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Fuzzy Structural Analysis Using Improved Jaya-based Optimization Approach

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Abstract

A new approach to performing the α-level optimization in the fuzzy analysis of structural systems is developed in this study. The method uses a simple global optimizer, the Jaya algorithm, together with an innovative dimension reduction technique. The dimension reduction technique aims to transform the original large α-level optimization problem into a low-dimension one by making use of the monotonic behavior of the system output with respect to the input variables. Then, the Jaya algorithm is applied to solve the reduced max/min α-level optimization problems to determine the bounds of the fuzzy output. Two numerical examples, including a 2D truss and a 3D truss, with a relatively large number of fuzzy input variables are analyzed and the fuzzy displacements under static loads are predicted. It is demonstrated that the proposed approach can save a significant computational amount and also estimate the fuzzy displacement with high accuracy.

Keywords

fuzzy structural analysis, Jaya algorithm, α-level optimization

1 Introduction

The structural analysis should be based on information about the structure, the effects on the structure, etc. In practice, this information often contains randomness, uncertainty, and inaccuracy (uncertain information). Besides probabilistic methods based on uncertain information modeled as random quantities with given distribution functions, the non-probabilistic approach for analysis and evaluation of structural systems by fuzzy model [1] has also attracted many studies [2, 3].

In structural analysis by fuzzy model, the α-cut method is often applied, in which all input fuzzy variables are discrete according to some similar membership levels (the α-cuts). Corresponding to each α-cut of the input variables, the interval of the fuzzy response (or the α-cut of the fuzzy output) is determined through the interval analysis. To determine the interval of the fuzzy response, two main approaches are commonly used: 1) interval arithmetic and 2) optimization. In particular, the optimization approach gives accurate results (in theory), is convenient in implementation and can be combined with available computational tools, for example, finite element analysis programs [2]. Some early studies on fuzzy structural analysis according to the optimization approach can be mentioned such as Möller et al. [4] with Modified Evolutionary Algorithm, Degrauwe et al. [5] with the gradual α-level decreasing(GαD) optimization algorithm, Farkas et al. [6] with reduced global optimization method.

In [4], the modified evolution strategy combines Monte Carlo Method, gradient search, and evolutionary algorithm. The algorithm allows us to find the global optimal solution, regardless of the type or behavior of the objective function. One disadvantage of the algorithm is that its structure is quite complex.

The GαD developed in [5] also allows for finding a globally optimal solution without using special characteristics of the objective function. The algorithm searches from the highest α level and gradually moves down to the lower α levels. At each level α, the extreme of the objective function is searched in the vicinity of the extremes defined at the previous α level. Depending on the problem, the algorithm can lead to different levels of computational complexity. In the case of multiple local extremes (multimodal problem), GαD will generate many search directions which increase the computational cost.

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The reduced global optimization approach [6] based on the monotonic behavior of the output with some input variable can reduce the number of function evaluations compared to conventional optimization. This method works on the principle of "blocking" the input variables at the lower/upper bound values if they have a monotonic effect on the output. Just like in the Modified Evolutionary Algorithm in [4], the analysis results at a level α will be saved so that they can be reused if needed when performing optimization at a lower α level, avoiding the need for re-analysis. However, to identify whose input variables can be "locked", the algorithm requires determining the objective function at 3^n samples (n is the number of input variables) before performing the optimization. Thus, the computational volume increases exponentially with the number of fuzzy variables. In addition, the use of 3^n samples which is a combination of the values of the lower bound, the upper bound, and the midpoint also does not completely guarantee the accurate determination of the monotony of the objective function according to the input variable.

A new strategy for fuzzy structural analysis is the combination of the α-cut method with a metaheuristic algorithm [7–9]. Thanks to the advantage of metaheuristics, this approach can be suitable for many different types of problems, regardless of the behavior or type of the objective function, and has high reliability (the ability to find the exact bounds of the fuzzy response). In addition, metaheuristic algorithms are quite easy to implement and combine with existing analysis tools [10]. The major obstacle is that metaheuristics often require a large volume of computation due to the huge number of function evaluations. Some techniques have been applied to reduce the number of function evaluations compared to using the traditional corresponding metaheuristics, such as the α-level subspace technique and the nearest neighbor comparison techniques in [7], reuse of samples and information within the sublevels in [9]. However, the amount of computation to be performed is still large [11–13]. Furthermore, expert knowledge of setting algorithm-specific parameters is often required for metaheuristics.

