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A Simple Differential Evolution with Random Mutation and Crossover Constants for Constrained Optimization

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Abstract

The article proposes a simple version of the differential evolution algorithm (abbreviated as *sDE*) in which the mutation factor and crossover constant are chosen randomly in the range (0,1) during the search for the optimal solution. The *sDE* is the same as the original version of the differential evolution algorithm, except the user does not have to choose the best values of mutation constant and crossover constant for each optimization problem. Therefore, the optimization process is now very simple as it remains only one parameter (i.e. the population size) in the algorithm, besides the stopping criterion (e.g. number of iterations). It also consumes less computation time than the original differential evolution as it is not necessary to tune the mutation and crossover constants. In this study, the proposed technique is applied to three constrained optimizations, three engineering design problems, and six planar and spatial trusses under frequency constraints. Despite the very simple characteristics of the proposed technique, *sDE* gives promising results in comparison with other results in the literature.

Keywords

differential evolution, control parameter, constrained optimization, truss optimization

1 Introduction

For almost thirty years, the differential evolution algorithm (DE) [1] has proven its efficiency in solving optimization problems in engineering. Based on the original form of differential evolution, there are various versions to improve its capacity. However, one of the drawbacks of DE versions is their sensitivity to control parameters. Therefore, one of the modification trends is related to the main control parameters of DE such as mutation constant F and crossover constant CR. These values are known to affect the capacity of the algorithm in exploration (affected mainly by mutation constant F) and exploitation (affected mainly by crossover constant CR). Normally, the larger value of F will increase the probability of escaping a local minimum, and a larger value of CR will make the convergence faster. In the early period, some authors suggested the good ranges of CR and F [1, 2]. Later, the control parameters are determined using deterministic rules. For example, a simple relation among F, CR, and the dimension of the problem in [3], two DE variants with random values of F in the range (0.5, 1) or time-varying values of F from a predetermined maximum to a predetermined minimum value in [4]. The tuning to choose the fittest parameters for each optimization problem is often tedious, computation time-consuming, and more importantly, impossible to cover all combinations of F and CR. This leads to some studies that employ various mechanisms for automatic tuning of these control parameters instead of hand tuning. For example, the introduction of self-adapting control parameters F and CR in [5], versions of DE such as the Fuzzy Adaptive Differential Evolution (FADE) [6], the Self-adaptive Differential Evolution Algorithm for Numerical Optimization (SaDE) [7] and its updated version SaDE2 [8]. It is noted that all the above-mentioned variants also need extra parameters for the choice of F and CR. The readers can find more details of control parameters in DE in the recent reviews [9, 10]. However, to the best of our knowledge, there is no study on the simultaneous use random values of F and CR without any extra parameters. This encourages our study to experiment with the random values of F and CR to simplify the optimization process without compromising result quality. The article is structured as follows. Section 2 will briefly introduce the

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differential evolution algorithm, Section 3 will propose our technique, using random F and CR, as mentioned above. Section 4 is for experiments with constrained optimization problems and the Section 5 is for some conclusions.

2 Differential evolution

Differential evolution is a population-based algorithm that can self-adjust its search direction during the optimization process. Three operators of DE include:

1. mutation using the information within the population to alter the search space;
2. crossover to mix components of individuals; and
3. selection to reserve the best individual for the next generation.

They are defined as follows:

- Mutation: This study uses 5 available mutation strategies [11]

$$DE / rand / 1\text{: } v_{i,G+1} = \boldsymbol{x}_{r1,G} + F\left(\boldsymbol{x}_{r2,G} - \boldsymbol{x}_{r3,G}\right) \quad i = 1,2,\ldots,N \tag{1}$$

$$DE / best / 1\text{: } v_{i,G+1} = \boldsymbol{x}_{\text{best},G} + F\left(\boldsymbol{x}_{r1,G} - \boldsymbol{x}_{r2,G}\right) \quad i = 1,2,\ldots,N \tag{2}$$

$$DE / rand / 2\text{: } v_{i,G+1} = \boldsymbol{x}_{r1,G} + F\left(\boldsymbol{x}_{r2,G} - \boldsymbol{x}_{r3,G}\right) + F\left(\boldsymbol{x}_{r4,G} - \boldsymbol{x}_{r5,G}\right) \quad i = 1,2,\ldots,N \tag{3}$$

