reservoir.

The earthquake is introduced by the pseudo-static method, considering both the horizontal and the vertical components of the seismic loading. The pseudo-static

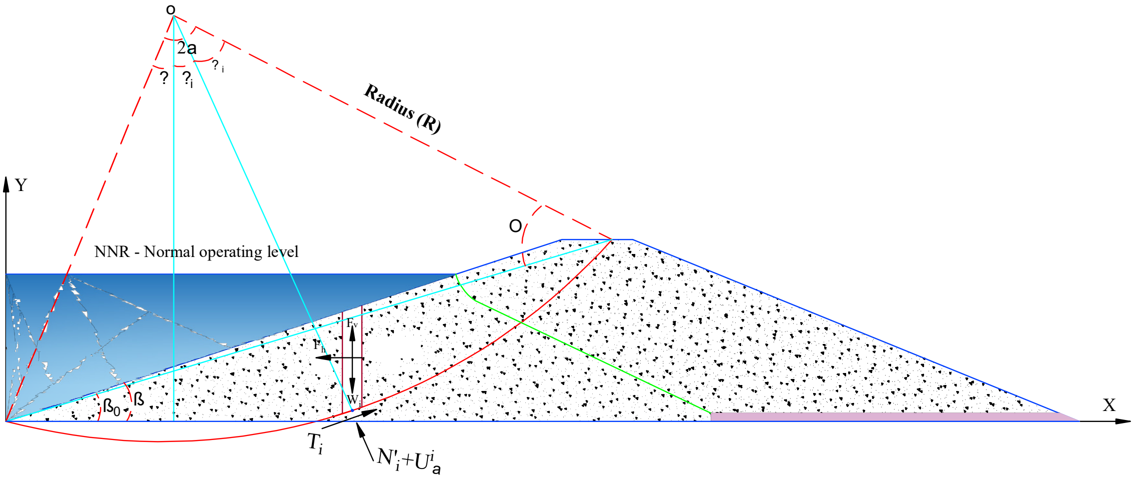


Fig. 1 Sliding circle of the upstream slope of an earth dam

method for analyzing the seismic stability of dams is accepted in Canada. The National Code of Canada and USACE [15], however, recommend adopting a design seismic acceleration greater than 0.4 g. In Great Britain, it is possible to use the pseudo-static method for the analysis of structures built in clay. The choice of the pseudo-static coefficient seems however, to be left to the choice of the engineer, depending on the nature of the structure and its location. In India, for each seismic zone, a basic coefficient is assigned to be used by the pseudo-static method. An important aspect of the pseudo-static method lies in the choice of the factor of safety from which it is possible to conclude that the structure is safe [16, 17].

In this research, several project situations are considered based on the recommendations of the French Committee for Dams and Reservoirs (called in French CFBR) [18]. These situations model a physical set representing the real conditions to which the dam is subjected, and which occur during a certain period during which the actions and the resistances are considered constant, as will be explained in detail in Section 3. In addition to uncertainties related to geotechnical parameters of the embankment dam, as analyzed by the various authors cited above, seismic acceleration coefficient is introduced as a random variable in the reliability analysis developed in this study. After a brief description of the basic concepts of the reliability theory in Section 4, the probabilistic modeling and the corresponding numerical results are presented and analyzed in Section 5. The slope stability analysis (deterministic and reliability approach) of earth dam is implemented in a computer program developed with Matlab® [19]. Finally, the conclusions of the study and suggestions for further research are presented in Section 7.

2 Material and methods

The ordinary method of slices assumes that failure takes place along a circular-shaped sliding surface, and that it occurs instantaneously along the entire surface of the slope (Fig. 1). The method also considers that, there is no interaction in the third dimension of the dike.

Let us consider, a vertical slice of order (*i*) (Fig. 1), the inventory of the forces acting along the direction perpendicular to the sliding surface (Fig. 2) can be written as:

$$\sum F_{\alpha} = N_i^{'} + U_{\alpha}^{i} + F_h.\sin\alpha_i - (W_i - F_v).\cos\alpha_i \\ -U_{\beta}^{i}.\cos(\beta - \alpha_i) = 0. \quad (1)$$

One thus deduces the resultant normal component of slice (*i*) at the level of the slide circle:

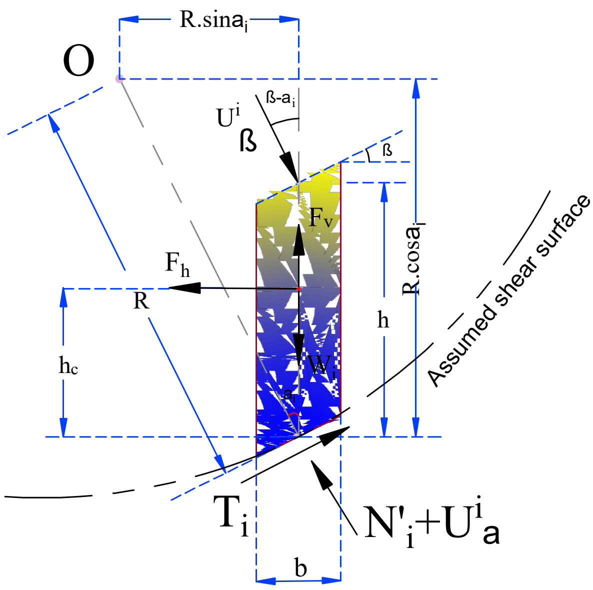


Fig. 2 Inventory of the forces acting on slice (*i*)

$$N_i' = -U_\alpha^i - F_h.\sin\alpha_i + (W_i - F_v).\cos\alpha_i + U_\beta^i.\cos(\beta - \alpha_i), \quad (2)$$

where:
N_i' refers to the effective normal force;
W_i refers to the slice weight (i);
F_h refers to the horizontal component of the seismic loading;
F_v refers to the vertical component of the seismic loading;
U_β^i, refers to the hydrostatic load on the slice upper surface;
U_α^i, refers to the component of the interstitial force;
α_i, Inclination of the base of the slice (i);
β_i, Inclination of the top of the slice (i).

The driving moment with respect to the centre of the circle O, due to the different forces acting on the slice (i) is:

$$\begin{aligned} M^i{}_m &= \left[(W_i - F_v) + U_\beta^i.\cos\beta\right].R\sin\alpha_i \\ &-U_\beta^i.\sin\beta.(R.\cos\alpha_i - h) + F_h.(R.\cos\alpha_i - h_c), \end{aligned} \quad (3)$$

where:

- b, refers to the slice width.
- h, refers to the slice mean height.
- h_c, refers to the height at the slice gravity centre.