There are various metaheuristic algorithms available in the literature, such as those in [14]. In this study, Jaya algorithm [15], an algorithm-specific parameter-less algorithm is considered. Due to its simplicity and effectiveness, the algorithm has been widespread among the optimization research community [16, 17]. There have been successful applications of the Jaya algorithm and its variants to solve different problems in engineering, such as energy [18, 19], electrical engineering [20], fracture mechanics [21, 22], structural engineering [23-29], and environmental engineering [30, 31].

Recently Jaya has been applied successfully by the author for determining fuzzy displacements of a 2D truss structure [32]. In [32], Jaya was integrated with the α-level subspace technique and the nearest neighbor comparison techniques in [7] to reduce the number of structural analyses in solving α-level optimization problems.

In this paper, the Jaya algorithm is integrated with a dimension reduction technique to significantly save the computational cost, while ensuring the accuracy of the obtained fuzzy response of a structure. The effectiveness of the proposed Jaya-algorithm-based fuzzy procedure is investigated by the fuzzy static analysis of a planar truss and a space truss involving a relatively large number of fuzzy inputs.

The remaining part of the paper is organized as follows. Section 2 presents the proposed methodology for fuzzy structural analysis, with an emphasis on a dimension reduction technique. Section 3 gives a brief description of Jaya algorithm. Section 4 shows the numerical results and discussions of the case studies. Finally, Section 5 provides some conclusion remarks.

2 Fuzzy structural analysis

2.1 The α-level approach

According to the α-level optimization method [4], the bounds of the α-cut of the fuzzy output , are determined by solving two optimization problems:

$$\underline{y}_\alpha = \min_{x_i \in X_{i,\alpha}} f\left(x_1, x_2, \ldots, x_n\right), \tag{1}$$

$$\overline{y}_\alpha = \max_{x_i \in X_{i,\alpha}} f\left(x_1, x_2, \ldots, x_n\right), \tag{2}$$

where $x_i, i = 1 \ldots n$, are n fuzzy input variables; $X_{i,\alpha}$ is the α-cut of the fuzzy variable x_i; $\underline{y}_\alpha$ and $\overline{y}_\alpha$ are the lower and upper bounds of the α-cut of the fuzzy output y, respectively.

In theory, the α-level optimization method gives accurate results but often requires a large amount of computation because of many function evaluations (model analyses). Researches to overcome this limitation can be divided into two ways: 1) Using a simpler alternative model (The most commonly used method is the response surface method), and 2) Reducing the number of model analyses required when performing the optimization. The advantage of the former method is that there is no need to perform analyses on the original complex model. However, the accuracy of this strategy completely depends on the accuracy of the replacement model. For the latter way, researches focus on building suitable optimal algorithms, with the requirement of reducing the number of model analyses while ensuring the accuracy of the optimal results.

2.2 Dimension reduction strategy

One way to lower the number of function evaluations in optimization, especially when applying metaheuristics, is to reduce the dimension of the problem by "locking" the input variables at the lower/upper bound values if they have a monotonic effect on the output. In this study, we propose an effective way to reduce the problem dimension when performing the α-level optimization in fuzzy structural analysis. The method uses partial derivatives to identify the monotony of the output. The strategy is presented as follows.

Step 0: At the nominal value of the fuzzy input (value with $\alpha = 1$), evaluate the output and its partial derivatives. The partial derivative of the output is approximated by the central difference method as follows:

$$f_i' = \frac{f\left(x_i + \delta x_i\right) - f\left(x_i - \delta x_i\right)}{2\delta x_i}, \tag{3}$$

where δx_i is the variation of the fuzzy variable x_i, taken as $0.001*x_i$. Then, at each level α:

Step 1: Let the interval (α-cut) $X_{i,\alpha}$ of x_i corresponding to the level α is bounded by the lower bound $\underline{x}_{i,\alpha}$ and the upper bound $\overline{x}_{i,\alpha}$. Two input value sets are determined as follows:

- Value set for the lower bound, $\underline{C}_\alpha$: if f_i' at the lower-extreme point of the previous α-level is greater than zero, $\underline{x}_{i,\alpha}$ is included in $\underline{C}_\alpha$, otherwise $\overline{x}_{i,\alpha}$ is used.

