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Comparison of Three Chaotic Meta-heuristic Algorithms for the Optimal Design of Truss Structures with Frequency Constraints

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Abstract

The main reasons for the success of using chaos maps in meta-heuristic algorithms are fast optimization of non-linear and non-convex problems. One of these cases is the control of the natural frequencies of structures to prevent the destructive and dangerous phenomenon of resonance. Natural frequencies have useful information about the dynamic behavior of structures, and by applying dynamic constraints, a significant improvement is achieved in the optimal design of structural weight. Applying frequency limits with traditional and gradient-based methods is very difficult and time-consuming, and in most cases, the calculation process stops at local optima. Recent research shows that chaos maps play a major role in escaping from local optima and reaching global optima. By combining these maps with meta- heuristic algorithms, while avoiding premature convergence, the access to global optima is accelerated and improved, and the ideal state of balance between the exploration and exploitation stages is realized. Today, chaotic algorithms are widely accepted by researchers and are considered as a challenging topic. In a recent research, six chaotic meta-heuristic algorithms have been investigated for the formation and improvement of results with the optimal design of truss structures. In this part the chaotic algorithms include Chaotic Water Evaporation Optimization (CWEO), Chaotic Tug-of-War Optimization (CTWO) and Chaotic Thermal Exchange Optimization (CTEO) are examined.

Keywords

meta-heuristic algorithms, chaos maps, exploration, exploitation, premature convergence

1 Introduction

Truss structures have special features that cause their extensive use in the field of engineering. This type of structures is a unique choice for covering large openings such as industrial buildings and light structures such as telecommunication towers. In terms of behavior, they have unique capabilities, such as the participation of most of the members in sustaining the load and not being destroyed due to the collapse of a limited number of members. Therefore, their popularity and variety in their use is more than impressive. Their use in important cases such as bridges, airplanes, ships, power transmission towers and astronaut structures has attracted the engineers to the optimization of this group of structures. In optimization with traditional gradient-based methods, the derivative of the objective function is formed and then the optimal answers are searched. Regarding optimization with frequency limitation, we do not have a clear and direct relation of the objective function, so we have to investigate partial derivatives. Also, in a number of optimization problems, the search space is discrete and it will not be practical to form their derivatives. The inefficiency of gradient-based methods reaches its peak when the number of decision variables become high. In recent decades, special categories of optimization methods have been born in the field of artificial intelligence, known as meta- heuristic algorithms. These algorithms are based on decisions and principles of probabilistic and random search [1]. In these algorithms, the value of the objective function itself is evaluated instead of derivatives, and with inspiration from natural phenomena, it gets an improving process in successive iterations. The main reason for this choice is the characteristics of natural phenomena that if there is a need for empowerment, nature does it in the best way. The first choice of researchers for inspiration is the genetic evolution of living things over millions of years from the

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beginning of their life. In this evolution, the characteristics of living things are improved by the act of crossing and mutation so that by adapting as much as possible to the surrounding environment, they win in the competition with other living things in the courtyard of life. Examples of these algorithms include Genetic Algorithms (GA) [2], Differential Evolution (DE) [3] and Evolutionary Strategy (ES) [4]. The second inspiration for meta- heuristic algorithms is based on the swarm intelligence of animals and their nature in searching and accessing food. The components of this inspiration include population, cooperation, communication, information exchange, information flow and self-organizing, which is evident in the collective life of birds, fish, ants and other animals. Examples of these algorithms include Particle Swarm Optimization (PSO) [5], Ant Colony Optimization (ACO) [6], Artificial Bee Colony (ABC) [7], Cyclical Parthenogenesis Algorithms (CPA) [8], Whale Optimization Algorithm (WOA) [9] and Gray Wolf Optimization (GWO) [10]. Inspired by physical laws, the third group of meta- heuristic algorithms forms. Examples of these algorithms include: Water Evaporation Optimization (WEO) [11], Tug-of-war Optimization (TWO) [12], Thermal Exchange Optimization (TEO) [13], Charged System Search (CSS) [14], Particle Colliding Bodies Optimization (CBO) [15], Harmony Search (HS) [16], Vibrating Particles System (VPS) [17] and Big Bang-Big Crunch (BB-BC) [18]. Meta-heuristic algorithms with the origin of physical inspiration are more popular among researchers and have played a major role in improving the results of structural optimization. Today, the scope of inspiration has expanded on a wide level and has created remarkable successes in the state-of-the-art. Based on the traditional Nelder and Mead method, the Shuffled Complex Evolution (SCE-UA) [19] was proposed. In this algorithm, the geometric operators of contraction and reflection are inspired. This algorithm is the basis of the Shuffled Frog-Leaping Algorithm (SFLA) [20], which is classified as a memetic algorithm. Other algorithms, such as Imperialist Competitive Algorithm (ICA) [21], Teaching-Learning-Based Optimization (TLBO) [22] and Biogeography-Based Optimization (BBO) [23] respectively, are inspired by the political performance of emperors, the process of education in the classroom and the migration process in wildlife habitats from various perspectives. They have gained inspiration and significant improvement in engineering optimization. Despite the significant progress in meta-heuristic algorithms in the optimization of engineering problems, pests still threaten these algorithms. These pests can be things such as early maturity, falling into the trap of local optima and stopping the calculation process, etc. Research shows that by benefiting from chaos maps and replacing them in the exploration and exploitation stages, it greatly improves the weaknesses of the algorithms. Although there are no signs of random phenomena in chaos maps, their dynamic state leads to the emergence of very disordered behaviors in the environment. This behavior can be very useful to escape and jump from the trap of local optima. The salient features of a chaos function can be summarized in the following. These functions are sensitive to the initial conditions, their periodic rotation is dense, they have a quasi-random but non-periodic behavior, and finally, the rule governing their function is such that they do not have an inverse [24]. In the recent research, chaos signals are distributed in scattered points of the decision space, and then the search is performed around their neighborhood, and if they are trapped in the local optima, they escape from them and jump to the global optima. In this way, the balance between the exploration and exploitation stages is provided and a significant improvement in the optimization results is achieved [25]. In this research, Logistic and Gaussian chaos maps have been combined with three meta-heuristic algorithms and chaotic meta-heuristic algorithms have been created. In order to access broad statistical communities and increase diversity in the search space, three scenarios have been considered. In the first and second scenario, the chaos maps replace the exploration and exploitation phase of meta-heuristic algorithms, respectively, and in the third scenario, they are applied simultaneously in both phases. Due to the wide statistical space of the answers, the global optimality cannot be far from the target of the chaotic algorithms, and eventually "absolute pseudo-optimal" answers are obtained. Chaotic algorithms can be developed for Topologies optimization in the case of probabilistic loading [26], Reliability based topology optimization of thermoelastic structures [27], and elasto-plastic limit analysis of reliability based geometrically [28]. In these cases, the design with the limit of the minimum penalized weight is practical.

2 Formulation of the optimization problems

In this section, the dynamic analysis of the free vibration of the structure and access to natural frequencies is performed first, and then the optimization model is presented with the formation of the objective function, constraints, penalty function and the combination of the objective function with the penalty function.

2.1 Free vibration and natural frequencies

When a structure is affected by dynamic loads such as earthquakes or storms, in order to prevent the phenomenon of resonance, the natural frequency should be limited to a certain range [29]. In order to apply this type of constraints, the natural frequencies of the structures contain all the required information about the dynamic behavior of the structures. In the low frequency vibration problems, the response of the structure depends on the basic frequencies and mode shapes, and by applying the frequency constraints, the dynamic behavior of the structure can be easily controlled. In a number of optimization problems, the effects of some modes can be reduced by using these relations. For example, in the design of airplane wings, the effort is to reduce bending and torsion modes. To calculate the natural frequencies of the structure, the matrix form of the free vibration equation of the multi-degree-of-freedom system is investigated. In these relationships, M is the mass matrix, K is the stiffness matrix, Y is the displacement equation, $\boldsymbol{\varphi}_n$ is the shape vector of the nth mode, $\boldsymbol{\varphi}_n(t)$ is the time coordinate of the nth mode, A_n and B_n are the integration constants that are determined from the initial conditions. Dynamic relationships of free vibration are presented from Eqs. (1) to (5).

$$M \times \ddot{Y}(t) + K \times Y(t) = 0 \tag{1}$$

$$Y(t) = \phi_n \times q_n(t);\ \ n = 1,2,\ldots,N \tag{2}$$

$$q_n(t) = A_n \cos\omega_n t + B_n \sin\omega_n t \tag{3}$$

$$Y(t) = \phi_n \times (A_n \cos\omega_n t + B_n \sin\omega_n t) \tag{4}$$

In these relationships, ωn and φn are unknowns which results in the placement of Y(t) in the free vibration differential equation.

$$\left[K - \omega_n^2 \times M\right] \times \phi_n = 0 \ \Rightarrow\ \left|K - \omega_n^2 \times M\right| = 0 \tag{5}$$

With the expansion of these determinants, the frequency equation is obtained. This equation has N real and positive roots for ${\omega_n}^2$, which determines the N natural frequency of ω_n. The roots of this characteristic equation are known as eigenvalues. Also, by reflecting on these equations, it is concluded that the design variables are not explicitly present in these equations and their presence is implicit. Therefore, if optimization is carried out with mathematical methods and implicit derivatives, one will encounter strongly non-linear and non-convex equations the Solution will be very difficult and time consuming. As a result, if we want to solve the frequency constraints in an optimization using traditional methods, we must perform a sensitivity analysis. Thus, the derivatives of the eigenvalues and eigenvectors must be calculated, and this usually encounters various approximations. In addition, in some cases, we will find repetitive amounts and repetitive frequencies that are not distinguished by conventional investigations and can only be determined by directional derivatives. When analyzing the sensitivity of the structures, it is created by repetitive frequencies of particular complexity because it is mainly "unique in specific values. Another limit that greatly affect the traditional mathematical optimization methods is the choice of a good starting point. In cases where the starting point is not appropriate, these methods stop by reaching the local optimization, and there is no solution to escape these local optimists. Today, with the complication of issues and increasing the number of decision variables, the lack of accountability of classical methods is evident. Therefore, in order to overcome these challenges over the past decade, various types of powerful optimization methods have been invented, some of which have been optimized by frequency constraints. In most of these optimization methods, they are inspired by meta-heuristic techniques. Meta-heuristic algorithms are widely accepted by researchers and are considered as powerful tools for engineering optimization problems. The main features of these methods can be stated as the following:

- These are based on the population.
- These are independent of the specific problem.
- These are inspired by natural phenomena.
- These do not need any information about the gradient of the objective function and constraints.
- The quality of the final solution does not depend on the starting point.
- These are based on decisions and principles of random search.

In these algorithms, the value of the objective function itself is used instead of its derivatives, and these have global search capabilities, and are also suitable for complex, non-linear, discrete and non-convex search spaces.