As for the resisting forces moment, one has to distinguish on the dike embankment, two distinct behaviors: a short-term behavior and a long-term one.

2.1 Short-term behavior

In the short-term, the stability calculation is carried out using a total stress analysis, and the shear parameters used will be the undrained parameters of the embankment, denoted (C_u and φ_u) [18]. The shear stress at the base of slice (i) is written:

$$\tau_{\max}^i = C_u + \sigma_i.\mathrm{tg}\varphi_u. \quad (4)$$

The total stress at the base of slice (i) can be expressed as follows:

$$\sigma_i = \sigma_i' + u^i = \frac{1}{b}.\left(N_i' + U_\alpha^i\right). \quad (5)$$

The resisting forces moment at the base of slice (i) is given by:

$$M_s^i = R.\int_{A_i}(C_u + \sigma_i.tg\varphi_u).dA = R.\int_b\left(C_u + \frac{1}{b}.\left(N_i' + U_\alpha^i\right).tg\varphi_u\right).dl. \quad (6)$$

After integrating:

$$\mathrm{M}_s^i = R.\left[C_u.b + \left(N_i' + U_\alpha^i\right).\mathrm{tg}\varphi_u\right]. \quad (7)$$

The factor of safety (F_S) is then given by:

$$Fs = \frac{\sum_{i=1}^{n}\left[C_u.\mathrm{b} + \left[-F_h.\sin\alpha_i + (W_i - F_v).\cos\alpha_i + U_\beta^i.\cos(\beta - \alpha_i)\right].tg\varphi_u\right]}{\sum_{i=1}^{n}\left[\left[(W_i - F_v) + U_\beta^i.\cos\beta\right].\sin\alpha_i - U_\beta^i.\sin\beta.(\cos\alpha_i - \frac{h}{R}) + F_h.(\cos\alpha_i - \frac{h_c}{R})\right]}. \quad (8)$$

2.2 Long-term behavior

In the long-term, the effective stress analysis is justified, and the shear parameters used will be the drained or effective parameters of the embankment, noted (C' and φ'). The resisting shear stress which is developed along of the sliding circle is given by the Mohr-Coulomb's law and is written:

$$\tau_{\max}^i = C' + \sigma_i'.tg\varphi'. \quad (9)$$

The resisting forces moment at the base of slice (i) is given by:

$$M_s^i = R.\int_{A_i}\left(C' + \sigma_i'.tg\varphi'\right).dA = R.\int_b\left(C' + \sigma_i'.tg\varphi'\right).dl. \quad (10)$$

The effective stress σ_i' can be expressed as follows:

$$\sigma_i' = \frac{N_i'}{b}. \quad (11)$$

This gives:

$$M_s^i = R.\left[C'.b + N_i'.tg\varphi'\right]. \quad (12)$$

The factor of safety (F_S) is then given by:

$$Fs = \frac{\sum_{i=1}^{n}\left[C'.b + \left[-F_h.\sin\alpha_i - U_\alpha^i + (W_i - F_v).\cos\alpha_i + U_\beta^i.\cos(\beta - \alpha_i)\right].tg\varphi'\right]}{\sum_{i=1}^{n}\left[\left[(W_i - F_v) + U_\beta^i.\cos\beta\right].\sin\alpha_i - U_\beta^i.\sin\beta.(\cos\alpha_i - \frac{h}{R}) + F_h.(\cos\alpha_i - \frac{h_c}{R})\right]}. \quad (13)$$

3 Project situations to consider

The design situations model a set of physical conditions representing the real conditions to which the dam can be subjected, and which occur at different times in the life of the dam. Those different scenarios correspond to the loads to which the dike is likely to be subjected either in: transitional situation at the end of construction, normal level of operation, low level of the reservoir, rapid drawdown or extreme seismic situation.

3.1 Transitional situation at the end-of-construction

During construction, the material that constitutes the embankment of the dam is laid in thin, well-compacted layers. Adding water facilitates and improves compaction by working while wet. At the end of the embankment set up, before filling the reservoir dam and before any materials consolidation, the construction pore pressures are not entirely dissipated, particularly when the material is not very permeable. This situation may correspond to maximum pore pressures in the material, which would have developed during construction and would not have dissipated. The short-term stability of the two slopes (upstream and downstream) will be verified and carried out in total stresses on the basis of undrained characteristics [18].

3.2 Normal operating situation at full dam reservoir for long term

During the filling of the dam reservoir, a permanent flow is established in the body of the dike and its foundation. Under these conditions, the water is at its Maximum Operating Level (that we will call NNR), and the stability study is carried out for the downstream slope of the dike. The stability of the slope will be verified for long-term behavior, based on the drained characteristics (C', φ'), and the effective stresses, considering a stable flow.

3.3 Normal operating situation at low level for long term

The normal operating situation at low level is determined by reference to the upstream hydrostatic level corresponding to the reduced water level authorized for the normal operating situation of a dam, also known as the Minimum Operating Level. The stability of the slope will be verified for long-term behavior, based on the drained characteristics, and the effective stresses, considering a stable flow.

3.4 Transient situation of rapid drawdown

This rare project situation, rapid drawdown, is carried out for safety reasons. The French Committee for Large Dams (called in French CFGB) recommends being able to reduce the hydrostatic loading by half within a maximum of 8 days [20]. When rapid drawdown occurs in the reservoir dam, the water level drops sharply below the Maximum Operation Level (NNR). This situation concerns generally embankments made of poorly draining materials and is particularly restrictive for embankments made of impermeable clays. It aims at verifying the stability conditions of the upstream embankment in a saturated condition. The failure mechanism that produces the slip is as follows: After establishing the flow network from the NNR, a decrease in the water level of the reservoir dam occurs, while in the body of the dam an equal decrease does not occur because its material is quite impermeable. The internal water level decrease is negligible compared to that of the external water level. For this situation, the stability of the upstream slope is checked in the long term, and in effective stress.

3.5 Extreme seismic situation

For justification in an extreme seismic situation, the structural stability must be ensured, and the structure must not experience any damage likely to jeopardize its safety. In the extreme seismic situation, one generally adopts, for the hydraulic loading (Water level and uplift) and the geotechnical parameters, assumptions similar to those of the normal operate situation.