- Value set for the upper bound, $\overline{C}_\alpha$: if f_i' at the upper-extreme point of the previous α-level is greater than zero, $\underline{x}_{i,\alpha}$ is included in $\underline{C}_\alpha$, otherwise $\underline{x}_{i,\alpha}$ is used.

Calculate the new partial derivatives at $\underline{C}_\alpha$ and $\overline{C}_\alpha$, i.e., $f_i'(\underline{C}_\alpha)$ and $f_i'(\overline{C}_\alpha)$ by using Eq. (2).

Step 2: Compare the signs of the new partial derivatives calculated at Step 1 with the signs of the corresponding partial derivatives at the previous α-level.

For all the partial derivatives f_j' having a different sign, the output is non-monotonic with respect to x_j. Then, the α-level optimization will be performed with respect to x_j to find the extrema (lower bound or upper bound) of the output. The other variables are locked by the values in $\underline{C}_\alpha$ and $\overline{C}_\alpha$.

By locking all the variables having a monotonic effect on the output, the dimension of the α-level optimization problem to be solved is reduced. Thus, the optimization process can converse faster.

It is noted that if there is no difference in signs of the new partial derivatives, the output behaves monotonically, $\underline{C}_\alpha$ and $\overline{C}_\alpha$ will give the actual extrema for the current α-level, and there is no need to perform optimization.

Step 3: Compare the extrema obtained at this α-level by with those of the previous α-level to assure global optima. Move to the lower α-level and repeat Step 1.

3 Jaya algorithm

The Jaya algorithm is a parameter-less meta-heuristic algorithm and has a very simple structure. The briefs of the algorithm are presented below.

Initially, a population of NP solution candidates, $\boldsymbol{x}_p, p = 1...NP$, is randomly generated from the search space (the reduced search space in this context). Then, each candidate in the population will be updated in the optimization process through a survival selection based on the objective function value.

First, for each candidate, a new variant $\boldsymbol{x}_p^{new}$ is created through the mutation as follows:

$$\boldsymbol{x}_p^{new} = \boldsymbol{x}_p + \boldsymbol{r}_1 \cdot \left(\boldsymbol{x}_{best} - \left|\boldsymbol{x}_p\right|\right) + \boldsymbol{r}_1 \cdot \left(\left|\boldsymbol{x}_p\right| - \boldsymbol{x}_{worst}\right), \tag{4}$$

in which $\boldsymbol{x}_{best}$ and $\boldsymbol{x}_{worst}$ are the best and the worst candidates in the current population, respectively; $\boldsymbol{x}_p$ is the p-th candidate; $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are two vectors of uniformly distributed random numbers in the interval [0, 1].

The new alternative $\boldsymbol{x}_p^{new}$ is compared with the old one $\boldsymbol{x}_p$, and it will replace $\boldsymbol{x}_p$ in the population if its objective function value is better than that of $\boldsymbol{x}_p$. The entire population will be updated through many iterations until a stopping condition is satisfied. The best solution in the final population will be selected as the optimal solution for the problem.

In this study, the Jaya algorithm is used to solve the reduced α-level optimization at Step 2 (if required). The outcomes of Jaya optimization are the lower and upper bounds of the α-cut of the fuzzy output.

4 Numerical application

To demonstrate the effectiveness of the proposed fuzzy structural analysis procedure, two truss structures with fuzzy parameters are examined in this section. Jaya algorithm is utilized to find the extreme values of the structural response at each α-level in both cases: with and without the dimension reduction (DR) strategy. The parameter setting for Jaya is given in Table 1. The search process will stop when the relative error, defined as $e = f_{mean}/f_{best} - 1|$, where f_{mean} is the average value of the objective functions in the current population, and f_{best} is the smallest objective function value, reaches 10^{-6}.

Table 1 Parameter setting for Jaya

	w/o DR	with DR
Population size (NP)	50	20
Max. iteration (Tmax)	300	300
Relative error	10^{-6}	10^{-6}

The displacement and internal force are determined by the finite element method. All codes for the numerical analysis are implemented in MATLAB by the authors.