$$DE / best / 2\text{: } v_{i,G+1} = \boldsymbol{x}_{\text{best},G} + F\left(\boldsymbol{x}_{r1,G} - \boldsymbol{x}_{r2,G}\right) + F\left(\boldsymbol{x}_{r3,G} - \boldsymbol{x}_{r4,G}\right) \quad i = 1,2,\ldots,N \tag{4}$$

$$DE / target\text{-}to\text{-}best / 1\text{: } v_{i,G+1} = \boldsymbol{x}_{i,G} + F\left(\boldsymbol{x}_{\text{best},G} - \boldsymbol{x}_{i,G}\right) + F\left(\boldsymbol{x}_{r1,G} - \boldsymbol{x}_{r2,G}\right) \quad i = 1,2,\ldots,N \tag{5}$$

  where r_1, r_2, r_3, r_4, r_5 are random numbers in [1, N], integer, mutually different, and different from the running index i; F is mutation constant in (0, 1); N is the population size; G is the current generation.
  In the notation *DE/rand/1*, rand denotes a randomly chosen individual for mutation, and 1 is the number of difference individuals used. For *DE/best/1* and *DE/target-to-best/1*, *best* means the individual with the lowest objective function. Other strategies *DE/rand/2* and *DE/best/2* are similar except two difference individuals are used.
- Crossover:

$$u_{i,G+1} = \left(u_{1i,G+1}, u_{2i,G+1}, \ldots, u_{Di,G+1}\right) \tag{6}$$

  where

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if } (r \le CR) \text{ or } j = k \\ \boldsymbol{x}_{ji,G} & \text{if } (r > CR) \text{ and } j \ne k \end{cases}, \quad j = 1,2,\ldots,D$$

  CR is the crossover constant in (0, 1); r is a random number in (0, 1); k is a random integer number in [1, D] which ensures that $u_{i,G+1}$ gets at least one component from $v_{i,G+1}$; and D is the dimension of the problem.
- Selection:

$$\boldsymbol{x}_{i,G+1} = \begin{cases} u_{i,G+1} & \text{if } f\left(u_{i,G+1}\right) < f\left(\boldsymbol{x}_{i,G}\right) \\ \boldsymbol{x}_{i,G} & \text{otherwise} \end{cases} \tag{7}$$

  where f is the objective function.

It is noted that strategies with two difference individuals (*DE/rand/2* and *DE/best/2*) can generate more different trial individuals than those with only one difference individual (*DE/rand/1*, *DE/best/1*, and *DE/target-to-best/1*). On the other hand, using the *best* individual makes the population tend to the local minimum.

Differential evolution is originally for unconstrained minimization. To apply *sDE* to constrained minimization problems, a constraint-handling technique is based on the comparison between two individuals. For a minimization problem, the comparison is based on the following priority: feasibility (all constraints are satisfied), smaller objective function, or smaller violation (in case constraints are not satisfied).

3 Proposed technique

The study proposes a very simple technique, named as simple differential evolution (*sDE*). Instead of tuning to choose the most suitable values of the mutation constant F and the crossover constant CR for each optimization problem, the proposed technique uses random values in (0, 1) of F and CR during the search for the optimal solution. Therefore, the proposed *sDE* is almost unchanged in comparison with the original form of differential evolution, except that there remains only one parameter (the population size) in the algorithm, besides the stopping criterion which is the number of iterations in this study. The *sDE* will be engaged with five common mutation strategies (Eqs. (1)–(5) as mentioned above) to determine the most effective strategies associated with it. For convenience, we use the prefix *s*- for the corresponding strategy, i.e. notations *sDE/rand/1*, *sDE/best/1*, *sDE/rand/2*, *sDE/best/2*, and *sDE/target-to-best/1*. The study also surveys the influence of the number of iterations and population on the proposed technique.

For comparison purposes, the *sDE* will be tested with common problems that are available in the literature, including constrained optimization problems (three test functions), engineering design problems (pressure vessel, welded cantilever beam, and spring), and trusses under frequency constraints (three planar trusses and three spatial trusses). The results will be compared with known solutions with fixed mutation constant F and crossover constant CR.