The pseudo-static method is the method (in the case of small and medium dams) that is the most generally used, and in almost all parts of the world, for it takes into account the seismic action [21]. This method is well suited for the calculation methods of circular-base slices. This method principle is to present the seismic actions by two additional forces, a horizontal force denoted F_h and a vertical force denoted F_v, applied at the gravity center of displaced embankment volume. These forces applied to a moving slice (i) are given by the following equations [22]:

$$F_h^i = K_h.W_i \,, \tag{14}$$

$$F_v^i = K_v.W_i \,. \tag{15}$$

Where K_h et K_v refer respectively, to the horizontal and vertical pseudo-static coefficient, given as a function of the acceleration coefficient (A) of seismic zone.

$$K_h = 0.50A \tag{16}$$

$$K_v = 0.30K_h \tag{17}$$

The acceleration coefficient of seismic zone is determined according to the seismic zone and the usage group of the considered structure, as presented in Table 1. According to the Algerian seismic design code [22], dams are classified as being public structures of national interest and of a certain economic importance; they are referred to as usage group 1B.

4 Reliability approach

The stability limit state function used in the reliability analysis is defined with respect to a failure by sliding of the embankment. It is given by:

Table 1 Acceleration coefficient of seismic zone [22]

Seismic zone	I	IIa	IIb	III
Usage group 1B	0.12	0.20	0.25	0.30

$$G(x) = M_m - M_R = 0 \,. \quad (18)$$

The probability of failure will be used as a quantity for the evaluation of reliability. Given a vector $\boldsymbol{X}$ of n random variables and a performance function defined by $G(x)$, the failure probability P_f can be defined by:

$$P_f = \int_{G(x)\leq 0} f_x(x).dx \,, \quad (19)$$

where $f_x(x)$ is the probability density function of random variables (x).

Analytical evaluation of the integral is very difficult, if not impossible in most cases. For this integral evaluation, the Monte Carlo simulation method, also known as the significance draw method, will be used. This old and intuitive technique assesses the failure probability by simulation of a sequence of a large number (N_t) of independent random variables identically distributed according to their appropriate law of distribution. Thus, a ruin indicator Id is used to define the failure state of slopes embankment, and it is given as follows:

$$I_d(x) = \begin{cases} 1 & si \quad G(x) \leq 0 \\ 0 & si \quad G(x) > 0 \end{cases} . \quad (20)$$

The probability of failure P_f can be obtained by:

$$P_f = \frac{\sum_{i=1}^{N_t} I_d^i}{N_t} \,. \quad (21)$$

5 Results and discussion

The practical application presented here deals with a small earth dam of homogeneous type, made of local material, whose design is realistic. This development intended for irrigation is located in region of Médéa (Western of Algeria); classified as a high-seismicity zone (zone III). The small dam has been designed so that it fulfills the role of a seasonal regulator, in order to provide support for irrigation needs during the driest periods. The dam spillway level corresponding to the normal operating level (NNR) of the reservoir dam is located at 634.25 m NGA (General Leveling of Algeria), i.e., nearly 14.65 m higher than the dam base. The small dam is equipped with a single steel outlet pipe with a diameter of Φ 300 mm, characterized by the presence of a strainer at the 625.48 m NGA level (Fig. 3) [23].

This pipe plays a dual role, firstly, in restitution of the stored water for irrigation and secondly, of course, in emptying the reservoir to allow maintenance and especially the possibility of rapid drawdown in a few days in the event of danger or an unforeseen incident. The length of the horizontal drain was adopted in accordance with the United Nations Development Program (called in French PNUD) [12], i.e., 1/3 to 1/4 of the dike width, so as to maintain the saturation line at, at least 2 m from the downstream face.

The study presented here deals with the stability analysis of this dike. Its geometric characteristics are summarized in Table 2 and illustrated in Fig. 3.

The material parameters of the dam embankment were obtained from the report established by the engineering department of the African Geosystem Company [23] in which the average values of the embankment parameters are available. These average values are required for the reliability analysis, whereas for the characteristic values of the parameters, which will be used in the deterministic approach, a transformation has been carried out according to their distribution laws. The characteristic value of the resistance parameter (C and φ) implies a calculated probability of a more unfavorable value controlling the occurrence of the limit state not exceeding 5%.

Table 2 Dike geometrical characteristics [23]

Designation of parameter	value	unit
Dike height	18.00	m
Crest width	7.00	m
Slope of the upstream embankment	1:3	PNUD [12]
Slope of the downstream embankment	1:2.5	PNUD [12]
Drain length	31.00	m

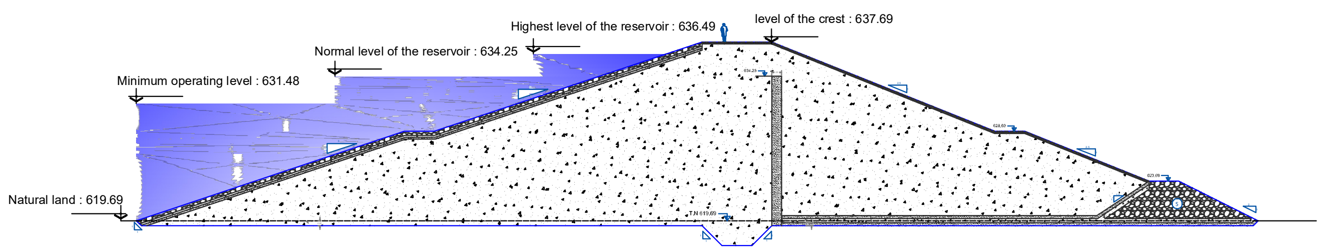


Fig. 3 Cross section of the earth dam [23]

The distribution types and coefficients of variation (COV) for each parameter were chosen according to the recommendations given by several studies [24]. The distribution laws as well as the COV of the different parameters are given in Table 3. The wet density γ_h varies between the actual dry density and the saturated density γ_{sat} depending on the saturation degree (Table 4). A maximum difference, about 4 kN/m^3, is estimated between γ_h and γ_{sat}.

5.1 Limit-equilibrium analysis

The saturation line is introduced in the factor of safety calculation in order to take into account the wet and saturated mass of each moving slice of the dike body. In the case of a drained homogeneous dam, which rests on an impermeable base, Kozeny showed that the saturation line takes the shape of a parabola [12], of which the horizontal axis focal point is the upstream end of the drain, and to which the water line connects (Fig. 1).

Analysis of the upstream embankment stability showed that the F_S increases proportionally to the height of water in the reservoir dam (Fig. 4). Analysis of these results shows that the normal operating situation (NNR) gives the highest factor of safety (F_S = 3.04). This result can be explained by the fact that when the water surface of the reservoir dam is at its NNR level, a permanent flow is established in the dike body. This flow generates pore pressures in the upstream embankment, and simultaneously the hydrostatic loading on the face of the upstream embankment plays a stabilizing effect, which results in an increase in the effective stresses. The effect is an increase in the shear strength of the embankment material and therefore stability improve of the upstream slope.