4.1 2D truss

The truss is adapted from [33]. The structure consists of 31 members as depicted in Fig. 1. The fuzzy parameters include the Young's modulus of materials E_i (i = 1,…, 31), the cross-section areas A_i (i = 1,…, 31) and the applied load P_j (j = 1,…, 5), whose membership functions are shown in Fig. 2, respectively. Therefore, the total number of fuzzy parameters is 67. The displacements at node 8 are considered, including horizontal displacement u_8 and vertical displacement v_8.

Fig. 3 illustrates the membership functions of u_8 obtained by different methods, including the direct optimization using Jaya (JAYA), the reduced optimization using Jaya (R-JAYA), Taylor's approximation-based method (TAM) [11, 12], and Taylor's expansion with extrema management (TEEM) [34]. It is seen that the result of R-JAYA is the same as that of JAYA. That means R-JAYA can estimate the fuzzy displacement with high accuracy. The advantage of R-JAYA in comparison with JAYA is that it requires much fewer function evaluations. In this case, R-JAYA needs only 2030 structural analyses, while JAYA calls 55001 analyses, i.e., R-JAYA saves more than 96% computational cost. On the other hand, methods like TAM and TEEM are approximation methods that can somewhat predict the behavior of the structural response. The advantage of TAM and TEEM is their computation efficiency (in this example, TAM uses 141 and TEEM uses 1449 structural analyses). However, they cannot capture the "true" fuzzy membership function as shown in Fig. 3.

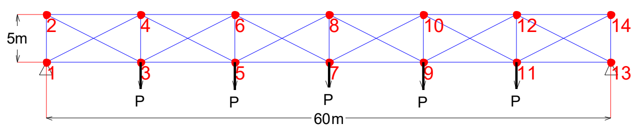

Fig. 1 The layout of 31-bar planar truss

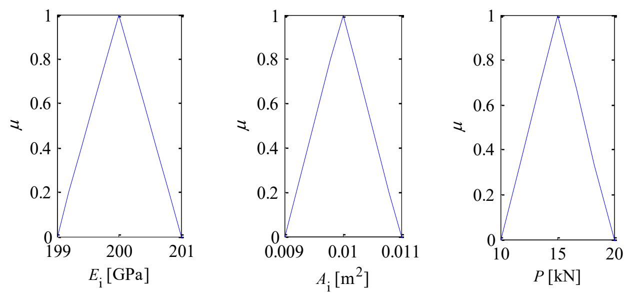

Fig. 2 Membership functions of modulus of elasticity, cross-section area, and load

In Fig. 4, the membership functions of v_8 are shown. In this case, all methods TAM, TEEM, JAYA, and R-JAYA give similar results. In this case, v_8 is monotonic with respect to the fuzzy parameters. The numbers of structural analyses required by JAYA, R-JAYA, TEEM, and TAM are 43701, 945, 945, and 141, respectively.

The extreme values, including the lower bound (LB) and the upper bound (UB), of the fuzzy displacements obtained by TAM, TEEM, JAYA, and R-JAYA are shown in Table 2. The results for u_8 by R-JAYA are the same as those by JAYA, and better than those by TAM and TEEM.

4.2 3D truss

The second example is a space truss structure as shown in Fig. 5. The structural model consists of 160 bar elements. The fuzzy parameters include the elastic modulus of the