2.2 Formation of the objective function, constraints and penalty function

In optimization problems for the cross-sectional area and geometric shape of truss structures that are associated with frequency Constraints, the goal is to minimize the weight of the structure in such a way that the limitations of a number of natural frequencies for vibration modes are met. The cross-sectional areas of the members along with the coordinates of some nodes are introduced as decision variables. These variables are selected as continuous. Upper and lower bounds are also determined for variables in some cases. These optimization problems are formulated in mathematical form according to Eqs. (6):

$$\begin{aligned} & Find\ X=\{A,S\},\ A=\{A_1,A_2,...,A_{na}\},\ S=\{S_1,S_2,...,S_{ns}\} \\ & to\ minimize:\ W(A,S)=\sum_{i=1}^{nm}\rho_i\times l_i(S)\times A_i \\ & subjected\ to:\begin{cases} \bar{\omega}_i^L\le\omega_i\le\bar{\omega}_i^U, & i=1,2,...,n\omega \\ A_j^L\le A_j\le A_j^U, & j=1,2,...,na \\ S_k^L\le S_k\le S_k^U, & k=1,2,...,ns \end{cases} \end{aligned} \tag{6}$$

where, $\boldsymbol{X}$ is the vector of decision variables, A is the variable related to the cross-sectional area of the members, na is the symbol for represent the number of cross-sectional area variables, A_i is the value of the cross-sectional area of the ith variable, S is the variable related to the shape and arrangement, ns is the number of shape variables that have the same coordinates, S_i is the numerical value of the ith shape variable, W expresses the weight of the truss, nm specifies the total number of members, ρ_i is the specific gravity of the materials belonging to the ith member of the truss, L_i the length of the ith member which can be determined through the variables. The shape and initial shape of the structure should be determined, ω_i represents the ith natural frequency of the truss, ϖ_i^L and ϖ_i^u respectively represent the lower limit and upper limit of the ith fixed base frequency, $n\omega$ represents the sum of the total frequency limits, A_j^L and A_j^u respectively represent the lower limit and upper limit of the jth variable of the cross section A_j, and similarly S_k^L and S_k^u, respectively, the lower limit and the upper limit of the kth shape variable S_k. Due to the fact that meta-heuristic algorithms are used for unconstrained problems, we use the penalty function to convert the constrained state to unconstrained in modeling. In this method, if there is no violation, the amount of the fine will be zero, and if there is a violation, the value of the penalty function is obtained from the following relationships:

$$V_i=\begin{cases} 0 & if \quad \bar{\omega}_i^L\le\omega_i\le\bar{\omega}_i^U \\ 1-\dfrac{\omega_i}{\bar{\omega}_i^L} & if \quad \bar{\omega}_i^L>\omega_i \\ 1-\dfrac{\bar{\omega}_i^U}{\omega_i} & if \quad \omega_i>\bar{\omega}_i^U \end{cases} \tag{7}$$

$$\gamma=\sum_{i=1}^{n\omega}V_i \tag{8}$$

$$F_{penalty}=(1+\varepsilon_1\times\gamma)^{\varepsilon_2} \tag{9}$$

$$to\ minimize\ Mer(A,S)=W(A,S)\times F_{penalty} \tag{10}$$

In these relationships, y represents the set of violations and ε_1 and ε_2 are chosen based on the ratio of search and extraction. In this research, ε_1 unit and ε_2 are selected with incremental linear changes in the range of 1.3 to 3 at the end of the iteration. Finally, "*Mer*" is the merit function or the objective function after applying the penalty.

3 Meta-heuristic algorithms and applying the chaos functions

In meta-heuristic algorithms, two important stages of exploration and exploitation are considered in order to converge towards optimal answers. In the exploration phase, one settles on the points of the search space that have a privileged strategy, and in the exploitation phase, one carefully examines the area related to the neighborhood of the selected points. The Random Search algorithm (RS) is a fully exploration algorithm, while the Hill Climbing algorithm (HC) and Tabu Search algorithms (TS) are fully exploitation. in order to expand and cover the search space in both stages, we will have to use random parameters. The distribution function related to these parameters is different according to the proposed algorithm and can include uniform, normal, logistic, levy, etc., distribution. Although the selection of these parameters is completely random, it has a significant effect on the efficiency of the algorithm. Random parameters play a major role in increasing or decreasing the speed of convergence and escaping from the trap of local optima, and in some cases these key factors can control the balance between exploration and exploitation. Today, the random selection of these parameters is accompanied by doubts, and researches show that some of the ineffectiveness factors of meta-heuristic algorithms are random parameters, so dynamic chaos series, whose values are definite, will be a good alternative for these parameters. Chaotic series can accelerate the convergence towards global optima, and play a major role to avoid falling into the trap of local optima and premature convergence. The series created by chaos maps are similar to random processes, but their values are deterministic, non-linear, dynamic and non-repetitive and non-convergent towards a certain limit. We will not have inverse chaos functions and we can consider different situations to apply them [30]. In some cases, these functions are suitable substitutes for the possible parameters related to the exploration, and in some other cases, we will gain improved results by replacing the parameters related to the exploitation part. Also, in a small number of algorithms, applying chaos functions in both cases simultaneously will lead to improved results. In order to reach the desired state, we have to examine the algorithms in three different scenarios. Which scenario works best for a particular algorithm varies from example to example. In fact, the non-linear and non-convex behavior of the objective functions in the optimization of truss structures has created these conditions for meta- heuristic algorithms. In recent research to investigate the effects of chaos maps in improving the optimization results, first these algorithms are examined in a standard way and then the results will be compared with the three proposed scenarios.

3.1 Standard Water Evaporation Optimization (WEO)

The inspirations in this algorithm are based on the evaporation of tiny water molecules on the solid surface with different humidity states. This algorithm was presented by Kaveh and Bakhshpoori [11] and it is population-based like most meta- heuristic algorithms. In this algorithm, dynamic simulation is considered to model the molecular evaporation of water. Based on this simulation, the desired surface changes from the state of repelling to the absorbing state of water, but the rate of evaporation does not decrease uniformly in this sudden change, but increases first and starts to decrease when it reaches a maximum volume. Also, when the moisture level of the desired layer is not enough, the water molecule becomes a spherical droplet. In cases where the moisture level of the desired layer is sufficient, the water molecule spreads and spreads as a single layer. This performance has been used in the water evaporation optimization.

3.1.1 Basic steps of the Water Evaporation Optimization

Step 1 Selecting the parameters of the algorithm, which are: determining the number of water molecules, which is the initial population nWM, maximum number of repetitions of the "maxNITs" algorithm, minimum and maximum probability of evaporation of the single-layer state MEPmin and MEPmax, which we usually choose 0.03 and 0.6, respectively, minimum and maximum probability of evaporation of droplet state DEPmin and DEPmax which we usually choose 0.6 and 1, respectively. This algorithm with n number of proposed answers which constitute the same water molecules is randomly selected from the search space and the matrix of molecules It forms water. Then, the responses are applied to objective functions and penalized objective functions, and the corresponding vectors of those functions are formed.

Step 2 The formation of the evaporation phase based on the single-layer strategy of water, which is carried out in the form of Eq. (11) and the probability in the range of 0.03 to 0.6.

$$if\ NFE_s \leq \frac{\max NFE_s}{2} \rightarrow \begin{cases} if\ \boldsymbol{rand}_{ij} < \exp(E_{sub}(i)) \Rightarrow MEP_{ij} = 1 \\ if\ \boldsymbol{rand}_{ij} \geq \exp(E_{sub}(i)) \Rightarrow MEP_{ij} = 0 \end{cases} \tag{11}$$

The formation of the evaporation phase based on the water drop strategy, which is carried out in the form of Eq. (12) probability relations in the range of 0.6 to 1.

$$if\ NFE_s > \frac{\max NFE_s}{2} \rightarrow \begin{cases} if\ \boldsymbol{rand}_{ij} < J(\theta_i) \Rightarrow DEP_{ij} = 1 \\ if\ \boldsymbol{rand}_{ij} \geq J(\theta_i) \Rightarrow DEP_{ij} = 0 \end{cases} \tag{12}$$

Step 3 Determining the range of step-size is determined according to Eq. (13).

$$\begin{aligned} Stepsize = rand \otimes (&WM[permute1(i)(j)] \\ &-WM[permute2(i)(j)]) \end{aligned} \tag{13}$$

Step 4 The formation of new water molecules (offspring) that arise according to Eq. (14).

$$\begin{cases} if\ NFE_s \leq \dfrac{\max NFE_s}{2} \Rightarrow newWM = WM + Stepsize \times MEP \\ if\ NFE_s > \dfrac{\max NFE_s}{2} \Rightarrow newWM = WM + Stepsize \times DEP \end{cases} \tag{14}$$

The new evaluated molecules are replaced if they are better than the previous answers.

Step 5 Termination conditions are checked and if necessary, the operation from step 2 is repeated.

3.1.2 Chaos-Embedded Water Evaporation Optimization (CWEO)

In this algorithm, the formation of the evaporation phase is based on two important strategies, which include the single-layer and water droplet strategies. Therefore, these two steps will play an important role in exploration and exploitation. By replacing the chaos maps in the random selections related to these steps, we will witness a significant improvement in the performance of the algorithms. The suggested scenarios for this replacement are as follows:

Scenario 1 - Replacing the chaos map in the evaporation phase based on a monolayer strategy: In this case, the first chaos map CHM1 is replaced in Eq. (11) and the result of Eq. (15) is as follows:

$$if\ NFE_s \leq \frac{\max NFE_s}{2} \rightarrow \begin{cases} if\ \mathbf{CHM1}_{ij} < \exp(E_{sub}(i)) \Rightarrow MEP_{ij} = 1 \\ if\ \mathbf{CHM1}_{ij} \geq \exp(E_{sub}(i)) \Rightarrow MEP_{ij} = 0 \end{cases} \quad (15)$$

Scenario 2 - Replacing the chaos function in the evaporation phase based on the drip strategy: In this case, the second chaos map CHM2 is replaced in Eq. (12) and the result of Eq. (16) is as follows:

$$if\ NFE_s > \frac{\max NFE_s}{2} \rightarrow \begin{cases} if\ \mathbf{CHM2}_{ij} < J(\theta_i) \Rightarrow DEP_{ij} = 1 \\ if\ \mathbf{CHM2}_{ij} \geq J(\theta_i) \Rightarrow DEP_{ij} = 0 \end{cases} \quad (16)$$

Scenario 3 Placing the chaos function in both steps simultaneously: In this case, two chaos maps are simultaneously replaced in Eqs. (11–12).

3.2 Standard Tug of War Optimization (TWO)

This algorithm was presented by Kaveh and Zolghadr [12]. In this algorithm, it is inspired by the game of tug-of-war and is based on the population. Like other meta- heuristic algorithms, a set of initial answers is selected from the decision space and each solution is considered as a team, and all of them form a league. In each iteration of the algorithm, the evaluation of the teams is determined and sorted based on merit. In this competition, the best team has the most weight and the worst team has the least weight. Both competing teams are pulling the rope, the light team loses the competition and moves to the heavy team.