As for the downstream slope, the most critical stability condition is reached in a normal operating situation (F_S = 1.61), because the flow that establishes through the dike generates, in the downstream slope, pore pressures, which are more and more significant as the water surface level of the reservoir dam increases, reaching the Maximum Operation Level (NNR). As a result, a reduction in the effective stresses is noted, thus leading to a reduction in the shear resistance and, consequently, to the reduction of the F_S value (Fig. 4). As far as the low-level operating situation is concerned, the lowering of the water surface level causing the drawdown of the saturation line and its distancing from the downstream slope produces the increase of the factor of safety in the downstream slope and the opposite in the upstream slope.

It is noteworthy, that the factors of safety of slopes are significantly higher to the admissible F_S values (Table 5) in the case of small earth dams. This allows concluding that the dike is stable as designed [25].

Table 3 Distribution laws of the embankment parameters

Designation of parameter	Distribution law	COV	unit
Wet density γ_h	Normal	5%	KN/m^3
Saturated density γ_{sat}	Normal	5%	KN/m^3
ungauged density γ_d	Normal	5%	KN/m^3
Effective cohesion of the embankment C'	Log-normal	20%	KN/m^2
Effective friction angle of the embankment ϕ'	Log-normal	10%	(°)
Undrained cohesion of the embankment C_u	Log-normal	20%	KN/m^2
Undrained friction angle of the embankment ϕ_u	Log-normal	10%	(°)

Table 4 Statistics of the dike constituting material

Symbol	Mean value	COV	Standard deviation	Characteristic Value	unit
γ_h	19.00	5%	0.95	17.43	KN/m^3
γ_{sat}	21.00	5%	1.05	19.27	KN/m^3
C'	20.36	20%	4.07	13.64	KN/m^2
ϕ'	26.55	10%	2.66	22.17	(°)
C_u	97.63	20%	19.53	65.41	KN/m^2
ϕ_u	9.48	10%	0.95	7.92	(°)

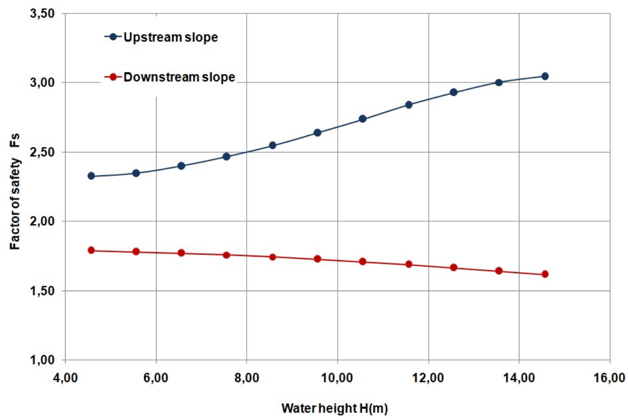


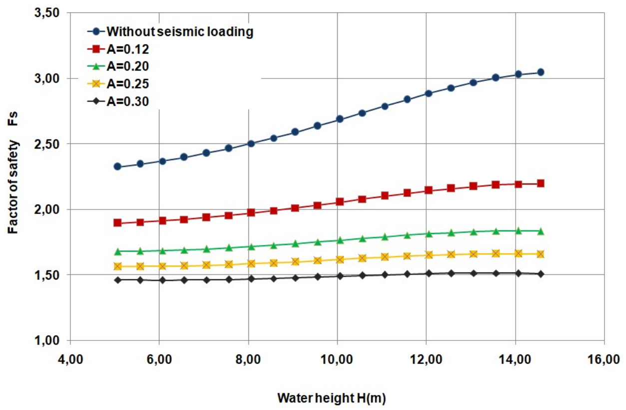
Fig. 4 Factor of safety depending on water height

Table 5 Admissible factors of safety according to project situation [25]

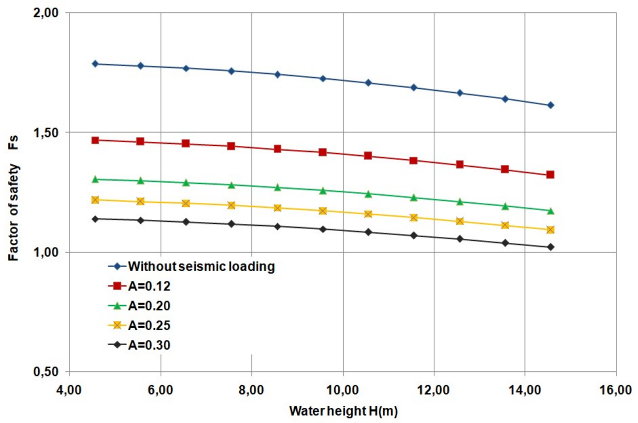
Project situation	without seismic loading	with seismic loading
End of construction	1.3	1.1
Normal operating	1.5	1.1
Rapid drawdown	1.3	

The analysis of the two slopes, under seismic loading, reveals that for a given water height, the factor of safety decreases with the increase of the acceleration coefficient of seismic zone (Fig. 5(a) and 5(b)).

The upstream slope is stable regardless of the seismicity level and the situation of project. In the other hand the downstream slope, is only stable in low and medium seismicity zone. The slope instability appears at a Maximum Operating Level (NNR) in a high and very high seismicity zone, and at a low operating level situation in a very high seismicity zone. This instability is nevertheless interesting to analyze, as it is known that the studied dike slopes design complies with PNUD recommendations [12] for homogeneous earth dam with a high percentage of clay whose height is greater than 10 m. This instability appeared when the seismic loading was introduced (Table 6). This result is also confirmed by Li et al. [26], who conclude that the FS values of earth dams stability obtained under normal conditions decrease under seismic conditions.



(a)



(b)

Fig. 5 Factor of safety depending on water height for various acceleration coefficient of seismic zone; (a) Upstream slope, (b) Downstream slope

A civil engineer who neglects or omits the seismic effect in the analysis of slope stability would lead to consider only the static analysis. This very regrettable negligence, for an engineer, inexperienced or less experienced in structural calculations, will lead to underestimate the driving moment of the embankment block in motion, due to the presence of a seismic additional force. One can conclude that not taking the seismic action into account will result, in making the dike vulnerable to greater intensity seismic stresses, in a normal operating situation.

For the verification of the embankment stability under the rapid drawdown scenario, the analysis within the framework of this study will not be carried out by considering an instantaneous emptying, as conventional calculations are usually carried out, which moreover is not far from reality. To get closer to reality, one will evaluate the water surface level variation in the reservoir dam as a function of time. To do so, we will use the Height-Surface-Volume curve (Fig. 6) which defines the variation of the storage volume, depending on the level of the water surface, which is carried out, for its part, on the basis of the reservoir dam realistic topographic data.