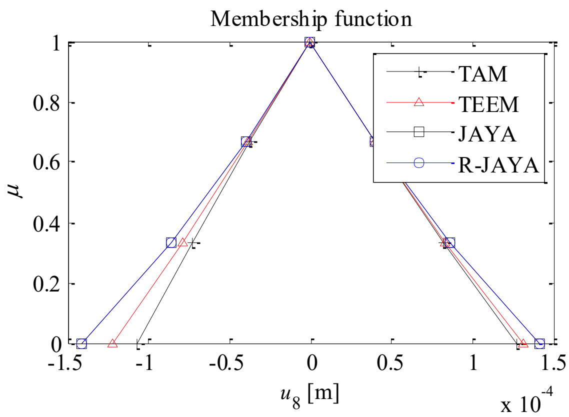

Fig. 3 Membership functions of u_8 by different methods

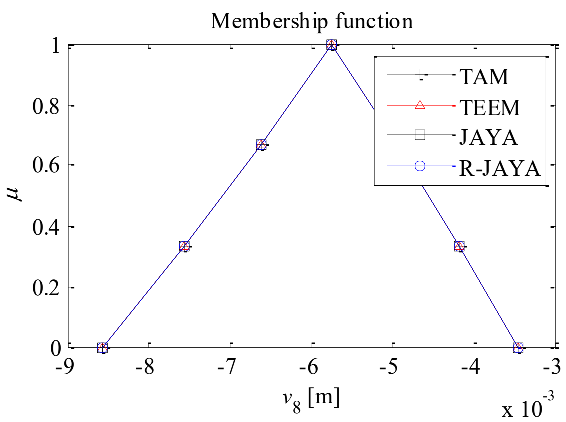

Fig. 4 Membership functions of v_8 by different methods

Table 2 The extreme values of fuzzy displacements of node 8 for the 31-bar truss obtained by different methods

Output		TAM	TEEM	JAYA	R-JAYA
u_8 (mm)	LB	-0.1072	-0.1219	-0.1413	**-0.1413**
	UB	0.1271	0.1319	0.1413	**0.1413**
v_8 (mm)	LB	-8.5664	-8.5664	-8.5664	**-8.5664**
	UB	-3.4618	-3.4618	-3.4618	**-3.4618**

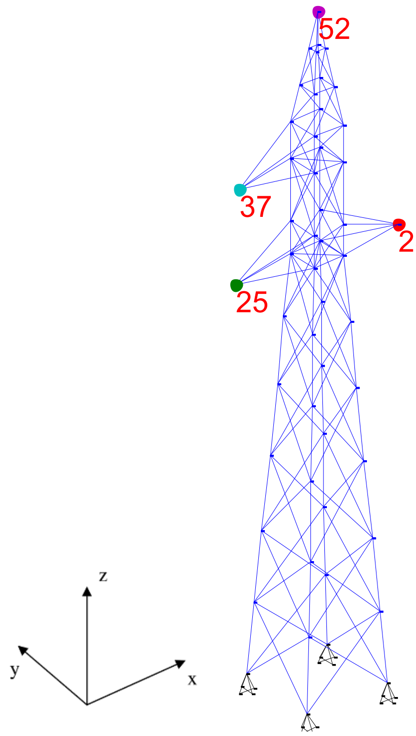

Fig. 5 The layout of 3D truss structure

material of each bar, the cross-sectional areas of the bars, and the loads acting at nodes 25, 28, 37, and 52. Assume the maximum variation of the elastic modulus ±0.5%, the dimension ±5%, and the load ±10%, from the confidence (nominal) values. All fuzzy variables are assumed to have triangular membership functions with the nominal data given in Table 3. Thus, the problem has a total of 328 fuzzy variables and can be considered a large-scale problem.

By using JAYA, R-JAYA, TAM, and TEEM, the membership functions of the displacements at node 52 are obtained as shown in Figs. 6, 7, and 8. The results obtained by R-JAYA and JAYA are quite similar and enclose those of TAM and TEEM.

Table 4 gives the extreme values of the fuzzy displacements at node 52 according to TAM, TEEM, JAYA, and R-JAYA. The results show that the bounds obtained by R-JAYA are wider than those determined by the other methods. That means R-JAYA can capture the non-monotonic behavior of the displacement responses effectively.

Table 3 Nominal values of the fuzzy parameters for the 3D truss

	Parameters	Nominal value
Elastic modulus	E_i (kgf/cm^2)	2.047e+6
Cross-section	Ai (cm^2)	10.0
Node 25	*Fx* (kgf)	-1091
	Fz (kgf)	-546
Node 28	*Fx* (kgf)	-1091
	Fz (kgf)	-546
Node 37	*Fx* (kgf)	-996
	Fz (kgf)	-546
Node 52	*Fx* (kgf)	-868
	Fz (kgf)	-491