3.2.1 Basic steps of the Tug of War Optimization Algorithm

Step 1 Determining the parameters of the algorithm: The parameters include the following.

- The number of teams participating in the competition, which is represented by the symbol *nT*,
- the number of members of each team or *T*, which is known as the league matrix.

In this way, in each step, it consists of evaluating the values of the vectors corresponding to the objective function and the penalized objective function.

Step 2 Determining the weights in the tug-of-war competition: Each solution is known as a team from the league, and the numerical value of its weight in these competitions is determined according to the Eq. (17):

$$W_i = \frac{PFit_i - \min(PFit)}{\min(PFit) - \max(PFit)} + 1 \quad (17)$$

In this regard, the penalized objective function along with its maximum and minimum value is considered to determine the weight of each team. Based on this relationship, the weight of each team is placed between 1 and 2. Numerical value 2 is the best and heaviest team.

Step 3 Competing between teams: Each team in the league competes with all other teams. To move to its new position in each period of repetition, the tensile force applied by each team is proportional to the frictional force at rest. In the modeling, the value of the coefficient of friction is assumed to be one, and the pulling force between the two teams i and j can be the maximum of the following two values:

$$F_{p,ij} = \max\{W_i\mu_s, W_j\mu_s\} \quad (18)$$

Finally, the resulting force on team *i* in the face of heavier team *j* is obtained as follows:

$$F_{r,ij} = F_{p,ij} - W_i\mu_k \quad (19)$$

The amount of acceleration in the movement of team *i* towards team *j* results from the following equation:

$$a_{ij} = \frac{F_{r,ij}}{W_i\mu_k} g_{ij}, \quad g_{ij} = T_j - T_i \quad (20)$$

In this relationship, *g* is the acceleration constant of gravity, which is obtained from the difference of the position vector of the two teams. Also, the displacement amount of team *i* after competing with team *j* is determined according to the following relationship:

$$stepsize_{ij} = \frac{1}{2} a_{ij} \Delta T^2 + \alpha\beta(Lb - Ub) \odot \boldsymbol{randn} \quad (21)$$

The second term of this relationship considers random cases in the amount of team displacement. This term can consider that part of the search space that team *i* travels before being stopped by team *j*. The *α* coefficient gradually reduces the possible effects and is selected in the range of [0.99, 0.9]. Larger values for *α* slow down the convergence rate and give enough time for the proposed answers to fully explore the search space. Also, $\boldsymbol{\beta}$ is the scaling factor that can be selected in the range of [1, 0]. This parameter controls the steps of the proposed answers, and in cases where we need more accuracy for searching, the $\boldsymbol{\beta}$ parameter is selected with smaller steps. Also, L_b and U_b are the lower limit and upper limit of the search space and their differences express the allowed range for design variables. The multiplication used is of member-by-member type and random values with standard normal distribution are selected for ***randn*** vector. In this relationship, the multiplication used is member-by-member type and a random vector with normal distribution is selected. We can define ΔT time periods as 1. It should be noted that in cases where j is lighter than i, the values related to the displacement of team i are considered zero. Finally, the total relocation of team i is summarized as follows:

$$stepsize_{ij} = \sum_{j=1}^{nT} stepsize_{ij},\ i \neq j \tag{22}$$

The new position of team i is as follows:

$$T_i^{new} = T_i + stepsize_i \tag{23}$$

Step 4 Updating the position of league teams: After the competition of the league teams, the results should be updated. For this, the new results are replaced if they are better than the current values.

Step 5 The possibility of the results leaving the range of the control decision space and if in some cases the positions of the teams are out of this search range, it is reconstructed based on the following relationship.

$$T_{ij} = bestT_j + (\frac{\boldsymbol{randn}}{NIT_s})(bestT_j - T_{ij}) \tag{24}$$

In this equation, *bestT* is the best team so far and NIT_s is a repeat number.

Step 6 The termination conditions are checked and if necessary, the competition between the league teams is repeated.

3.2.2 Chaos-Embedded Tug of War Optimization (CTWO)

In order to determine the new position of the league teams, two exploration and exploitation strategies have been considered in the scale factor β and the search space limitation. By replacing the chaos maps in the random selections related to these steps, we will see a significant improvement in the performance of the algorithm. The suggested scenarios for this replacement are as follows:

Scenario 1 Replacing the chaos function in choosing the scale factor β: In this case, the first CHM1 chaos map is replaced in Eq. (21). The result will be as below:

$$stepsize_{ij} = \frac{1}{2} a_{ij} \Delta T^2 + \alpha \cdot \mathbf{CHM1} \otimes (Lb - Ub) \otimes \boldsymbol{randn} \tag{25}$$

Scenario 2 Replacing the chaos map in applying the search space limitation: In this case, the second CHM2 chaos map is replaced in Eq. (24). The result will be as below:

$$T_{ij} = bestT_j + (\frac{\mathbf{CHM2}}{NIT_s})(bestT_j - T_{ij}) \tag{26}$$

Scenario 3 placing chaos functions in both stages: In this case, two chaos maps are simultaneously replaced in Eqs. (21–24).

3.3 Standard Thermal Exchange Optimization (TEO)

This algorithm was presented by Kaveh and Dadras [13], inspired by Newton's law of cooling. In this algorithm, the physical relations related to the thermal exchange between the object and the surrounding environment are used and it is based on the population. The use of this algorithm for structural problems has brought significant improvement. Newton's law of cooling states that the rate of heat loss between an object and the surrounding environment is proportional to their temperature difference. Therefore, the population of heat exchanging particles is separated into two parts and the first half is placed in heat exchange with the second half.

3.3.1 Basic steps in Thermal Exchange Optimization

Step 1 Determining the initial values of the algorithm: The thermal exchange algorithm starts with the introduction of the initial proposed solutions. These solutions are selected within the scope of the search space. The number of elements selected for thermal exchange is equal to *nTO*. Then the objective functions and penalized objective functions corresponding to each of the solutions are determined.

Step 2 Creating groups: In order to carry out the thermal transfer process, first the introduced components are sorted in ascending form based on the penalized objective function and then they are classified into two groups with the same number of components. In this step, the components of the first group ($i = 1, 2, ..., nTO/2$) exchange thermal with the components of the second half ($i = nTO/2 + 1, ..., n$).

Step 3 thermal transfer and updating the position of the components: Based on the thermal exchange relationship of the elements with the surrounding environment, the final position of the elements is determined by the following important heat exchange relationship.

$$newTO(i) = envTO(i) + (TO(i) - envTO(i))\exp(-\beta(i)t) \tag{27}$$

In this regard, *newTO*(i) expresses the new position after thermal exchange and *envTO*(i) expresses the thermal position of the surrounding environment before thermal exchange. Also, in order to apply the parameters of time and β, we use the following suggested relationships.

$$t = \frac{NITs}{\max NITs},\ \beta(i) = \frac{PFit(i)}{\max PFit(i)} \tag{28}$$

In these relationships, *NITs* and max*NITs* express the number of iterations in each step and the maximum number of iterations, respectively. Also, *PFit* and max*PFit* consider the values of the penalized objective function and its maximum value.

Step 4 Applying possible conditions: Up to this stage, the calculations are based on specific and definite relationships. In order to escape the trap of local optima and premature maturity in the algorithm, two mechanisms of search and discovery in the thermal exchange of elements should be considered. In the following relations, this possible condition is provided by the parameters C, C1, C2 and a choice of rand.

$$nevTO = (1 - c \times \boldsymbol{rand})TO \tag{29}$$

$$C = C1 + C2 \times (1 - t) \tag{30}$$

$$C1 = round(\boldsymbol{rand}),\ C2 = round(\boldsymbol{rand}) \tag{31}$$

Step 5: Elitist thermal exchange algorithm: In order to achieve the elitism of the algorithm, memories are allocated for the best results. For this purpose, the best thermal exchange (TO-M) and the objective function (Fit-M) and the corresponding penalized objective function (PFit-M) are compared with the previous periods in each period, and replaced if the results are improved.

Step 6: The termination conditions are checked and if necessary, the thermal exchange of the elements with the surrounding environment is repeated.

3.3.2 Chaos-Embedded Thermal Exchange Optimization (CTEO)

In order to determine the new position of the heat flow as a result of the heat exchange of each element with the surrounding environment, two exploration and exploitation strategies have been considered in Eqs. (29–31).

By replacing the chaos functions in the random selections related to these steps, we will have a significant improvement in the performance of the algorithm. The suggested scenarios for this replacement are as follows:

Scenario 1 Replacing the chaos map in the exploration phase: At this stage, the first CHM1 chaos map replaces the random term in Eq. (29). The results of applying the chaos map will be as follows:

$$nevTO = (1 - c \times \mathbf{CHM1})TO \tag{32}$$

Scenario 2 Replacing the chaos map in the exploitation phase: In this stage, the second chaos map CHM2 is replaced to determine the values of C1 and C2 in Eq. (31). For this purpose, we have:

$$C1 = round(\mathbf{CHM2}),\ C1 = round(\mathbf{CHM2}) \tag{33}$$

Scenario 3 placing the chaos map in both stages: In this case, the chaos map is simultaneously applied to the random selections of both stages.

4 Introduction of the Selected Chaos Map

In most of the meta-heuristic algorithms, the optimization results stagnate and stop when they reach the local optimal position, in such conditions premature convergence occurs. In order to escape from the trap of local optima, chaos functions create suitable conditions that by creating disorder in the search space, it is possible to jump to most positions of the search space. Therefore, general optima will not have the chance to escape from the shooter of chaotic functions. These functions do not have any traces of random behavior in them, but they cause the emergence of very irregular behaviors in the search space. One of the most important features of these functions is sensitivity to initial conditions and non-periodic and ergodic behaviors, and these functions do not have an inverse. How to apply these functions in meta-heuristic algorithms is presented in the flowchart of Fig. 1. Using chaotic series to evolve variables in COA meta-heuristic algorithms has significant advantages over other methods. In deterministic search, compared to random search, more speed and convergence towards the general answer is achieved [31]. In this research, logistic and Gaussian chaos function has been chosen. In the logistic chaotic function, the search space converges from the local minima to the global minimum with a very high probability. Therefore, this function is suitable for improving exploration conditions of algorithms. But the Gaussian chaotic function moves the search space towards local minima with a very high probability and is suitable for improving the exploitation conditions. Therefore, by choosing these chaos functions, the weakness of algorithms of any kind is improved. The numerical distribution of these chaos functions for 100 iterations is presented in Fig. 2.