Based on the Height-Capacity curve, one can deduce the curve presented in (Fig. 7) giving the behavior of the water surface level as a function of the rapid drawdown time, corresponding to the total opening of the valve. It appears that the total time for lowering the water level from the NNR level to the dead level is estimated at 77.77 hours, i.e., 3.25 days. The permeability of the material constituting the dike being low, the drawdown duration measured, nearly half a week, is rapid compared to the dissipation

Table 6 Factor of safety depending on water height and seismic zone

F_S		$A = 0$	$A = 0.12$	$A = 0.20$	$A = 0.25$	$A = 0.30$
Upstream slope	NNR ($H = 14.56$ m)	3.04	2.19	1.83	1.66	1.51
	Low operating level ($H = 11.79$ m)	2.79	2.10	1.79	1.64	1.50
	Dead level ($H = 5$ m)	2.32	1.89	1.68	1.57	1.46
Downstream slope	NNR ($H = 14.56$ m)	1.61	1.32	1.17	**1.09**	**1.02**
	Low operating level ($H = 11.79$ m)	1.69	1.38	1.23	1.14	**1.07**
	Dead level ($H = 5$ m)	1.78	1.46	1.30	1.21	1.13

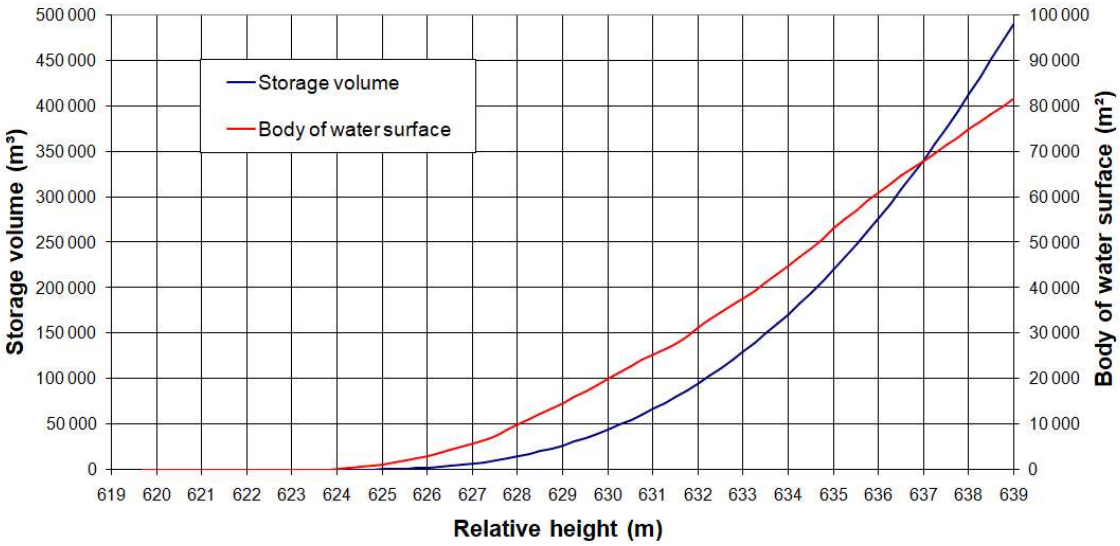


Fig. 6 Height- water Surface - Volume curve of the reservoir dam

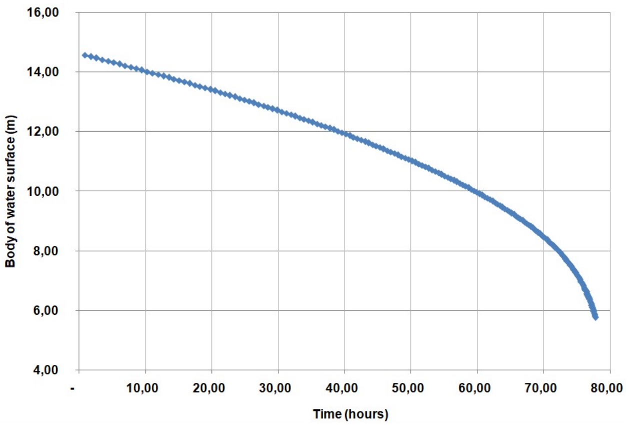


Fig. 7 Body of water surface depending on time during rapid drawdown

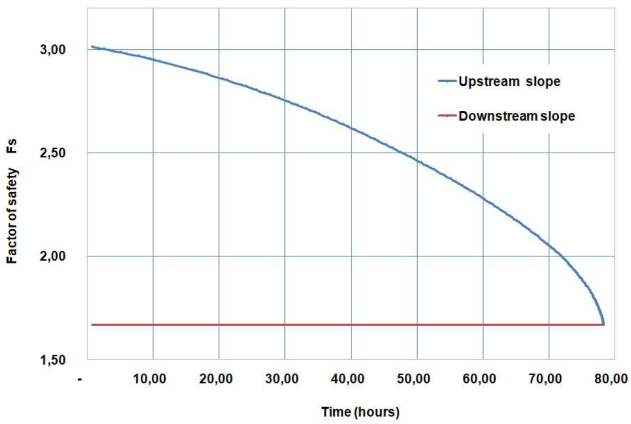


Fig. 8 Factor of safety evolution during rapid drawdown

time of the pore pressure, and the variation of the latter will take place under undrained conditions. The effect of this storage level drop will be to cause a change in the distribution of the pore pressure inside the dike body.

During rapid drawdown, the saturation line is maintained at the top of the NNR level to take into account of the realistic emptying. The stability analysis of the upstream slope, in this situation, reveals that the factor of safety (F_S = 3.04) drops sharply, with the water body level reaching the most critical value (F_S = 1.65) at the end of emptying time (Fig. 8). It is easy to deduce that this scenario remains the worst case scenario, given that the base of the dike is potentially submerged, while the top exerts its full dead weight. Furthermore, the factor of safety remains constant (F_S = 1.67) in the case of the downstream slope, given that the pore pressures do not change.

5.2 Numerical analysis

In this section, the Flac2D© software [27] based on the finite difference method, is used for the analysis of the embankment stability. The results obtained are compared with those given by the limit-equilibrium analysis. For this analysis, the foundation block under the dike is modeled as shown in Fig. 9, where the horizontal and vertical translations are blocked at its boundaries to get as close as possible to reality. The depth of the foundation block is four times the height of the dike, while its width is chosen equal to the width of the dike on either side of the dam slope base [28].

