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Multi-objective Colliding Bodies Optimization Algorithm for the Obnoxious *p*-median Problems

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Abstract

The obnoxious *p*-median problem consists of locating *p* facilities among a set of sites such that the sum distance from any demand to its nearest facility and the dispersion among facilities are maximized. In this paper, the multi-objective colliding bodies optimization algorithm (MOCBO) is utilized to obtain the trade-off curve of the obnoxious *p*-median problems. The performance of the developed optimization method is investigated for locating obnoxious facilities through two case studies to maximize the two conflicting objectives. The performance of the MOCBO algorithm is further compared with those of the MPSO and NSGA-II algorithms representative of the state of the art in the field of multi-objective optimization. In this study, the MOCBO algorithm showed suitable convergence performance and generalization abilities compared to the MPSO and NSGA-II algorithms.

Keywords

optimal locating, obnoxious *p*-median, colliding bodies optimization, graph methods, multi-objective algorithms

1 Introduction

Locating facilities is a challenging optimization problem and the p-median is probably one of the most tackled optimization problem. The objective of the p-median problem is to select p locations out of the potential facility locations as medians and assign each demand point to its closest median in a way to minimize the total cost or the total distances. The total cost is calculated as the sum of the distances or costs between each demand point and its corresponding median [1–4].

The obnoxious p-median problem (OpM) is a well-known optimization problem in facility location theory that specifically addresses hazardous locations. It involves selecting the best locations for facilities given certain criteria in order to minimize the negative impact on the surrounding areas. In hazardous location scenarios the obnoxious p-median problem aims to maximize the sum of the minimum distance between each client and their nearest facility and, on the other hand, the dispersion among facilities. Both objective values should be kept as large as possible for a convenient location of dangerous facilities [5].

Some algorithms have been developed for the OpM to determine a set of opened facilities such that the sum of distances between each client and the opened facilities (out-transmission) are maximized [6–9]. However, the problem is computationally challenging because it is known to be *NP*-hard. Therefore, various mathematical models and algorithms have been proposed to tackle this problem [10]. Some studies have modeled the problem as bi-objective p-median problems, where the out-transmission and the facility dispersion are varied. These typically involve formulating the problem as the heuristic optimization algorithms [10]. At present, there are many multi-objective algorithms are available, most of which are developed based on recent meta-heuristic optimization techniques [11–14]. These meta-heuristic optimizers can provide the decision maker with a various set of best trade-off scheduling plans, termed as Pareto optimal solutions.

The formulations of this paper expand upon the obnoxious p-median problem with discrete point facility site locations and demands. In this paper, an algorithm based on the

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p-median concept is presented for finding optimal location of the obnoxious facilities using the MOCBO algorithm. Computer programs have been developed to perform this method and two numerical examples are presented to illustrate the application/efficiency of the proposed method.

2 Problem formulations

The aim of the obnoxious p-median problem is to find a node set, N, into p node, $N_p \in N$, such that the sum of the distances of nodes to the median nodes and the dispersion among facilities is maximized. The problem of obnoxious p-median can be stated as optimizing two objective functions which decomposes the domain G into p subdomain $G_1, G_2, \ldots, G_p$, where p is the number of subdomains. The first objective function which must be maximized is formulated as:

$$f_1\left(N_p\right) = \sum_{j \in N} d\left(N_p, j\right) \tag{1}$$

where $f_1(N_p)$ is called the out-transmission of nodes N_p, N_p is the median (or facility) node number and $d(N_p,j)$ is defined as:

$$d\left(N_p, j\right) = \min\left\{d\left(i', j\right)\right\} : \left(i' \in N_p\right) \tag{2}$$

Let i' be the node of N_p which produces the minimum in Eq. (2), then we say the node j is allocated to i'. A SRT has been rooted from nodes for obtain the shortest distance between nodes [9].

The second objective function, dispersion of the facilities, is computed as the sum of the minimum distances from each facility to the rest of the facilities in N_p. In order to find the nodes number of medians of graph, the coordinates of medians are considered as the variables of the optimization problem. Then, the nearest nodes from this coordinate are selected as the medians of the graph. Otherwise, if we consider the node number as the optimization variable; firstly, the number of meshes in finite element are high and therefore the search space become very large, secondly the optimization process should be proposed as discreet variable problem. Therefore, in this work, the proposed optimization algorithm is considered as continues variables [6].