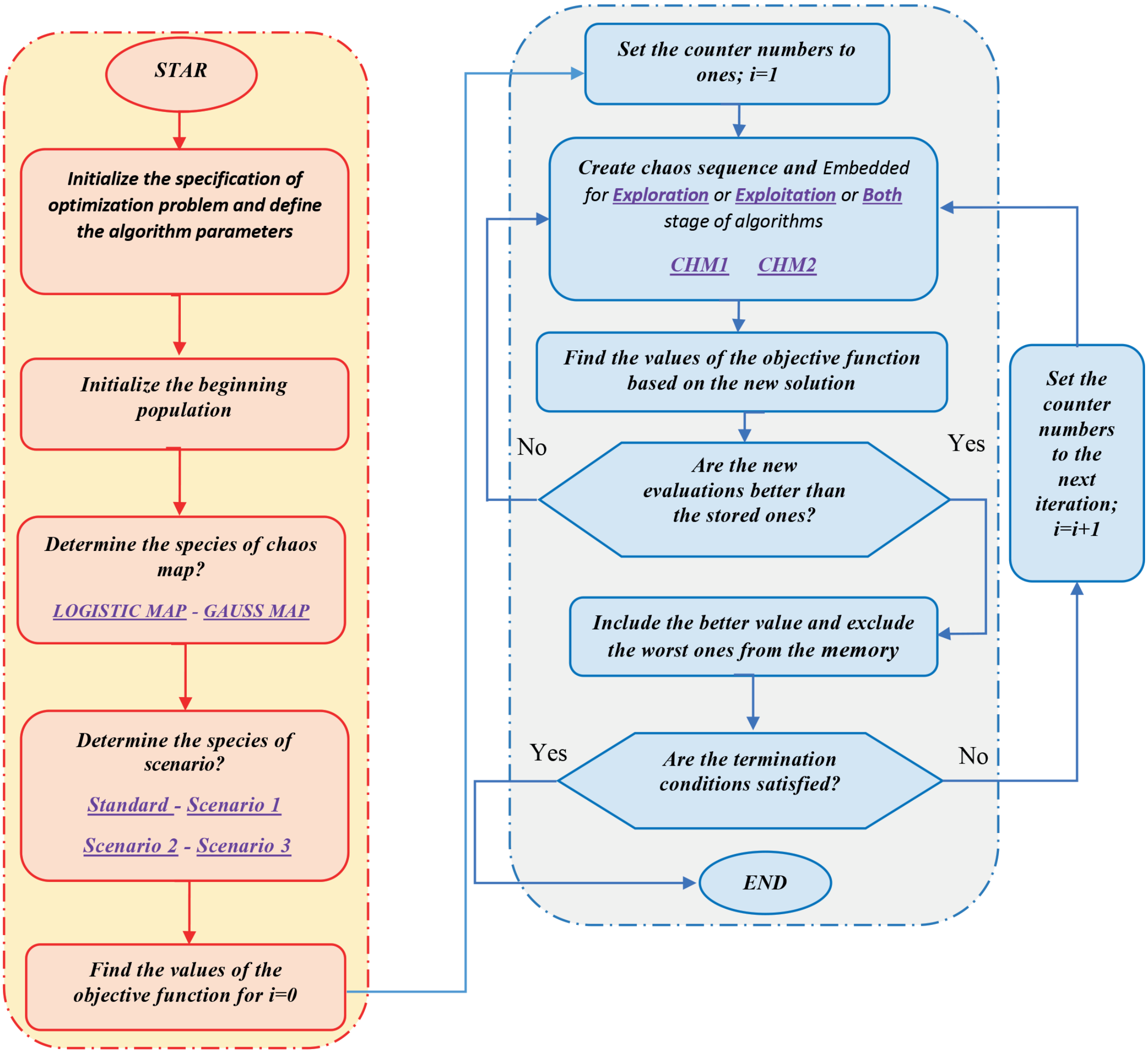

Fig. 1 Flowchart for the chaos algorithm

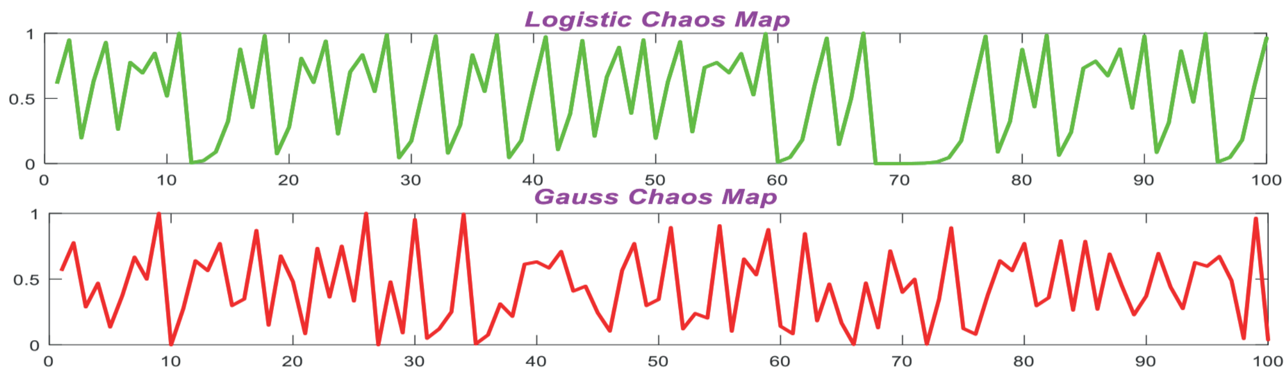

Fig. 2 The chaotic value distribution during 100 iterations

4.1 Logistics chaotic map

This function appears in the non-linear dynamic behaviors related to the biological population [32]. The statements of chaotic sequences are obtained in the logistic function according to the group of Eqs. (34):

$$\begin{aligned} &CHM_{k+1} = a \times CHM_k(1 - CHM_k) \\ &CHM_{k+1} \in (0,1);\ CHM_k \in (0,1) \\ &CHM_0 \notin (0, 0.25, 0.50, 0.75, 1) \end{aligned} \tag{34}$$

In the studies conducted, $a = 4$ is considered.

4.2 Gauss chaotic map

Using this function in nonlinear dynamic behaviors brings good result [33]. The statements of chaotic sequences in the Gaussian function are obtained according to Eq. (35):

$$CHM_{k+1} = \begin{cases} 0 & CHM_k = 0 \\ \dfrac{1}{CHM_k} - \left[\dfrac{1}{CHM_k}\right] & CHM_k \neq 0 \end{cases} \tag{35}$$

5 Numerical examples of Optimal Truss Design

In this research, in order to compare the efficiency of algorithms in standard and chaotic mode, truss structures with oscillation frequency limit have been selected. This restriction is to avoid the destructive phenomenon of resonance and high vibration of the structure. In the optimal design of the trusses, in addition to the cross-sectional area of the members, the geometric shape of the structures is also included in the objective function. In order to increase diversity and repair the weakness of exploration and exploitation in algorithms, different chaos maps with different scenarios have been formed. Also, in order to reach the statistical population and introduce the optimal values for the weight, the average and the coefficient of variation of each of the examples have been examined 20 times independently. In each table, the three chaotic modes are compared with the standard mode and the final optimal results for the optimal chaotic meta-heuristic algorithm are introduced. In another table, the comparison between the optimal chaotic algorithms is done and the best value among them is presented by introducing the assigned values for each cross section and unknown geometric shape. Considering that each example is associated with 18 independent answers, therefore, the possibility of accessing optimal answers has increased.

5.1 A 52-bar dome-like truss

The dimensional specifications of the 52-bar dome-like truss are according to Fig. 3. In this truss, in addition to the optimization of the size of the members' sections, the optimization of the geometric coordinates of the nodes is also considered, and the geometric shape of the structure is designed during the optimization process. The decision variables related to the size of the section are classified into 8 groups according to the symmetry in the geometric

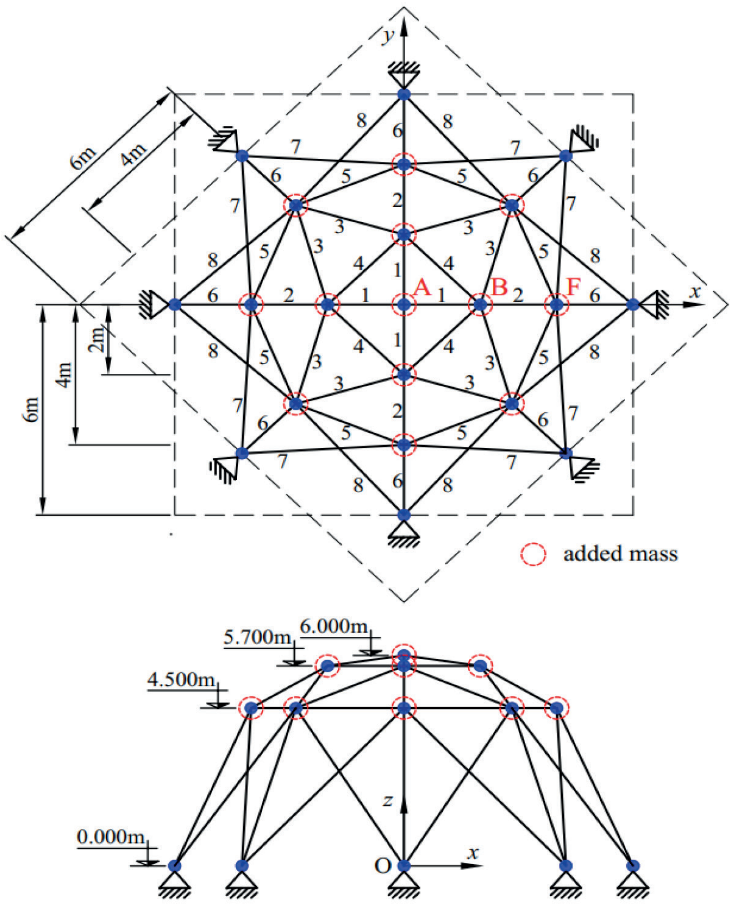

Fig. 3 Schematic of A 52-bar dome-like truss

shape. The geometric coordinates of all symmetrical free nodes can change by ±2 meters from the initial position along the coordinate axes. In this case, the number of decision variables related to the shape of the structure and the geometric coordinates of the nodes is limited to 5 variables, and the total number of variables, including shape and size, will be 13 variables. In all the free nodes, the lumped non-structural mass of 50 kg has affected all the free nodes. The mechanical specifications of the structure are: The density of the materials is 7800 kg/m^3, the modulus of elasticity is 210000 MPa, the frequency limits of the structure are less than 15.916 Hz in the first mode and greater than 28.648 Hz in the second mode. For the cross-sectional area of the members, the lower bound is 1 cm^2 and the upper bound is 10 cm^2. In order to form statistical samples, 20 independent surveys were conducted and the results related to the best weight, average value and coefficient of variation are presented in statistical Table 1. In Table 2, a comparison has been made between the chaotic meta-heuristic algorithms and the details of the cross-sectional area of the members in the optimal state for the states that have high efficiency are presented as the final result of the optimization.