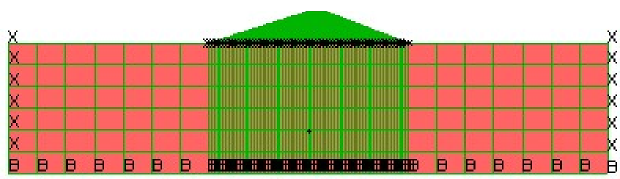


Fig. 9 Adopted mesh for the dike and the foundation block

The detailed geological section of the core drilling is presented in the form of a stratigraphic profile of the soil columns. This core drilling, of 30 m depth, was carried out in order to know the nature of the soils constituting the foundation site, and it revealed the presence of a layer of marl clay of 10 m thick resting on a layer of clay marl dark brown 20 m thick (Fig. 10).

The perfectly plastic linear Mohr-Coulomb model [29] is adopted to model the behavior of the dike embankment. This latter is considered as being either in a partially saturated state with a coupled fluid-mesh flow calculation scheme or decoupled depending on the project situation, in accordance with CFBR recommendations [18].

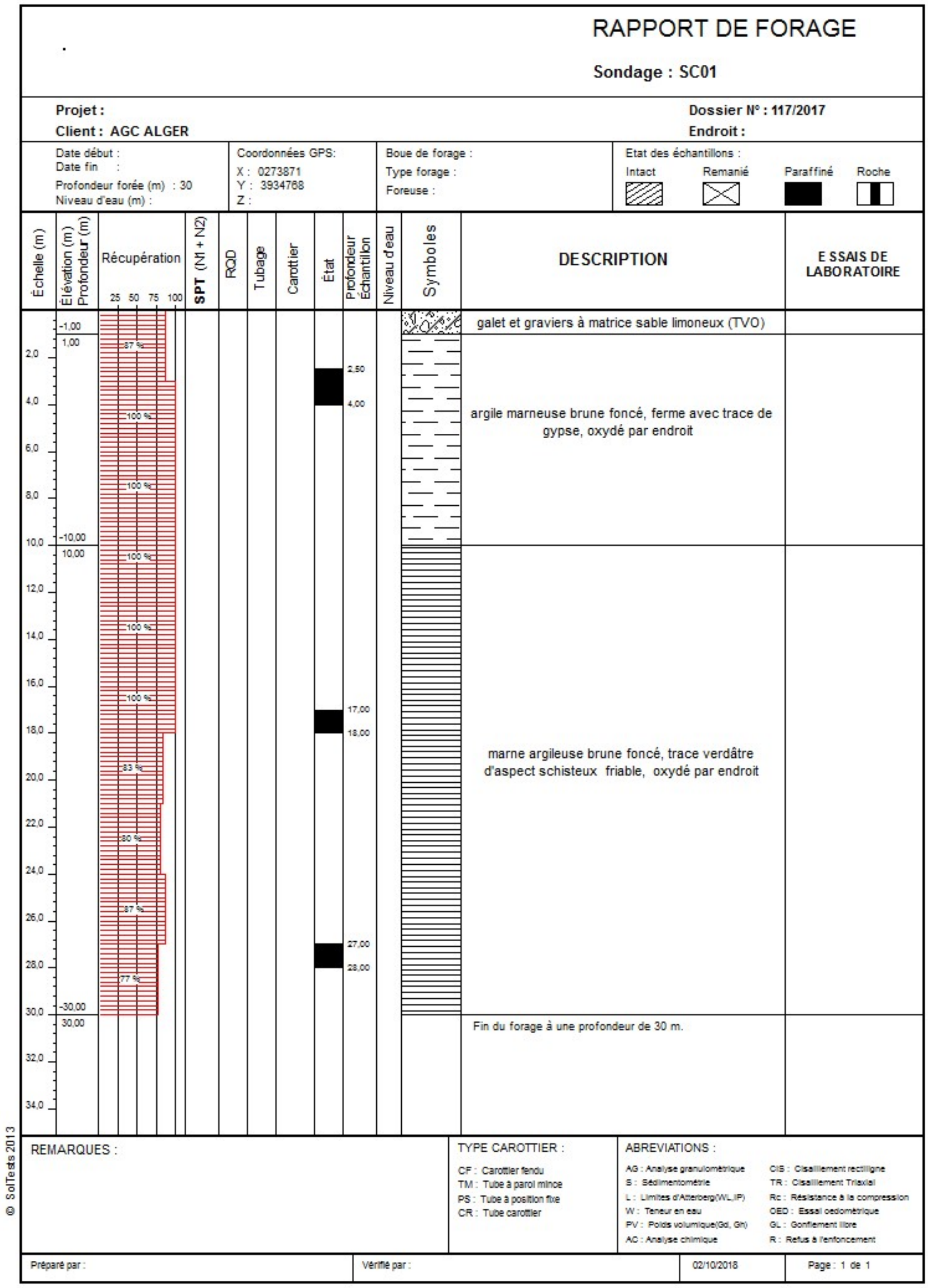


Fig. 10 The stratigraphic profile of the soil columns

The sliding plane analysis of the downstream embankment, obtained by the Flac2D© software [27], confirms the circular failure mode accepted by the method of slices (Fig. 11), this result is also confirmed by Mouyeaux [30] and Guo [31].

The assessment of the partial factor of safety by the Flac2D© software uses the parameter reduction method. This consists of gradually reducing the shear strength of a soil. The reduction is performed on the tangent of the effective friction angle ϕ' and the effective cohesion C' in the Mohr-Coulomb model [32]. At failure, the reduction factor is assimilated to F_S of the stability analysis.

The comparison between the results obtained in the case of different project situations by the method of slices and by the numerical method is presented in Table 7. The results clearly show that the values of the factor of safety are very close for all the considered project situations, and that the maximum recorded deviation does not exceed 7% (Fig. 12). These results are furthermore confirmed in the literature by Fredlund and Krahn [33] and Kourdey and Al Heib [34], who noted deviations of 4% and 7% respectively, justified by the hypotheses accepted in the limit-equilibrium analysis.

5.3 Probabilistic analysis by Monte Carlo simulation

In this section, a probabilistic evaluation of the dike stability is performed, by considering three random variables, namely the acceleration coefficient of seismic zone (A) and the mechanical resistances parameters of the dike embankment (C' and ϕ'). The variability of the geotechnical parameters is linked to the possible disparity in the extraction site of the embankment as well as the quality of its implementation when using the technique of compaction in successive layers on the dike body. These variables are generated using a log-normal distribution law from the mean values and the coefficients of variation defined in Table 4. The probability density function as well as the cumulative function of this distribution law for the acceleration coefficient (A) of the very high seismic zone are illustrated in Fig. 13 and Fig. 14, respectively.