3 The proposed multi-objective algorithm

Section 3 introduces a MOO algorithm, which is a single solution search method [11]. The proposed algorithm (Algorithm 1 [11, 15, 16]) is named as "multi-objective colliding bodies optimization (MOCBO)" because of utilizing the formulation of the CBO for the search process of the algorithm. As previously discussed, the multi-objective algorithms aim to achieve two main goals:

Algorithm 1 Multi-objective colliding bodies optimization (MOCBO)

Step 1. Similar to the meta-heuristic algorithms, the initial positions of populations are calculated with random initialization in the search space:

$$X_i^0 = X_{\min} + R\left(X_{\max} - X_{\min}\right), \; i = 1, 2, \ldots, N$$

where X_i^0 determines the initial value vector of the i^{th} population; $X_{\min}$ and $X_{\max}$ are respectively the minimum and maximum allowable values vectors of variables; R is a random vector in the interval [0, 1] from a uniform distribution; and N is the number of populations.

Step 2. For all populations $X_i = 1, 2, \ldots, N$, the objective functions $\{f_1(X_i), f_2(X_i), \ldots, f_M(X_i)\}$ are calculated.

Step 3. An empty external archive is first defined and then it is updated in each iteration. This update consists of inserting all the currently non-dominated solutions into the archive and the dominated solutions from the archive are eliminated. Since the size of the archive is limited, one should apply a secondary mechanism for keeping this limit: first, the objective function space is divided into grids (hypercubes), *ngrid*. Then, the non-dominated solutions to their corresponding hypercube are located according to their objective function values. Afterwards, hypercubes that contain more than one population, the nearest population to grids is kept and the remining populations are eliminated.

Step 4. The maximin value of each solution is computed as introduced by [15].

Step 5. The arrangement of the populations is performed in ascending order based on the maximin values. Hence, if a population has the smallest maximin value, it is assigned as the best rank. This ranking approach is beneficial to the convergence and sparsity of algorithm.

Step 6. In this step, first the sorted populations are equally divided into two groups:
1. The lower half of sorted populations (stationary group),
2. The upper half of sorted populations (moving group).

With regard to ranking the populations based on maximin value, the stationary group is close to the true Pareto front as well as located in sparse regions of the resulting Pareto front than the moving group. Then, the populations of moving group are move toward to stationary group and mating and collision process takes place.

Step 7. After the collision, the velocities of populations in each group are calculated as explained in the CBO algorithm with different definition of the population mass [11, 16].

Step 8. After calculating the new velocities of populations, the new positions of the populations are calculated as described in the standard CBO algorithm [16].

Step 9. The optimization is repeated from Step 2 until a termination criterion, as the maximum number of iterations, is satisfied.

1. extracting a non-dominated front that is close to the true Pareto front (convergence), and
2. maintaining the diversity of the solutions along the resulting Pareto front (sparsity) [17].

In the proposed method, two operations different from the standard single solution CBO are performed:

1. ranking the solutions based on maximin value to push the agents to low-crowded region of Pareto front;
2. incorporating an archive to algorithm for saving the non-dominated solutions forming the Pareto front.

The steps of Algorithm 1 outline the main procedure for the implementation of the MOCBO.

There are other multi-objective optimizations [18, 19]. The concepts of optimization can be extended including reliability, Plastic analysis and design [20–23].

4 Numerical examples

In Section 4, two numerical examples are studied. The topological properties of the regions are transferred to the connectivity properties of graphs, by the clique graphs. In both examples the weights of all the edges and the demands of all nodes are taken as unity, and the meshes has been considered as the four nodes rectangular meshes. In both examples, the number of obnoxious facilities (p) is considered as 4.