4 Experiments and results

4.1 Constrained optimization problems

A constrained optimization problem is formulated as:

$$\begin{array}{lll} \text{Minimize} & f(\boldsymbol{x}) & \boldsymbol{x} \in Rn \\ \text{Subject to:} & g_j(\boldsymbol{x}) \geq 0, & j = 1,\ldots,q \\ & h_k(\boldsymbol{x}) = 0, & k = 1,\ldots,p \\ & \boldsymbol{x}_i^l \leq \boldsymbol{x}_i \leq \boldsymbol{x}_i^u, & i = 1,\ldots,n \end{array} \tag{8}$$

where $\boldsymbol{x}$ is a vector of size n, $f(\boldsymbol{x})$ is the objective function, $g_j(\boldsymbol{x})$ is the j^{th} inequality constraint, $h_k(\boldsymbol{x})$ is the k^{th} equality constraint, and $\left[\boldsymbol{x}_i^l, \boldsymbol{x}_i^u\right]$ are the lower and upper bounds of the variable $\boldsymbol{x}_i$, respectively.

The *sDE* will be tested on three test functions [3, 12] as follows:

1. Test function 1
   Objective function:
   $f(\boldsymbol{x}) = 5.04\boldsymbol{x}_1 + 0.035\boldsymbol{x}_2 + 10\boldsymbol{x}_3 + 3.36\boldsymbol{x}_5 - 0.063\boldsymbol{x}_4\boldsymbol{x}_7$
   Constraints:
   $g_1(\boldsymbol{x}) = 35.82 - 0.222\boldsymbol{x}_{10} - 0.9\boldsymbol{x}_9 \geq 0$
   $g_2(\boldsymbol{x}) = -133 + 3\boldsymbol{x}_7 - 0.99\boldsymbol{x}_{10} \geq 0$
   $g_3(\boldsymbol{x}) = -g_1(\boldsymbol{x}) + \boldsymbol{x}_9(1/0.9 - 0.9) \geq 0$
   $g_4(\boldsymbol{x}) = -g_2(\boldsymbol{x}) + (1/0.99 - 0.99)\boldsymbol{x}_{10} \geq 0$
   $g_5(\boldsymbol{x}) = 1.12\boldsymbol{x}_1 + 0.13167\boldsymbol{x}_1\boldsymbol{x}_8 - 0.00667\boldsymbol{x}_1\boldsymbol{x}_8^2 - 0.99\boldsymbol{x}_4 \geq 0$
   $g_6(\boldsymbol{x}) = 57.425 + 1.098\boldsymbol{x}_8 - 0.038\boldsymbol{x}_8^2 + 0.325\boldsymbol{x}_6 - 0.99\boldsymbol{x}_7 \geq 0$
   $g_7(\boldsymbol{x}) = -g_5(\boldsymbol{x}) + (1/0.99 - 0.99)\boldsymbol{x}_4 \geq 0$
   $g_8(\boldsymbol{x}) = -g_6(\boldsymbol{x}) + (1/0.99 - 0.99)\boldsymbol{x}_7 \geq 0$
   $g_9(\boldsymbol{x}) = 1.22\boldsymbol{x}_4 - \boldsymbol{x}_1 - \boldsymbol{x}_5 = 0$
   $g_{10}(\boldsymbol{x}) = 98000\boldsymbol{x}_3/(\boldsymbol{x}_4\boldsymbol{x}_9 + 1000\boldsymbol{x}_3) - \boldsymbol{x}_6 = 0$
   $g_{11}(\boldsymbol{x}) = (\boldsymbol{x}_2 + \boldsymbol{x}_5)/\boldsymbol{x}_1 - \boldsymbol{x}_8 = 0$
   $0.00001 \leq \boldsymbol{x}_1 \leq 2000$
   $0.00001 \leq \boldsymbol{x}_2 \leq 16000$
   $0.00001 \leq \boldsymbol{x}_3 \leq 120$
   $0.00001 \leq \boldsymbol{x}_4 \leq 5000$
   $0.00001 \leq \boldsymbol{x}_5 \leq 2000$
   $85 \leq \boldsymbol{x}_6 \leq 93$
   $90 \leq \boldsymbol{x}_7 \leq 95$
   $3 \leq \boldsymbol{x}_8 \leq 12$
   $1.2 \leq \boldsymbol{x}_9 \leq 4$
   $145 \leq \boldsymbol{x}_{10} \leq 162$
   Best-known solution :
   $f(\boldsymbol{x}_{\min}) = -1768.80696$
   $\boldsymbol{x}_{\min}$ = (1698.096, 15818.73, 54.10228, 3031.226, 2000.0, 90.11537, 95.0, 10.49336, 1.561636, 153.53535)
2. Test function 2
   Objective function:
   $f(\boldsymbol{x}) = \boldsymbol{x}_1\boldsymbol{x}_4(\boldsymbol{x}_1 + \boldsymbol{x}_2 + \boldsymbol{x}_3) + \boldsymbol{x}_3$
   Constraints:
   $\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4 - 25 \geq 0$
   $\boldsymbol{x}_1^2 + \boldsymbol{x}_2^2 + \boldsymbol{x}_3^2 + \boldsymbol{x}_4^2 - 40 = 0$
   $1 \leq \boldsymbol{x}_i \leq 5$, $i = 1, \ldots, 4$
   Best-known solution:
   $f(\boldsymbol{x}_{\min}) = 17.0140173$
   $\boldsymbol{x}_{\min}$ = (1, 4.7429994, 3.8211503, 1.3794082)
3. Test function 3
   Objective function:
   $f(\boldsymbol{x}) = \exp(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4\boldsymbol{x}_5)$
   Constraints :
   $\boldsymbol{x}_1^2 + \boldsymbol{x}_2^2 + \boldsymbol{x}_3^2 + \boldsymbol{x}_4^2 + \boldsymbol{x}_5^2 - 10 = 0$
   $\boldsymbol{x}_2\boldsymbol{x}_3 - 5\boldsymbol{x}_4\boldsymbol{x}_5 = 0$
   $\boldsymbol{x}_1^3 + \boldsymbol{x}_2^3 + 1 = 0$
   $-2.3 \leq \boldsymbol{x}_i \leq 2.3$, $i = 1, 2$
   $-3.2 \leq \boldsymbol{x}_i \leq 3.2$, $i = 3, 4, 5$
   Best-known solution:
   $f(\boldsymbol{x}_{\min}) = 0.0539498478$
   $\boldsymbol{x}_{\min}$ = (−1.717143, 1.595709, 1.827247, −0.7636413, −0.763645).