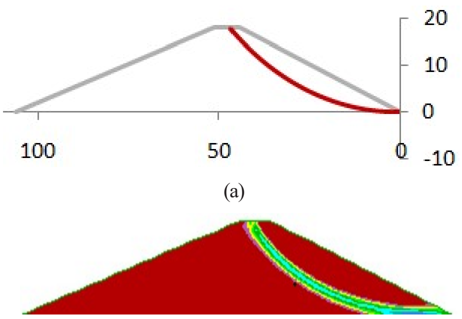


Fig. 11 Sliding plane of circular shape obtained by the method of slices (a) and Flac2D© (b)

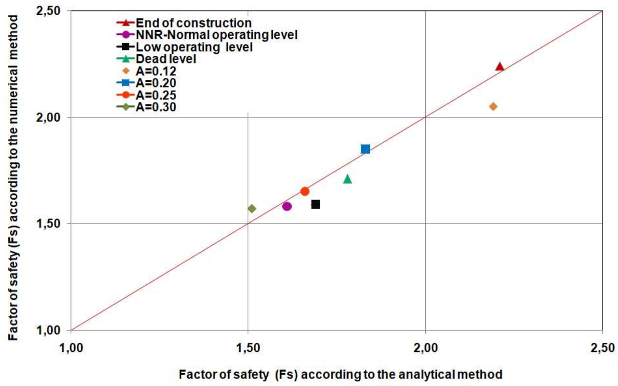


Fig. 12 Deterministic analysis – factors of safety comparison for different project situations

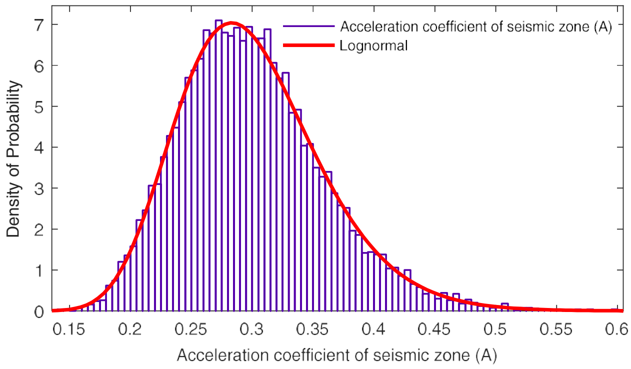


Fig. 13 Probability density function of the acceleration coefficient (A) of the very high seismic zone

Table 7 Critical factors of safety obtained through analytical and numerical methods

Concerned slope	Downstream	Downstream	Downstream	Upstream			
Project situation	End of construction	NNR A = 0	Low operating level	NNR – Seismic situation			
				A = 0.12	A = 0.20	A = 0.25	A = 0.30
Analytical method	2.21	1.61	1.69	2.19	1.83	1.66	1.51
Flac2D©	2.24	1.58	1.59	2.05	1.85	1.65	1.57
The gap (%)	1.339	1.899	6.289	6.829	1.081	0.606	3.822

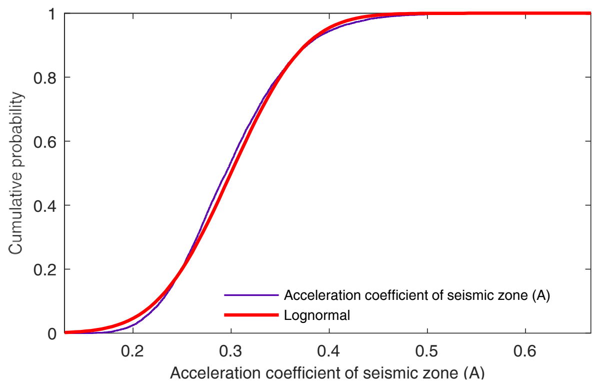


Fig. 14 Cumulative function of the acceleration coefficient (A) of the very high seismic zone

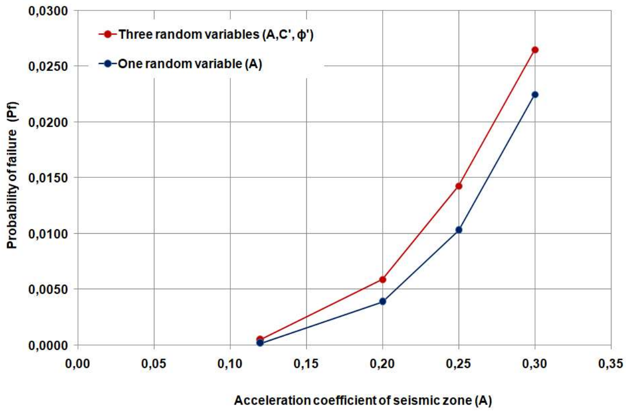


Fig. 15 Failure probability of the upstream slope depending on the seismic zone

The failure probability evolution of the upstream slope of the dike for the different seismic zones, taking into account the saturation line is summarized in Fig. 15, for NNR level of the water surface in the dam reservoir. Failure probability values increase with seismic intensity.

In addition, it is noteworthy, that all failure probability values exceed the admissible value ($P_f^{adm} = 3.10^{-5}$) recommended by the CFBR [21]. The results show that taking into account the variability of the embankment mechanical parameters further amplifies failure probabilities, which prove that the uncertainty on the mechanical parameters has a considerable influence on the slopes instability (Table 8).

Failure probability of the upstream slope decreases with the decrease of the water surface level (Table 9), in contrast with the factor of safety (FS). These variations follow a logical trend (Fig. 16).

While the deterministic analysis concluded that the embankment was stable with respect to the seismic loading, the reliability approach revealed failure probability values higher than those accepted for civil engineering structures (Table 10). As a consequence, the slope stability

Table 8 Probability of failure depending on the seismic zone of the upstream slope

	$A = 0.12$	$A = 0.20$	$A = 0.25$	$A = 0.30$
Analysis with only one random variable (A)	0.0002	0.0039	0.0103	0.0225
Analysis with three random variables (A, C', φ')	0.0050	0.0059	0.0143	0.0265

Table 9 Probability of failure depending on the seismic zone and the level of the water surface. Considering all three random variables – Upstream slope

Height of water surface H(m)	$A = 0.12$	$A = 0.20$	$A = 0.25$	$A = 0.30$
14.561	0.0005	0.0059	0.0143	0.0265
13.561	0.0005	0.0054	0.0134	0.0258
12.561	0.0005	0.0052	0.0128	0.0248
11.561	0.0005	0.0048	0.012	0.0233
10.561	0.0005	0.0042	0.0115	0.0226
9.561	0.0005	0.0032	0.0108	0.0212
8.561	0.0005	0.0031	0.0105	0.0203
7.561	0.0005	0.0030	0.0101	0.0189
6.561	0.0005	0.0026	0.0094	0.0181
5.561	0.0005	0.0026	0.0083	0.0165