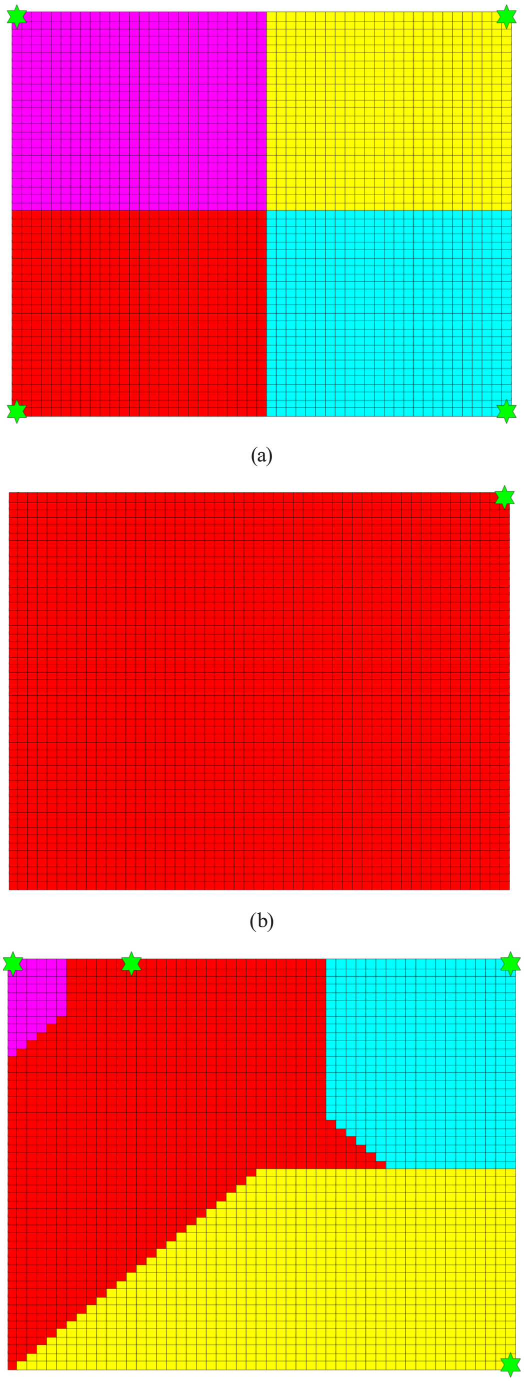
In order to compare with available meta-heuristic algorithms, all of examples are solved also using the multi-objective particle swarm optimization (MOPSO) [13] and the NSGA-II algorithm [14] in order to provide some comparison. Table 1 shows the parameters of these algorithms in the utilized examples. Comparisons are made through Pareto front produced. Capability and robustness of three algorithms are investigated for two models. The time of all computations is evaluated in clock time.

Table 1 Parameter settings for all the algorithms

Parameters	Algorithms		
	NSGA-II	MOPSO	MOCBO
Crossover probability (p_c)	0.9	-	-
Mutation probability (p_w)	0.4	0.5	-
Population size (N)	100	100	50
External archive size ($nrep$)	100	100	-
Number of adaptive grid ($ngrid$)	-	30	30
Inertia weight (ω)	-	0.4	-

4.1 Example 1

In the first case, suppose that decision maker intends to locate 4 obnoxious (e.g., noisy or polluting) factories in the rectangular region displayed in Fig. 1 such that sum distances of demands (graph's nodes) from the nearest factory and also the dispersion of factories be maximum.



(a) (b) (c)

Fig. 1 Configuration of facilities for the example 1; (a) case 1, (b) case 2, (c) case 3

The resulted trade-off between the out-transmission and dispersion from three multi-objective optimization methods are presented in Fig. 2. It can be seen that the PF set obtained using MOCBO is dominated those obtained using MOPSO and NSGA-II. Also, MOCBO has acceptable convergence performance and is able to cover all parts of PF and the obtained set of solutions is distributed uniformly. The average elapsed time per run of the MOCBO, MOPSO and NSGA-II were 101, 523 and 2950 sec, respectively.

The trade-off curve provides insights into the relationship between these conflicting objectives. By analyzing the trade-off curve decision-makers can make informed choices based on their preferences. They can assess the impact of different trade-offs and choose a solution that strikes a suitable balance between the competing objectives taking into account factors such as cost environmental impact and social concerns. In Fig. 2 the cases 1, 2 and 3 represent solutions with the maximum out-transmission, the maximum dispersion and the maximum both of these simultaneously, respectively. Fig. 1 shows the location of facilities for these three cases using the MOCBO algorithm.

4.2 Example 2

In this case, the rectangular region with four perforations is investigated, see Fig. 3. The resulted Pareto fronts from three multi-objective optimization methods are presented in Fig. 4. It can be seen that the PF set obtained using MOCBO is dominated those obtained using MOPSO and NSGA-II. Also, MOCBO has acceptable convergence performance and is able to cover all parts of PF and the obtained set of solutions is distributed uniformly. The average elapsed time per run of the MOCBO, MOPSO and NSGA-II were 109, 350 and 2471 s, respectively. Fig. 3 shows the location of the facilities for three cases using the MOCBO algorithm.