Table 1 Statistical results for the 52-bar dome-like truss

Algorithms	Statistical Information	WEO Standard	CWEO-21 Logist-1	CWEO-22 Logist-2	CWEO-23 Logist-3	CWEO-31 Gauss-1	CWEO-32 Gauss-2	CWEO-33 Gauss-3
1-WEO	Best	194.911	193.762	193.757	194.206	194.974	194.924	**193.701**
	Mean	200.138	**195.383**	199.064	196.563	199.353	201.493	200.338
	C.V(%)	2.2960	**1.3182**	5.8882	2.161	1.8123	1.9536	2.4699
Algorithms	Statistical Information	TWO Standard	CTWO-21 Logist-1	CTWO-22 Logist-2	CTWO-23 Logist-3	CTWO-31 Gauss-1	CTWO-32 Gauss-2	CTWO-33 Gauss-3
2-TWO	Best	194.066	**193.165**	193.290	193.660	193.256	193.431	194.658
	Mean	196.172	196.761	**193.531**	194.002	193.817	194.929	201.675
	C.V(%)	1.0556	2.2447	**0.1278**	0.1285	0.2736	1.2944	4.2939
Algorithms	Statistical Information	TEO Standard	CTEO-21 Logist-1	CTEO-22 Logist-2	CTEO-23 Logist-3	CTEO-31 Gauss-1	CTEO-32 Gauss-2	CTEO-33 Gauss-3
3-TEO	Best	193.886	**193.260**	193.428	193.303	193.994	193.367	193.373
	Mean	197.065	**194.078**	196.369	194.269	196.148	194.325	194.242
	C.V(%)	1.9229	**0.4104**	1.8252	0.8885	1.0123	0.8882	0.7835

Table 2 Optimal design comparison for the 52-bar dome-like truss

Decision Variable	WEO Stand	CWEO Gaus3	TWO Stand	CTWO Logis1	TEO Stand	CTEO Logis1
Z_A (m)	6.139	5.846	6.225	5.945	6.080	5.900
X_B (m)	2.112	2.104	2.313	2.265	2.140	2.268
Z_B (m)	3.888	3.740	3.806	3.722	3.844	3.730
X_F (m)	3.992	3.906	4.049	3.968	4.002	3.978
Z_F (m)	2.501	2.500	2.500	2.500	2.500	2.500
A_1 (cm^2)	1.007	1.000	1.001	1.000	1.000	1.000
A_2 (cm^2)	1.221	1.264	1.122	1.118	1.209	1.107
A_3 (cm^2)	1.244	1.273	1.247	1.222	1.259	1.209
A_4 (cm^2)	1.212	1.510	1.369	1.437	1.356	1.481
A_5 (cm^2)	1.343	1.415	1.494	1.403	1.428	1.410
A_6 (cm^2)	1.002	1.001	1.000	1.000	1.000	1.000
A_7 (cm^2)	1.629	1.528	1.478	1.592	1.502	1.558
A8 (cm^2)	1.436	1.406	1.400	1.371	1.433	1.397
Best Weight (kg)	194.9	193.7	194.1	**193.1**	193.9	193.2
Mean Weight (kg)	200.1	200.3	196.2	196.7	197.1	194.0
Coefficient Variation (CV)	2.296	2.469	1.056	2.245	1.923	0.410
NFE	24000	24000	27000	27000	28000	28000
ω_1 (HZ)	12.92	10.82	11.62	11.52	10.87	11.65
ω_2 (HZ)	28.65	28.64	28.65	28.65	28.64	28.64

Also, for quick access to optimization information, the bar chart of each component is displayed in Fig. 4. The analysis of the optimization results for different combinations of algorithms with chaos functions and comparing it with the standard mode shows a significant improvement in reducing the weight of the 52-bar dome-like truss. The results of each algorithm are: In the optimization algorithm based on Water Evaporation Optimization (WEO), the Gaussian chaos map with scenario 3 with a weight of 193.701 kg has an optimal response. In the optimization algorithm based on the Tug-of-War Optimization (TWO), the Logistic chaos map with scenario 1 with a weight of 193.165 kg has an optimal response. In the optimization algorithm based on Thermal Exchange Optimization (TEO), the Logistic chaos map with scenario 2 with a weight of 193.260 kg has an optimal response. In Table 2, all the results have been compared and among all the algorithms and chaos maps under investigation, the meta-heuristic algorithm based on the Tug-of-War Optimization (TWO), the Logistic chaos map with scenario 1 with a weight of 193.165 kg has obtained the most optimal results. In this table, the details related to the cross section of the elements, the average weight and the coefficient of variation are provided. The diagram of the convergence history of the algorithms to compare the standard and chaotic mode is presented in Fig. 5.

5.2 A 120-bar spatial dome

The dimensional specifications of the 120-bar spatial dome are according to Fig. 6. This truss is a well-known benchmark problem regarding weight optimization with frequency limitation. In this truss, only the optimization of the cross-section of the members is considered and the geometric shape of the structure is constant during the optimization process. The decision variables related to the cross-sectional area of the members are classified into 7 groups according to the symmetry in the geometric shape of the dome along the X and Y axes. Non-structural lumped mass

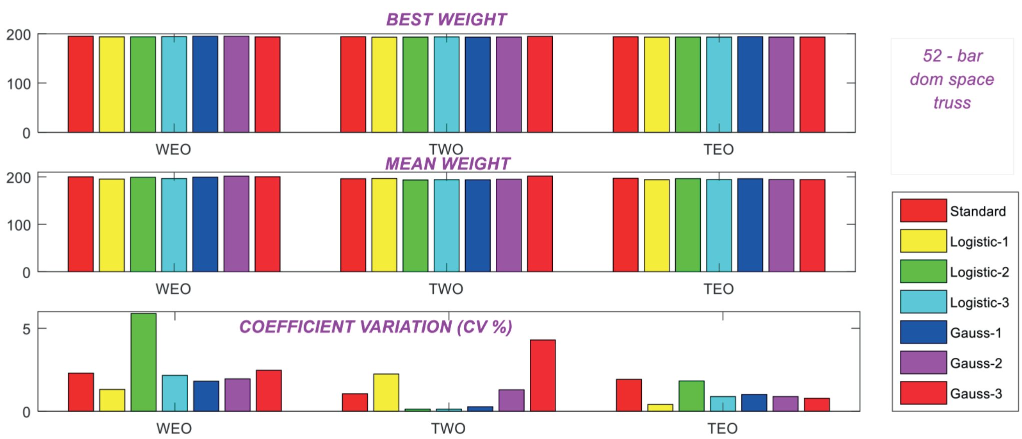

Fig. 4 Optimization results in standard mode and selection of chaos map for the 52-bar dome-like truss

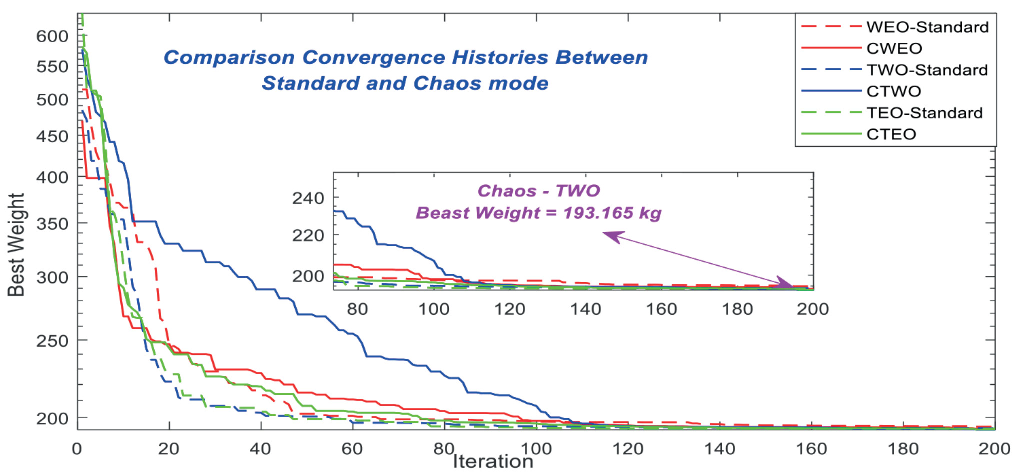

Fig. 5 The Convergence Histories for the 52-bar dome-like truss

affects all the free nodes of the structure. Their values are 3000 kg in node 1, 500 kg in nodes 2 to 13, and 100 kg in the rest of the nodes. The mechanical characteristics of the structure are: the density of the materials is 7971.81 kg/m^3, the modulus of elasticity is 210000 MPa, the frequency limits of the structure in the first and second modes are greater than 9 and 11 Hz, respectively. For the cross-section of members, the lower bound is 1 cm^2 and the upper bound is 129.3 cm^2. In order to form statistical samples, 20 independent surveys were conducted and the results related to the best weight, average value and coefficient of variation are presented in statistical Table 3. In Table 4, a comparison has been made between the chaotic meta-heuristic algorithms and the details of the cross-sectional area of the members in the optimal state for the states that have high efficiency are presented as the final result of the optimization.

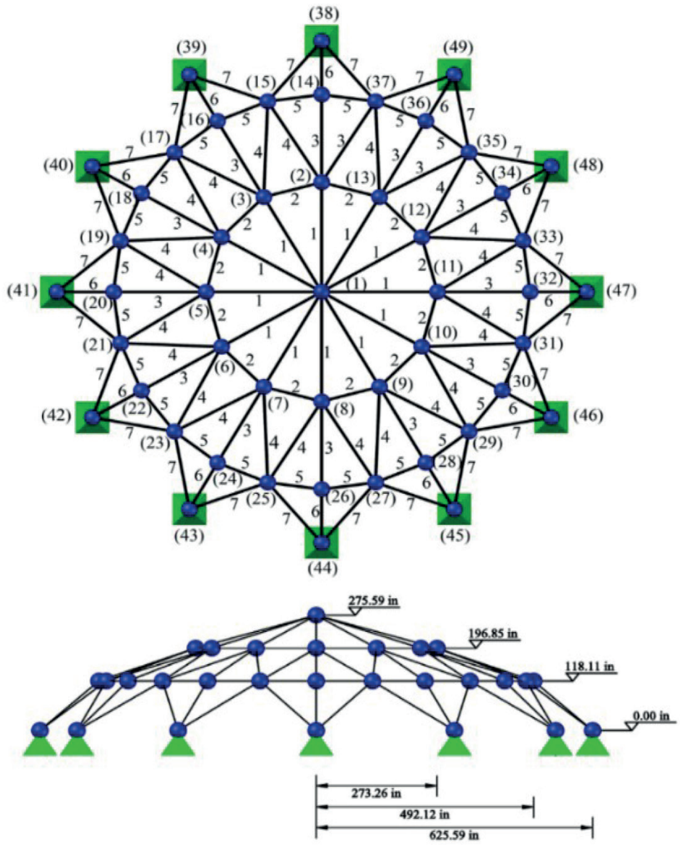

Fig. 6 Schematic of A 120-bar spatial dome

Table 3 Statistical results for the 120-bar spatial dome

Algorithms	Statistical Information	WEO Standard	CWEO-21 Logist-1	CWEO-22 Logist-2	CWEO-23 Logist-3	CWEO-31 Gauss-1	CWEO-32 Gauss-2	CWEO-33 Gauss-3
1-WEO	Best	8720.389	8712.838	8712.904	8715.883	8714.447	8715.201	**8711.427**
	Mean	8745.468	8725.251	8727.114	8723.127	8741.696	8748.133	**8718.489**
	C.V(%)	0.2454	0.1145	0.1778	**0.0586**	0.2539	0.3088	0.0763
Algorithms	Statistical Information	TWO Standard	CTWO-21 Logist-1	CTWO-22 Logist-2	CTWO-23 Logist-3	CTWO-31 Gauss-1	CTWO-32 Gauss-2	CTWO-33 Gauss-3
2-TWO	Best	8715.187	**8708.525**	8724.610	8710.925	8710.711	8708.553	8709.415
	Mean	8715.516	**8709.419**	8729.013	8714.213	8714.145	8709.620	8712.938
	C.V(%)	**0.0039**	0.0089	0.0316	0.0329	0.0245	0.01165	0.0229
Algorithms	Statistical Information	TEO Standard	CTEO-21 Logist-1	CTEO-22 Logist-2	CTEO-23 Logist-3	CTEO-31 Gauss-1	CTEO-32 Gauss-2	CTEO-33 Gauss-3
3-TEO	Best	8713.355	8714.335	8708.737	8716.217	**8708.667**	8724.145	8710.345
	Mean	8715.417	8721.072	8710.306	8720.087	**8709.095**	8729.086	8711.405
	C.V(%)	0.0222	0.559	0.0199	0.0633	**0.0061**	0.0495	0.0090