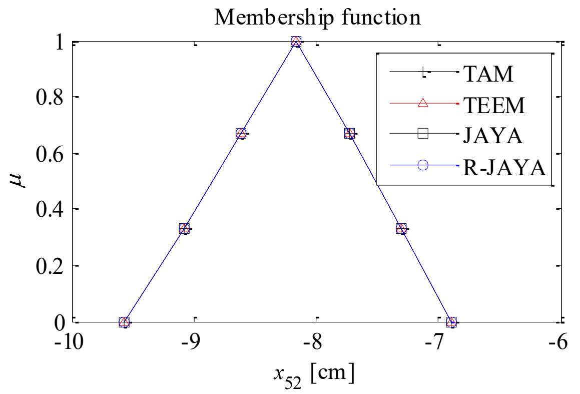

Fig. 6 Membership functions of x_{52} by different methods

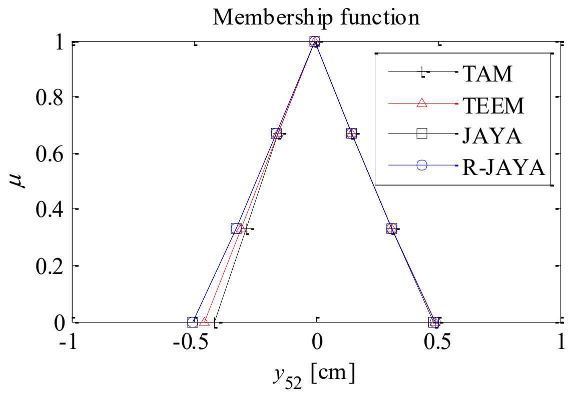

Fig. 7 Membership functions of y_{52} by different methods

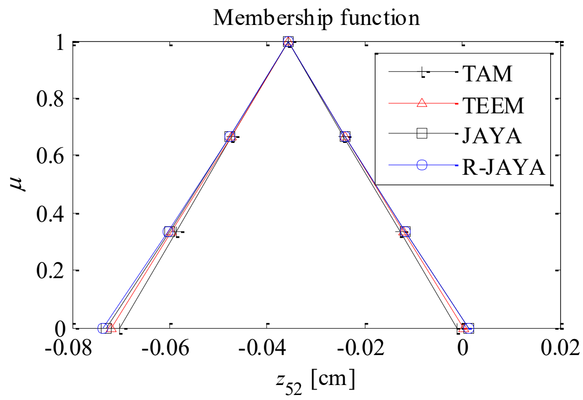

Fig. 8 Membership functions of z_{52} by different methods

Table 4 The extreme values of fuzzy displacements of node 52 for the 160-bar truss obtained by different methods

Output		TAM	TEEM	JAYA	R-JAYA
x_{52}(cm)	LB	-9.5713	-9.5713	-9.5685	**-9.5713**
	UB	-6.8928	-6.8928	-6.8971	**-6.8928**
	FEs	663	7744	82751	8646
y_{52} (cm)	LB	-0.4119	-0.4582	-0.5013	**-0.5044**
	UB	0.4873	0.4889	0.4848	**0.4905**
	FEs	663	8213	90301	9441
z_{52} (cm)	LB	-0.0702	-0.0718	-0.0731	**-0.0735**
	UB	-0.0010	0.0002	0.0013	**0.0014**
	FEs	663	7690	90301	8804

The results of TAM and TEEM are worse than those of R-JAYA. It is noted that JAYA can not find the bounds of R-JAYA within the given iteration (300 iterations). This is due to the large-scale problem considered.

In terms of computational efficiency, Table 4 also lists the number of function evaluations (FEs) called by these methods. Compared with JAYA, R-JAYA is much more efficient since R-JAYA reduces up to 90% computational cost. TAM is the most efficient method which uses 663 analyses. However, TAM cannot capture the non-monotonic behavior of the displacements.

From the two numerical examples above, it is deduced that the proposed strategy can save significant computational costs in solving the α-level optimization by Jaya. The accuracy of the obtained fuzzy outputs is maintained well by the reduced α-level optimization.

5 Conclusion

An efficient approach for fuzzy structural analysis has been presented. The method is based on the α-cut strategy and the Jaya algorithm. Instead of solving α-level optimization problems directly, Jaya has been suggested to combine with the dimension reduction strategy to reduce the computational volume. The technique can "freeze" those parameters having a monotonic effect on the output so that it transforms the original large α-level optimization problem into a low-dimension one. Numerical examples with a relatively large number of fuzzy variables have shown that the proposed methodology can capture the membership function of non-linear structural response with high accuracy. The Jaya when combined with the proposed dimension reduction technique saved significant computational volume compared with Jaya in the case of direct optimization. The Jaya-based optimization method does not require prior knowledge of the problem as well as the user's setting for algorithm-specific parameters. Therefore, the proposed method can ease the practice of fuzzy analysis of structures with non-monotonic behavior and many fuzzy variables.

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