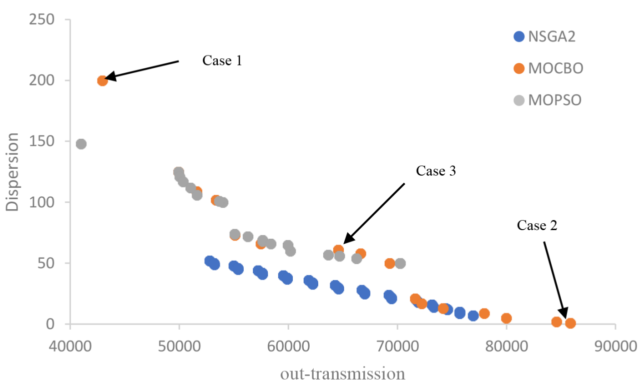


Fig. 2 Non-dominated fronts for the example 1

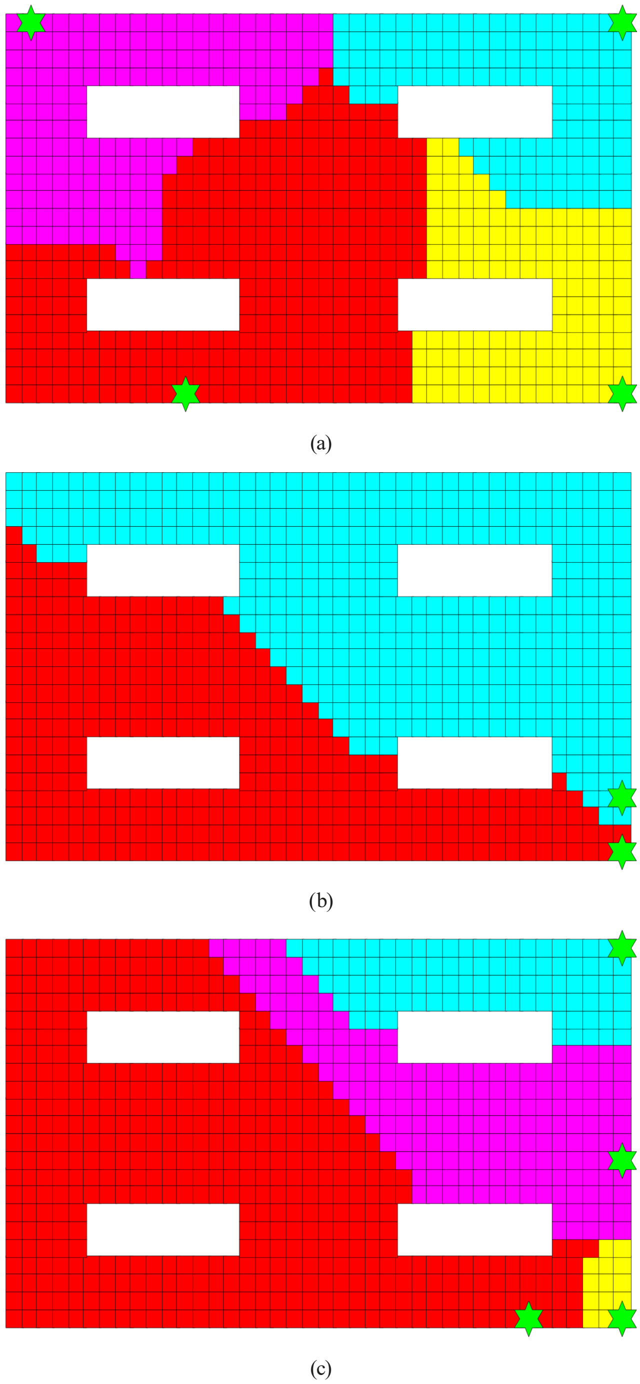


Fig. 3 Configuration of facilities for the example 2; (a) case 1, (b) case 2, (c) case 3

5 Conclusions

In this paper, a method is developed for the multi-objective optimization of the obnoxious facility location problem. In this method, both the sum distance from any demand to its nearest facility and the dispersion among facilities maximization are considered as the objectives. The multi-objective optimization problem is solved by the MOCBO, MOPSO and NSGA-II algorithms to select compromise

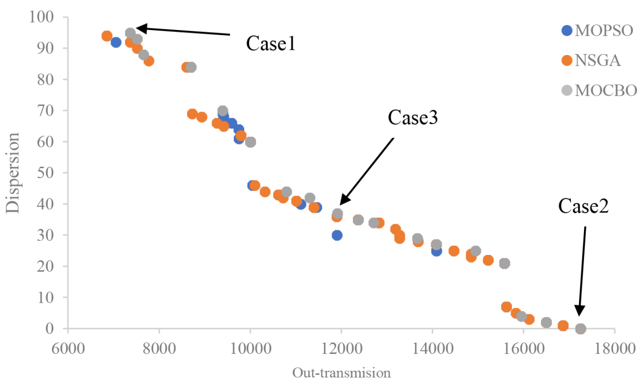


Fig. 4 Non-dominated fronts for the example 2

solutions from the obtained Pareto set. Despite the complexity of this type of problem, the MOCBO yielded better results, high convergence and uniformly distribution, than those found in MOPSO and NSGA-II, even for high-dimensional problems.

Future work will consist of studying real-size problems considering facility location and logical comparison of the multi-objective algorithms.

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