Table 4 Optimal design comparison for the 120-bar spatial dome

Decision Variable	WEO Stand	CWEO Gaus3	TWO Stand	CTWO Logis1	TEO Stand	CTEO Logis1
A_1 (cm^2)	19.831	19.842	19.356	19.521	19.621	19.515
A_2 (cm^2)	39.735	39.556	41.014	40.295	39.892	40.512
A_3 (cm^2)	10.486	10.729	10.569	10.603	10.834	10.637
A_4 (cm^2)	20.546	21.226	20.923	21.120	21.057	21.068
A_5 (cm^2)	9.739	9.611	9.941	9.872	9.852	9.747
A_6 (cm^2)	11.529	11.568	11.862	11.821	11.496	11.746
A_7 (cm^2)	15.748	14.945	14.931	14.819	15.021	14.893
Best Weight (kg)	8720.4	8711.4	8715.2	**8708.5**	8713.4	8708.6
Mean Weight (kg)	8745.5	8718.4	8715.5	8709.4	8715.4	8709.0
Coefficient Variation (CV)	0.245	0.076	0.0038	0.0088	0.0222	0.0061
NFE	20000	20000	24000	24000	40000	40000
ω_1 (HZ)	9	9	9	9	9.001	9
ω_2 (HZ)	11	11	11.001	11	11.002	11

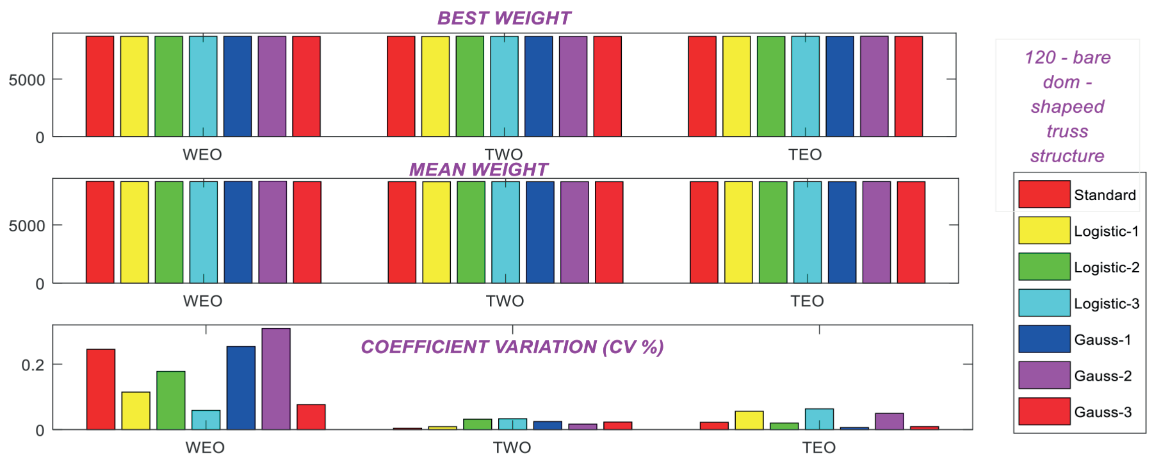

Fig. 7 Optimization results in standard mode and selection of the chaos map for the 120-bar spatial dome

Also, for quick access to optimization information, the bar chart of each component is displayed in Fig. 7. The analysis of the optimization results for different combinations of algorithms with chaos functions and comparing it with the standard mode shows a significant improvement in reducing the weight of the 120-bar spatial dome.

The results of each algorithm are: In the optimization algorithm based on Water Evaporation Optimization (WEO), the Gaussian chaos map with scenario 3 with a weight of 8711.427 kg has an optimal response. In the optimization algorithm based on the Tug-of-War Optimization (TWO), the Logistic chaos map with scenario 1 with a weight of 8708.525 kg has an optimal response. In the optimization algorithm based on Thermal Exchange Optimization (TEO), the Gaussian chaos map with scenario 1 with a weight of 8708.667 kg has an optimal response. In Table 4, all the results have been compared and among all the algorithms and chaos maps under investigation, the meta-heuristic algorithm based on the Tug-of-War Optimization (TWO), the Logistic chaos map with scenario 1 with a weight of 8708.525 kg has obtained the most optimal results. In this table, the details related to the cross section of the elements, the average weight and the coefficient of variation are provided. The diagram of the convergence history of the algorithms to compare the standard and chaotic mode is presented in Fig. 8.

5.3 A 200-bar planar truss structure

The dimensional specifications of the 200-bar planar truss structure are according to Fig. 9. This truss is a well-known benchmark problem regarding weight optimization with frequency limitation. In this truss, only the optimization of the cross-section of the members is considered and the geometric shape of the structure is constant during the optimization process. The decision variables related to the cross-sectional level of members are

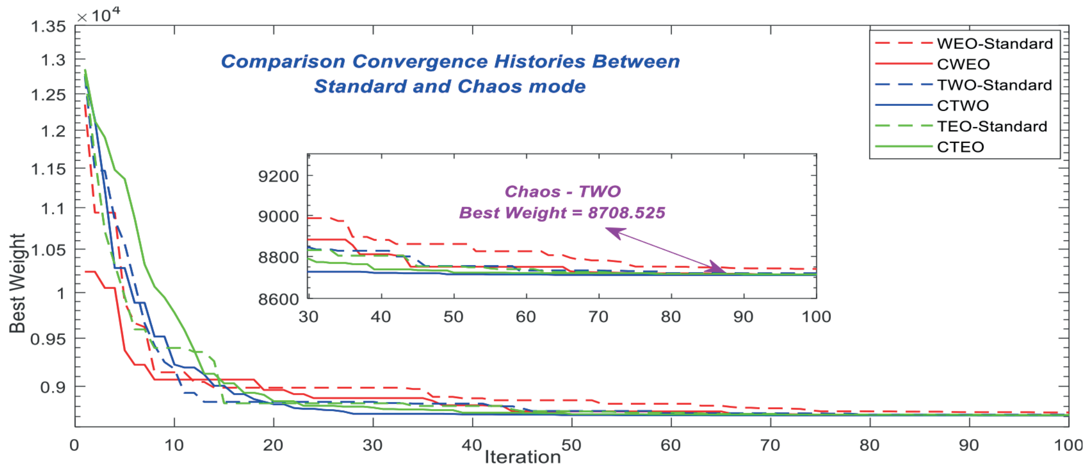

Fig. 8 The Convergence Histories for the 120-bar spatial dome

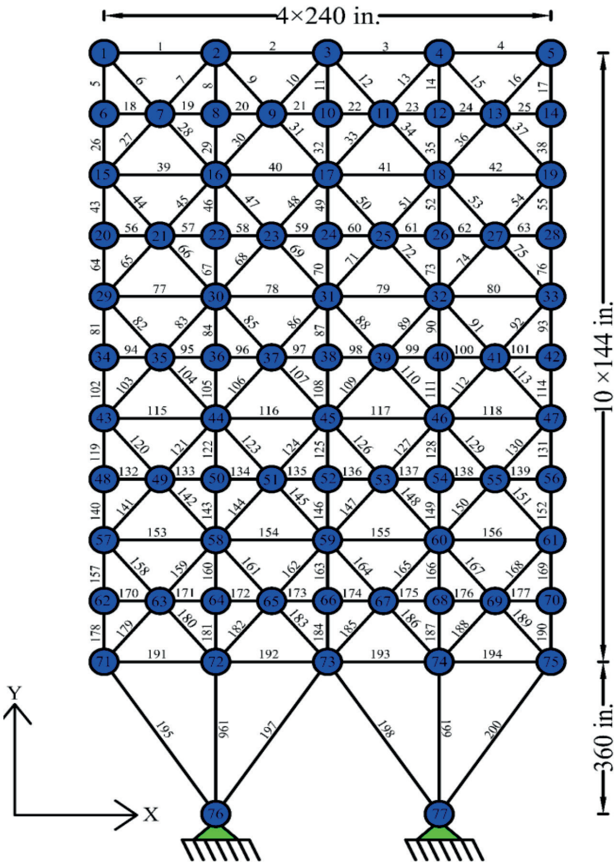

Fig. 9 Schematic of a 200-bar planar truss structure

classified into 29 groups. The lumped non-structural mass in nodes 1 to 5 and in the amount of 100 kg affect the structure. The mechanical specifications of the structure are: The density of the materials is 7860 kg/m^3, the modulus of elasticity is 210000 MPa, the frequency limits of the structure in the first, second and third modes are greater than 5, 10 and 15 Hz, respectively. For the cross-sectional area of the members, the lower bound is 0.1 cm^2 and the upper bound is 25 cm^2. In order to form statistical samples, 20 independent surveys were conducted and the results related to the best weight, average value and coefficient of variation are presented in statistical Table 5. In Table 6, a comparison has been made between the chaotic meta-heuristic algorithms and the details of the cross-sectional area of the members in the optimal state for the states that have high efficiency are presented as the final result of the optimization. Also, for quick access to optimization information, the bar chart of each component is displayed in Fig. 10.