It is difficult to find the optimal solution for the three test functions, as noted in [13]. For comparison, the parameters used in this study are similar to [13], i.e. population of 30, iteration of 10^5, and 50 independent runs for all five *sDE* strategies.

From the results of the three test functions in Table 1, it is seen that the strategy ***sDE/best/2*** generally yields the best results which are almost as same as the *DE/best/2* with chosen parameters (CR, F) in [13] and the best-known ones in the literature [12, 14]. The strategy *sDE/rand/2* also gives good results except for the function 3. On the other hand, *sDE/best/1*, and *sDE/target-to-best/1* almost fail in the experiments. Comparison between *sDE/rand/1* and the corresponding one *DE/rand/1* with chosen parameters (CR, F) in [12] also shows *sDE* is slightly superior.

Table 1 Optimal solutions by *sDE* strategies

Test func.	*sDE/rand/1*	*sDE/best/1*	*sDE/rand/2*	***sDE/best/2***	*sDE/target-to-best/1*	References: *DE/rand/1* in [12]	References: *DE/best/2* in [13]	References: Best known [12, 14]
1	−1598.930	fail	−1768.807	**−1735.440**	fail	−1278.079	−1690.003	−1768.807
2	17.017	fail	17.605	**17.014**	31.763	20.273	17.014	17.014
3	0.475	fail	0.441	**0.054**	fail	0.453	0.054	0.054

Table 2 Comparison between *sDE* and *DE* with combinations of (CR, F), iterations = 10^5

Test func.	*sDE/best/2* (random *CR* and *F*)	The original differential evolution *DE/best/2* with assigned values of (CR, F)								
		$CR = 0.1$, $F = 0.1$	$CR = 0.1$, $F = 0.5$	$CR = 0.1$, $F = 1$	$CR = 0.5$, $F = 0.1$	$CR = 0.5$, $F = 0.5$	$CR = 0.5$, $F = 1$	$CR = 1$, $F = 0.1$	$CR = 1$, $F = 0.5$	$CR = 1$, $F = 1$
1	**−1735.440**	fail	fail	fail	−1131.245	−1525.101	fail	fail	−1767.981	fail
2	**17.014**	33.822	fail	85.669	17.307	22.161	18.697	30.734	17.014	19.234
3	**0.054**	0.071	fail	fail	0.233	0.455	0.996	0.317	0.054	0.465

To survey the effect of iterations on the results, we compare the *sDE/best/2* and the corresponding *DE/best/2* with 9 combinations of the constants (CR, F). The results in Table 2 show that *sDE* with the random values (CR, F) is as good as the "best combination" (with $CR = 1$, $F = 0.5$) and better than other combinations (CR, F) for 10^5 iterations. As there is an uncountable combination of (CR, F), it will theoretically take timeless computation time to tune them for the best combination. Therefore, the effective use of random values (CR, F) in *sDE* will bring a considerable benefit.