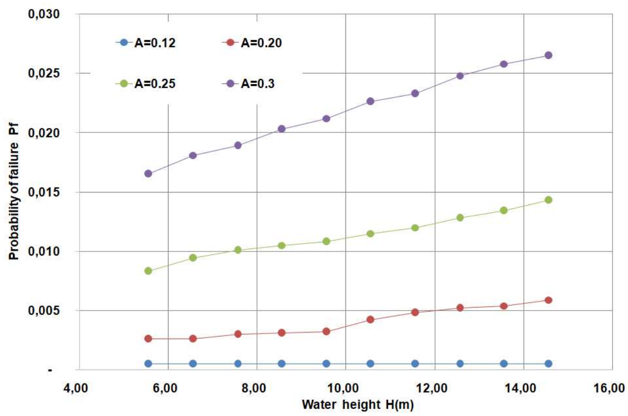


Fig. 16 Failure probability of the upstream slope depending on water height, considering the random variables A, C', and φ'

analysis of the earth dam using the factor of safety compared to an admissible FS can induce a misleading evaluation of the slopes stability of the dam, without resorting to a reliability analysis. This reliability approach, as seen above, makes it possible to introduce the variability that occurs in embankment characteristics and seismic action into the numerical process of stability analysis. It can be deduced that the failure probability results convey more information than the deterministic calculation based on the evaluation of the factor of safety.

Table 10 Summary table of the failure probabilities and factors of safety for the various project situations

Project situation	P_f	F_S (Analytical method)	F_S (Flac2D©)
End of construction situation	-	2.21	2.24
NNR – Normal operating level situation	-	1.61	1.58
Low operating level situation	-	1.69	1.59
NNR – Seismic situation – A = 0.12	0.0050	2.19	2.05
NNR – Seismic situation – A = 0.20	**0.0059**	1.83	1.85
NNR – Seismic situation – A = 0.25	**0.0143**	1.66	1.65
NNR – Seismic situation – A = 0.30	**0.0265**	1.51	1.57

6 Comparison with previous studies

The obtained results shown in Table 7 reveal that the main factor influencing FS is the seismic action. The increase of the acceleration zone coefficient to (A = 0.1) and (A = 0.3), leads to the decrease of Fs by up to 39% and 50%, respectively. The same observation is made by Khajehzadeh et al. [9], who conclude that increasing horizontal acceleration coefficient to 0.1 and 0.2, decreases FS by up to 19% and 32%, respectively.

The calculation results of critical FS shown in Table 6 were compared for the downstream slope in the case where the acceleration zone coefficient is (A = 0). The following conclusion can be drawn: the lowest factor of safety (F_s = 1.61) was obtained for the Maximum Operating Level, where as the greatest factor of safety (F_s = 1.78) was obtained for the dam Dead level. It is interesting to cite that the same conclusion is drawn by Panulinová and Harabinová [35] who concluded that the lowest factor of safety (F_s = 1.19) was recorded on the maximum retention level, whereas the greatest one (F_s = 1.98) was determined when analyzing the dam without water.

Agam et al. [6] conducted a slope sensitivity analysis by varying values of cohesion, friction angle, unit weight, and water table location. Among the four parameters concerned, the parameter that has the highest influence on the factor of safety is water table location. This was followed by friction angle, cohesion, and unit weight. Conversely, in this research, the most influential parameter is the seismic acceleration which causes the decrease of FS in the upstream slope, dropping by 50%, from 3.04 (A = 0) to 1.51 (A = 0.3). The second parameter is the water table location during the rapid drawdown, which causes in the upstream slope to decrease by 46%, going from 3.04 down to 1.67 after 3.25 days.

7 Summary and conclusions

The deterministic method based on the limit-equilibrium analysis of slope stability presents equations that are not too complex, and leads to simple resolutions which have been easy to implement using Matlab® programming language [19]. Through this study, we were able to demonstrate that:

- FS of upstream slope stability increases proportionally with water height in the reservoir dam, about 31% from the dead level to the Normal Operating level.
- FS of downstream slope stability decreases, by about 10%, when the water level changes from the dead level to the Normal Operating level.
- FS decreases under seismic loading by about 50% and 37% respectively for the upstream and the downstream slope for the maximum acceleration zone coefficient.
- FS of upstream slope stability decreases during the rapid drawdown, by about 46%, after 3.25 days.
- Among the three parameters concerned, the parameter that has the highest influence on the failure probability in the reliability analysis is the seismic acceleration. Followed by the soil parameters, i.e., friction angle and cohesion.

It can be concluded that, not taking into account the seismic loading can overshadow the instability risk of the downstream slope of a dike (Table 6), even if it is designed according to the recommendations of the Maghrebian guide [12]. The deterministic calculation of the slide stability with the method of slices, in such a situation, leads to an overestimation of the factor of safety by up to 50%. The recommended values of embankment slope according to the Maghrebian guide [12], should only be used for pre-design, and a seismic analysis must be undertaken, to validate these slopes values, while considering the saturation line across the dike body.

By means of the probabilistic analysis, it has been shown in Table 10 that the slope stability of the earth dams designed with the recommended factor of safety does not necessarily lead to a required level of reliability. To maintain a high level of reliability, dams require a higher admissible factor of safety than recommended. For a better accuracy of the safety margin, the reliability approach is necessary.

The failure probability values of the upstream slope of the embankment, taking into account the saturation line for different seismic zones, are greater than the admissible value, which confirms the importance of these two parameters.

Conventional deterministic methods such as those described in the Maghrebian guide [12], used by civil engineers in engineering design offices, are necessary at the preliminary design stage, but still insufficient and must be supplemented by probabilistic approaches. These later, take into account the variability parameters in the reliability calculation at the detailed design stage. The aim being is precisely understand certain stability phenomena. Assessing the reliability and failure probability of earth dams should be seen as complementary to the usual deterministic analyses. These probabilistic methods are tools that are currently being developed. One hopes that in the future, risk analysis could be integrated into design standards, so that it becomes an operational tool in the hands of civil engineers when designing projects, because it is fundamental for the interpretation of safety. It should be kept in mind, that the statistical and probabilistic analysis is only a tool, which is only as good as the quality of the analyzed data and the models that are used.

In order to get even closer to reality, the reliability analysis carried out in this study could be improved by considering as a fourth random variable, the density of the materials constituting the dike. As recommendations for future works, the introduction of the spatial variability of the embankment geotechnical parameters in order to analyze its influence on the factor of safety would be very interesting. A project within that scope, in the form of a PhD thesis in Civil Engineering is underway at the University of Tizi-Ouzou.

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