Table 5 Statistical results for the 200- bar planar truss structure

Algorithms	Statistical Information	WEO Standard	CWEO-21 Logist-1	CWEO-22 Logist-2	CWEO-23 Logist-3	CWEO-31 Gauss-1	CWEO-32 Gauss-2	CWEO-33 Gauss-3
1-WEO	Best	2184.383	2173.945	2157.010	2156.468	2158.276	2156.557	**2150.206**
	Mean	2206.390	2200.830	2161.293	**2156.502**	2203.995	2157.002	2200.136
	C.V(%)	1.1906	1.1570	0.3451	**0.0012**	1.7182	0.0184	1.5348
Algorithms	Statistical Information	TWO Standard	CTWO-21 Logist-1	CTWO-22 Logist-2	CTWO-23 Logist-3	CTWO-31 Gauss-1	CTWO-32 Gauss-2	CTWO-33 Gauss-3
2-TWO	Best	2184.379	**2072.411**	2118.812	2089.843	2081.136	2112.734	2147.054
	Mean	2412.920	2306.277	**2273.443**	2306.992	2354.293	2373.989	2382.391
	C.V(%)	8.324	9.535	**5.331**	6.2002	7.541	7.675	11.079
Algorithms	Statistical Information	TEO Standard	CTEO-21 Logist-1	CTEO-22 Logist-2	CTEO-23 Logist-3	CTEO-31 Gauss-1	CTEO-32 Gauss-2	CTEO-33 Gauss-3
3-TEO	Best	2158.103	2139.600	2018.500	2157.699	**2005.036**	2147.228	2141.909
	Mean	2161.8002	2285.482	2411.276	**2160.289**	2179.738	2384.314	2317.831
	C.V(%)	0.1871	3.8990	13.5340	**0.1165**	5.0617	8.6760	4.6620

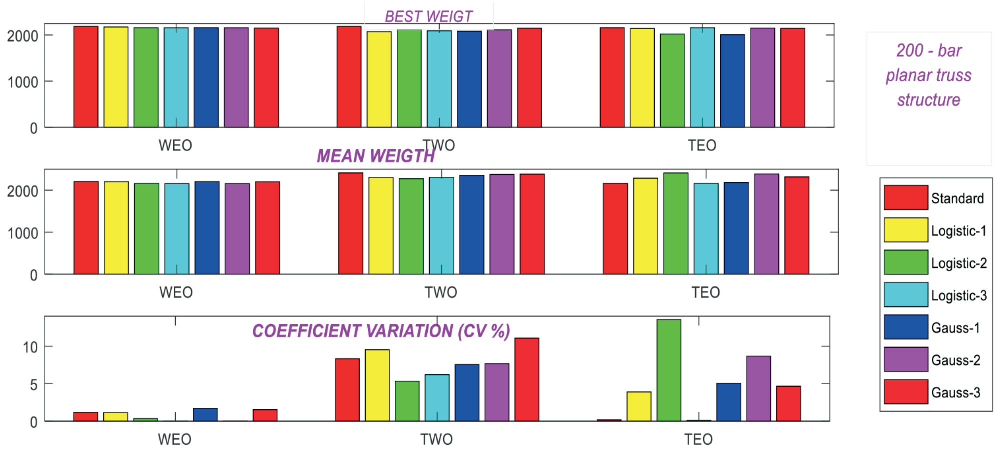

Fig. 10 Optimization results in standard and the chaos map for the 200-bar planar truss structure

Analyzing the optimization results for different combinations of algorithms with chaos functions and comparing it with the standard mode shows a significant improvement in reducing the weight of the 200-bar planar truss structure. The results of each algorithm are: In the optimization algorithm based on Water Evaporation Optimization (WEO), the Gaussian chaos map with scenario 3 with a weight of 2150.206 kg has an optimal response. In the optimization algorithm based on the Tug-of-War Optimization (TWO), the Logistic chaos map with scenario 1 with a weight of 2072.411 kg has an optimal response. In the optimization algorithm based on Thermal Exchange Optimization (TEO), the Gaussian chaos map with scenario 1 with a weight of 2005.036 kg has an optimal response.

In Table 6, all the results have been compared and among all the algorithms and chaos maps under investigation, the meta-heuristic algorithm based on Thermal Exchange Optimization (TEO) with Gaussian chaos map and scenario 1 and weight 2005.036 kg has obtained the most optimal results. In this table, the details related to the cross section of the elements, the average weight and the coefficient of variation are provided. The diagram of the convergence history of the algorithms to compare the standard and chaotic mode is presented in Fig. 11.

6 Discussions

In this research, samples have been selected from different groups of meta-heuristic algorithms and the reasons for the stagnation of algorithms in reaching overall optimality have been investigated. In general, for most meta-heuristic algorithms, the imbalance between the exploration and exploitation stage causes the algorithm to stop at local optima, and premature convergence occurs for them. In a number of algorithms, by applying the mutation stage, it is tried to escape from the trap of local optima and move towards global optima. Today, researches have shown that

Table 6 Optimal design comparison for 200- bar planar truss structure

Number group	WEO Stand	CWEO Gaus3	TWO Stand	CTWO Logis1	TEO Stand	CTEO Gaus1
A_1→1,2,3,4	0.342	0.390	0.320	0.699	0.316	0.322
A_2→5,8,11,14,17	0.476	0.394	0.555	0.562	0.438	0.485
A_3→19,20,21,22,23,24	0.164	0.125	0.100	0.100	0.100	0.100
A_4→18,25,56,63,94,101,132,139,170,177	0.154	0.111	0.100	0.100	0.100	0.100
A_5→26,29,32,35,38	0.456	0.714	0.595	0.609	0.514	0.564
A_6→6,7,9,10,12,13,15,16,27,28,30,31,33,34,36,37	0.861	0.766	0.770	0.725	0.821	0.724
A_7→39,40,41,42	0.187	0.127	0.100	0.120	0.100	0.100
A_8→43,46,49,52,55	1.174	1.709	1.400	1.378	1.430	3.019
A_9→57,58,59,60,61,62	0.177	0.464	0.100	0.100	0.100	1.639
A_{10}→64,67,70,73,76	1.571	1.489	1.539	1.448	1.593	2.139
A_{11}→44,45,47,48,50,51,53,54,65,66,68,69,71,72,74,75	1.322	1.206	1.034	0.978	1.140	0.995
A_{12}→77,78,79,80	0.152	0.433	0.104	0.100	0.130	0.100
A_{13}→81,84,87,90,93	2.903	2.892	2.945	2.819	2.979	2.799
A_{14}→95,96,97,98,99,100	0.298	0.177	0.100	0.100	0.101	1.291
A_{15}→102,105,108,111,114	2.801	3.155	3.713	6.776	3.244	2.890
A_{16}→82,83,85,86,88,89,91,92,103,104,106,107,109,110,112,113	1.491	1.500	1.357	1.282	1.599	1.309
A_{17}→115,116,117,118	0.401	0.349	0.198	0.100	0.297	0.379
A_{18}→119,122,125,128,131	5.492	4.378	5.403	4.291	5.144	6.262
A_{19}→133,134,135,136,137,138	0.154	0.497	0.240	1.541	0.102	3.517
A_{20}→140,143,146,149,152	5.422	5.754	4.375	5.994	5.610	4.776
A_{21}→120,121,123,124,126,127,129,130,141,142,144,145,147,148,150,151	2.010	2.071	1.812	1.789	2.081	1.815
A_{22}→153,154,155,156	0.736	0.423	0.434	0.955	0.799	0.387
A_{23}→157,160,163,166,169	8.089	8.895	10.57	11.646	7.716	6.454
A_{24}→171,172,173,174,175,176	0.429	1.545	4.299	3.383	0.166	0.398
A_{25}→178,181,184,187,190	8.400	8.038	7.403	10.357	7.791	7.632
A_{26}→158,159,161,162,164,165,167,168,179,180,182,183,185,186,188,189	3.073	2.833	2.787	3.144	2.827	2.455
A_{27}→191,192,193,194	9.275	9.879	10.51	5.245	9.980	7.176
A_{28}→195,197,198,200	20.928	20.115	16.26	13.585	20.96	17.18
A_{29}→196,199	12.133	10.466	21.49	17.626	11.77	12.35
Best Weight	2184.4	2150.2	2184.	2072.4	2158.	2005.
Mean Weight	2206.4	2200.1	2412.	2306.2	2161	2179
Coefficient Variation (cv)	1.191	1.534	8.324	9.535	0.187	5.062
NFE	26000	26000	31000	31000	33000	33000
ω_1 (HZ)	5.002	5	5	5	5	5
ω_2 (HZ)	12.645	13.187	12.58	14.69	12.31	12.81
ω_3 (HZ)	15.009	15.268	15	15	15.01	15

in most algorithms, the mutational cases of the algorithm itself are not effective and sufficient, and the application of chaos maps provides suitable conditions that escape from the trap of local optima is accelerated. These maps provide access to most positions of the search space by creating disorder in the search space. In this way, the global optimality will not have the opportunity to escape from the turmoil of chaotic functions.

It can be concluded that the most important role of chaos maps is to create a balance between the exploration and exploitation stage. By replacing these functions in the stages of exploration, exploitation or both, different scenarios for optimization are obtained. In this research, by applying chaos functions in several meta-heuristic algorithms, a significant improvement has been achieved in the process of weight and shape optimization of trusses. Also,

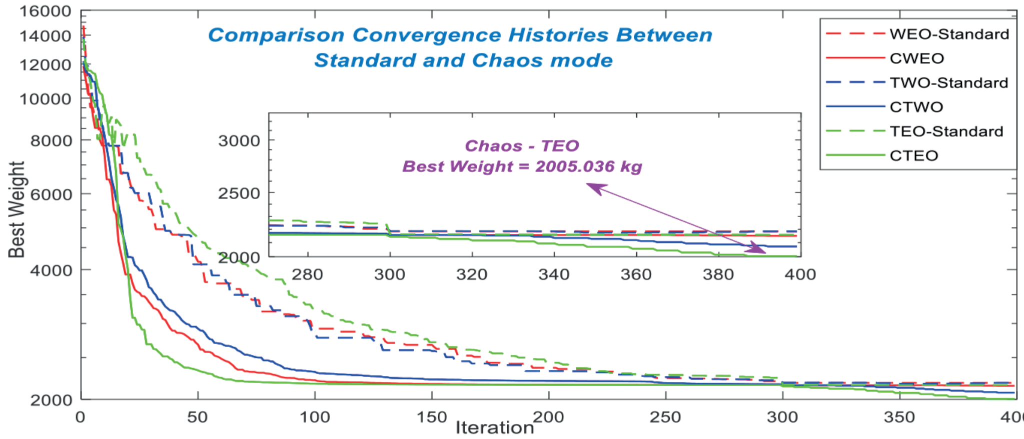

Fig. 11 The Convergence Histories for the 200-bar planar truss structure

in order to form statistical models and determine the best weight, the best average, and the best coefficient of variation, each structural model has been implemented with 20 independent repetitions. To extract the final results, the processes related to the previous tables are combined and then the results are normalized. The following relationship is intended to combine and summarize the information to contribute the results of the entire examples.

$$Val_{com}^{MV} = \frac{1}{S}\sum_{i=1}^{S}\left(\frac{Val^{MV}}{Val_{i,\min}} - 1\right) \tag{36}$$

Based on this relationship, the percentage of success of each of the 7 modes (standard mode along with 6 chaotic modes) is determined with the participation of all problems. With the participation of the results of all the examples in determining the efficiency of the standard mode and chaotic scenarios for each meta-heuristic algorithm, significant accuracy is achieved in introducing the most optimal scenario. In this regard, for each selected algorithm, Val^{Mv}, $Val_{i,\text{mi}}$n and Val_{com}^{Mv} respectively, the optimal values of the statistical tables for each example in each of the standard and chaotic modes, the lowest value among the 7 modes for the same example and the final result of the combination All the results are for the same mode of the algorithm. Also, i and S are the number of optimized structural examples and the number of examples, respectively. This relationship is formed for all three characteristics including optimum weight, optimum average and optimum variation coefficient. Up to this point, the criterion for determining the best efficiency is related to the modes that have the lowest value, and the results have not been normalized yet. Therefore, for the convenience of determining the optimal modes, two other cases are also considered, firstly, the highest percentages should be allocated for the most optimal results, and secondly, the results should be normalized. By considering the inverse functions and determining the contribution of each mode to the total modes, these two cases are provided. In Eq. (37), both cases are considered.

$$Val_{nor}^{MV} = \frac{\dfrac{1}{Val_{com}^{MV}}}{\sum_{j=1}^{nopt}\dfrac{1}{Val_{j,com}^{MV}}} \times 100 \tag{37}$$

In this regard, for each selected algorithm, Val_{com}^{Mv}, $Val_{j,com}^{Mv}$, *nopt* and Val_{nor}^{Mv}, respectively, the optimal values obtained for each of the 7 states from the previous relationship, the same values for applying summation, the total number of states, both standard and chaotic (This number here is 7) and finally, the optimal values are percentage and normalized.