As the number of iterations decreases to 10^4 and 10^3, the goodness of *sDE* solutions also decreases in comparison with the "best combination" (with $CR = 1$, $F = 0.5$) but is still better than other combinations (CR, F), as shown in Table 3 and Table 4. This implies that the use of *sDE* might be best suitable with enough iterations. On the other hand, it also means that the use of *sDE* is quite good if there is not available the "best combination" of CR and F.

4.2 Engineering design problems

Three popular engineering design problems, including a pressure vessel, a welded cantilever beam, and a coil spring, will be tested. As the number of variables in these problems is only 3 or 4, the population is chosen as 20. Two values of iterations (100 and 200) are surveyed to illustrate the effective use of the proposed technique.

4.2.1 Problem 1: Pressure vessel

A cylindrical vessel with both hemispherical ends [15] is shown in Fig. 1. The objective function is the total cost of the material and manufacturing. There are four variables,

Table 3 Comparison between *sDE* and *DE* with combinations of (CR, F), iterations = 10^4

Test func.	*sDE/best/2* (random *CR* and *F*)	The original differential evolution *DE/best/2* with assigned values of (CR, F)								
		$CR = 0.1$, $F = 0.1$	$CR = 0.1$, $F = 0.5$	$CR = 0.1$, $F = 1$	$CR = 0.5$, $F = 0.1$	$CR = 0.5$, $F = 0.5$	$CR = 0.5$, $F = 1$	$CR = 1$, $F = 0.1$	$CR = 1$, $F = 0.5$	$CR = 1$, $F = 1$
1	**−1230.697**	fail	fail	fail	−1012.949	−1255.028	−1213.540	fail	−1729.345	fail
2	**22.602**	88.706	fail	fail	25.976	105.722	24.855	25.064	19.662	20.862
3	**0.305**	0.987	fail	fail	0.488	0.440	fail	0.884	0.054	0.587

Table 4 Comparison between *sDE* and *DE* with combinations of (CR, F), iterations = 10^3

Test func.	*sDE/best/2* (random *CR* and *F*)	The original differential evolution *DE/best/2* with assigned values of (CR, F)								
		$CR = 0.1$, $F = 0.1$	$CR = 0.1$, $F = 0.5$	$CR = 0.1$, $F = 1$	$CR = 0.5$, $F = 0.1$	$CR = 0.5$, $F = 0.5$	$CR = 0.5$, $F = 1$	$CR = 1$, $F = 0.1$	$CR = 1$, $F = 0.5$	$CR = 1$, $F = 1$
1	**−950.146**	fail	fail	fail	−1271.340	fail	fail	fail	−1440.575	fail
2	**20.637**	fail	fail	fail	26.573	fail	31.388	18.855	26.971	17.800
3	**0.118**	fail	fail	fail	0.154	0.752	fail	0.069	0.260	0.812

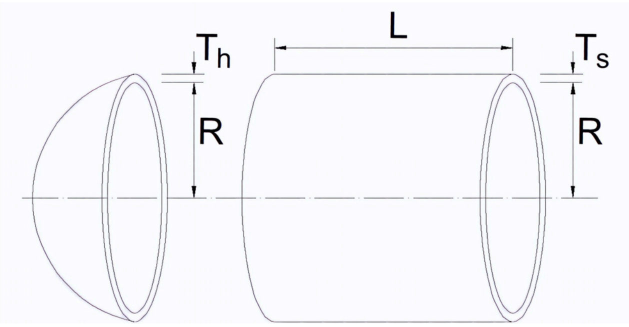


Fig. 1 Pressure vessel dimensions

including T_s (or $\boldsymbol{x}_1$) and T_h (or $\boldsymbol{x}_2$) are the thickness of the stem and the head, respectively. For manufacture requirements, they are of integer multiples of 0.0625; the radius R (or $\boldsymbol{x}_3$) and the stem length L (or $\boldsymbol{x}_4$) are continuous quantities. The problem is formulated as follows:

$$\begin{aligned}
&\text{Minimize} && f = 0.6224\boldsymbol{x}_1\boldsymbol{x}_3\boldsymbol{x}_4 + 19.84\boldsymbol{x}_1^2\boldsymbol{x}_3 + 1.7781\boldsymbol{x}_2\boldsymbol{x}_3^2 \\
& && +3.1661\boldsymbol{x}_1^2\boldsymbol{x}_4 \\
&\text{Subject to:} && g(1) = -\boldsymbol{x}_1 + 0.0193\boldsymbol{x}_3 \le 0 \\
& && g(2) = -\boldsymbol{x}_2 + 0.00954\boldsymbol{x}_3 \le 0 \\
& && g(3) = -\pi\boldsymbol{x}_3^2\boldsymbol{x}_4 - (4\pi/3)\boldsymbol{x}_3^3 + 1296000 \le 0 \\
& && g(4) = \boldsymbol{x}_4 - 240 \le 0 \\
& && 1 \le \boldsymbol{x}_1 \le 99,\ 1 \le \boldsymbol{x}_2 \le 99,\ 10 \le \boldsymbol{x}_3 \le 200, \\
& && 1 \le \boldsymbol{x}_4 \le 200.
\end{aligned}$$

4.2.2 Problem 2: Welded cantilever beam

The welded cantilever beam [16] with four design variables ($h = \boldsymbol{x}_1$, $d = \boldsymbol{x}_2$, $t = \boldsymbol{x}_3$, and $b = \boldsymbol{x}_4$) as shown in Fig. 2. The objective is to minimize the cost of the beam under constraints on shear stress τ, bending stress σ, buckling load P, and the end deflection of the beam δ. Given parameters $P = 6000$ lb, $L = 14$ in, $E = 30 \times 10^6$ psi, $G = 12 \times 10^6$ psi, $\tau_{\max} = 13600$ psi, $\sigma_{\max} = 30000$ psi, and $\delta_{\max} = 0.25$ in, the problem is formulated as follows:

$$\begin{aligned}
&\text{Minimize} && f = 1.10471\boldsymbol{x}_1^2\boldsymbol{x}_2 + 0.04811\boldsymbol{x}_3\boldsymbol{x}_4(14 + \boldsymbol{x}_2) \\
&\text{Subject to:} && g(1) = \tau - \tau_{\max} \le 0 \\
& && g(2) = \sigma - \sigma_{\max} \le 0 \\
& && g(3) = \boldsymbol{x}_1 - \boldsymbol{x}_4 \le 0 \\
& && g(4) = 0.10471\boldsymbol{x}_1^2 + 0.04811\boldsymbol{x}_3\boldsymbol{x}_4(14 + \boldsymbol{x}_2) \\
& && -5 \le 0 \\
& && g(5) = 0.125 - \boldsymbol{x}_1 \le 0 \\
& && g(6) = \delta - \delta_{\max} \le 0 \\
& && g(7) = P - P_c \le 0 \\
& && 0.1 \le \boldsymbol{x}_1 \le 2,\ 0.1 \le \boldsymbol{x}_2 \le 10,\ 0.1 \le \boldsymbol{x}_3 \le 10, \\
& && 0.1 \le \boldsymbol{x}_4 \le 2,
\end{aligned}$$

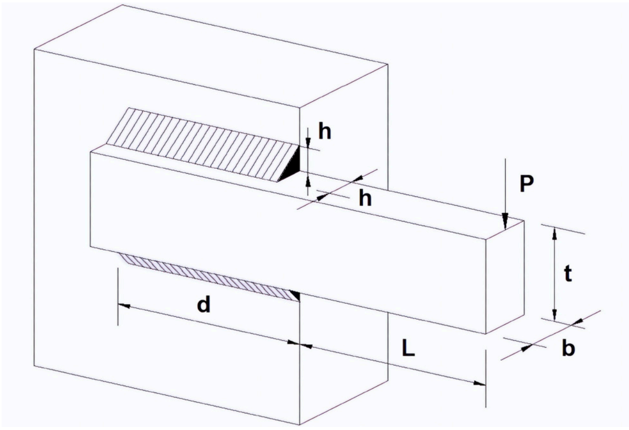


Fig. 2 Welded cantilever beam dimensions

where

$$\tau = \sqrt{\tau'^2 + \tau'\tau''\boldsymbol{x}_2/R + \tau''^2}$$

$$\tau' = \frac{P}{\sqrt{2}\boldsymbol{x}_1\boldsymbol{x}_2},\ \tau'' = \frac{MR}{J},\ M = P\left(L + \frac{\boldsymbol{x}_2}{2}\right)$$