By applying Eqs. (36) and (37), the final normalized results with the participation of all examples are obtained, and these results are presented in Table 7. Also, in order to quickly access the final results of the best weight, the best average and the best coefficient of variation, pie charts are a suitable option. With the formation of these charts, the following results have been obtained:

6.1 Results of optimal design for best weight

Based on the final results, the optimal design for determining the best weight in the water evaporation optimization belonging to the Gaussian chaos map with the third scenario, the algorithm based on tug-of-war optimization and the algorithm based thermal exchange optimization together, belonging to the Gaussian chaos function with the first scenario. The final results of the optimal design for introducing the best weight are displayed in Fig. 12.

6.2 Results of optimal design for best mean

Based on the final results, the optimal design for determining the best mean in the water evaporation optimization belonging to the logistic chaos function with the third scenario, the algorithm based on tug-of-war optimization belonging to the logistic chaos function with the second scenario and the algorithm based on thermal exchange optimization belonging to the logistic chaos function with the third scenario. The final results of the optimal design to introduce the best average is displayed in Fig. 13.

6.3 Results of optimal design for the best coefficient of variation

Based on the final results, the optimal design for determining the best coefficient of variation in the water evaporation optimization algorithm belongs to the logistic chaos function with the third scenario. The algorithm based on tug-of-war optimization algorithm belongs to the standard mode, and the algorithm based on thermal exchange optimization algorithm belongs to the logistic chaos function with the third scenario. The final results of the optimal design to introduce the best coefficient of variation are shown in Fig. 14.

7 Conclusions

Some of the considerable results in this research are as follows:

- In most cases, the combination of chaos functions with meta-heuristic algorithms have made a significant improvement compared to the standard mode. The main factor can be the effect of chaos functions in escaping from local optima and preventing premature convergence.
- In optimization problems based on frequency limitation and shape variables, chaos functions have caused significant improvement. Comparing the results of chaos functions with the standard value confirms this.
- In scenario 1 and 2, chaos functions have replaced the exploration and exploitation steps, respectively. Based on this, algorithms can be classified into two groups.
- The first group includes algorithms whose exploration phase is improved by applying chaos functions. Algorithms based on Tug-of-War Optimization (TWO) and Thermal Exchange Optimization (TEO) are from this group, and by applying chaos functions in the search phase, a significant improvement in the optimization results is achieved.

Table 7 Final normalized values with the participation of all the examples

Category	Algorithms	Standard	Logistic 21	Logistic 22	Logistic 23	Gauss 31	Gauss 32	Gauss 33
	WEO	5.001	6.918	8.484	9.082	12.49	8.736	**49.293**
Best Weight	TWO	3.176	24.362	8.685	10.392	**40.02**	7.147	6.217
	TEO	4.675	7.295	32.007	6.858	**34.21**	7.391	7.563
	WEO	8.388	15.067	14.555	**23.971**	13.49	12.657	11.868
Mean Weight	TWO	3.647	10.633	**52.155**	14.163	9.325	6.101	3.976
	TEO	7.735	7.048	3.531	**56.794**	14.94	4.154	5.793
	WEO	0.733	0.754	2.442	**71.159**	0.504	23.852	0.555
(CV %)	TWO	**24.155**	9.751	14.083	20.625	7.689	18.529	5.167
	TEO	18.938	10.675	4.617	**31.575**	13.71	6.609	13.871

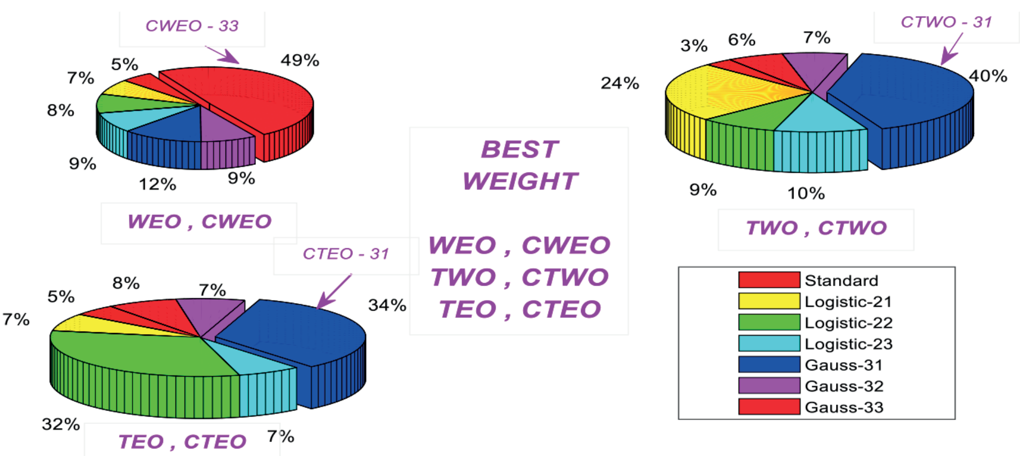

Fig. 12 The final results of the optimal design to determine the best weight

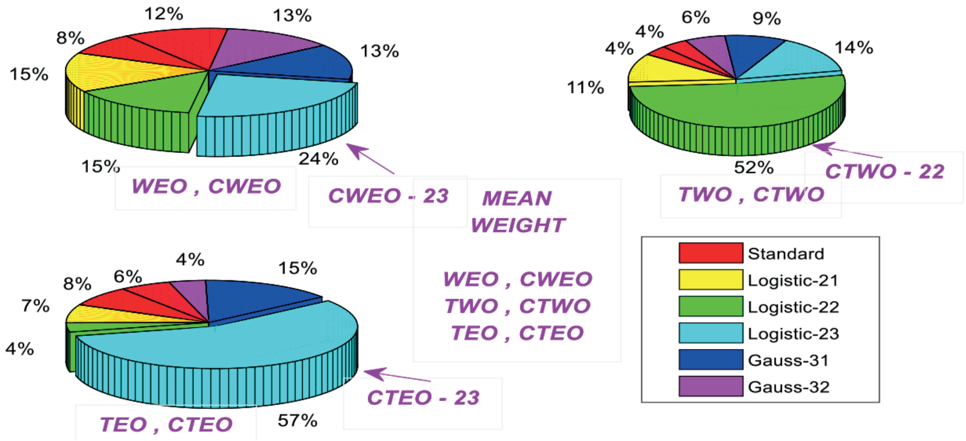

Fig. 13 The final results of the optimal design to determine the best mean

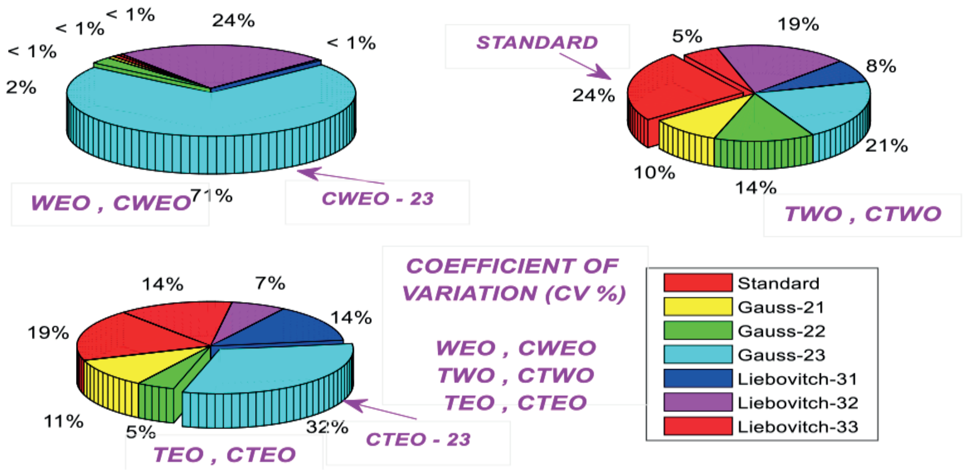

Fig. 14 The final results of the optimal design to determine the best coefficient of variation

- The second group includes algorithms that have good exploration and exploitation conditions in the standard mode, but there is no balance between them, which can be achieved by applying chaos functions in both stages simultaneously. In this research, Water Evaporation Optimization (WEO) is from the second group.
- By using chaos functions, determining the regulatory parameters of algorithms and sensitivity analysis is significantly improved. In fact, choosing the starting sentence in chaos functions replaces complex settings. It should be noted that in most cases, finding suitable adjustment parameters for each algorithm is more difficult than the optimization itself. Therefore, by using chaos functions, complex engineering problems such as shape optimization can be solved without the need to find parameters.
- In order to check the conditions of stability and reliability of the answers, the coefficient of variation, which is the dimensionless state of the standard deviation, has been used. In standard modes, the answers are uniform and have few changes, but in chaotic modes, the main factor to improve the results is to create sudden jumps to escape from local optima and move towards the global optimum, with these jumps, the diversity of the search space and the range of changes will increase. Therefore, in most chaotic situations, the improvement of the results is accompanied by an increase in the coefficient of variation and the average.
- In order to choose the initial sentence in the series of chaos functions, before the main iterations, performing several initial iterations and choosing the appropriate initial sentence improves the results by leaps and bounds.
- Among the chaos functions, the Gaussian function with scenario 1 has provided the most improvement in the optimization results.
- Finally, it might be interesting to mention that the force method of structural analysis can be used in place of the displacement method with great benefits for structures with smaller degrees of indeterminacy than the kinematical indeterminacy [34–37].

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