$$R = \frac{1}{2}\sqrt{\boldsymbol{x}_2^2 + (\boldsymbol{x}_1 + \boldsymbol{x}_3)^2}$$

$$J = \sqrt{2}\boldsymbol{x}_1\boldsymbol{x}_2\left[\frac{\boldsymbol{x}_2^2}{12} + \frac{(\boldsymbol{x}_1 + \boldsymbol{x}_3)^2}{4}\right]$$

$$\sigma = \frac{6PL}{\boldsymbol{x}_4\boldsymbol{x}_3^2}$$

$$\delta = \frac{4PL^3}{E\boldsymbol{x}_3^3\boldsymbol{x}_4}$$

$$P_c = \frac{4.013\sqrt{EG\boldsymbol{x}_3^2\boldsymbol{x}_4^6}}{6L^2}\left(1 - \frac{\boldsymbol{x}_3}{4L}\sqrt{E/G}\right).$$

4.2.3 Problem 3: Coil spring

The weight of a coil spring [17] is minimized. The spring is in tension/compression under axial force P, and constraints on deflection, shear stress, frequency of surge waves, and dimension relation. The design variables are the wire diameter d (or $\boldsymbol{x}_1$), the mean coil diameter D (or $\boldsymbol{x}_2$), and the number of active coils ($\boldsymbol{x}_3$) as in Fig. 3.

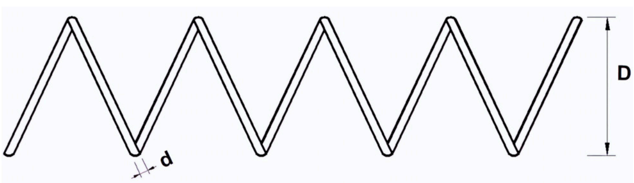


Fig. 3 Coil spring dimensions

The problem is formulated as follows:

Minimize $f = (\boldsymbol{x}_3 + 2)\boldsymbol{x}_2\boldsymbol{x}_1^2$

Subject to:

- Deflection constraint:

$$g_1 = 1 - \frac{\boldsymbol{x}_2^3\boldsymbol{x}_3}{71785\boldsymbol{x}_1^4} \le 0$$

- Shear stress constraint:

$$g_2 = \frac{4\boldsymbol{x}_2^2 - \boldsymbol{x}_1\boldsymbol{x}_2}{12566(\boldsymbol{x}_2\boldsymbol{x}_1^3 - \boldsymbol{x}_1^4)} + \frac{2.46}{12566\boldsymbol{x}_1^2} - 1 \le 0$$

- Surge wave frequency constraint:

$$g_3 = 1 - \frac{140.45\boldsymbol{x}_1}{\boldsymbol{x}_2^2\boldsymbol{x}_3} \le 0$$

- Outer diameter constraint:

$$g_4 = \frac{\boldsymbol{x}_1 + \boldsymbol{x}_2}{1.5} - 1 \le 0$$

- Variables bounds:

$$0.05 \le \boldsymbol{x}_1 \le 0.2,\ 0.25 \le \boldsymbol{x}_2 \le 1.3,\ 2 \le \boldsymbol{x}_3 \le 15.$$

The results of three engineering design problems in Table 5 [15–19] show that the strategy ***sDE/best/2*** again outperforms the other 4 strategies and gives better results compared with those in the literature. Similar to the previous test functions, the strategies *sDE/target-to-best/1* and *sDE/best/1* are not effective in these problems. The results improvement as the number of iterations increases from 100 to 200 is not much (Table 6). This implies that for these engineering design problems, the proposed technique does not need many iterations to reach good results. Despite the random values of the mutation and crossover constants, the convergence speeds in Fig. 4 of the proposed technique are quite fast in the first 20 iterations.

4.3 Trusses under frequency constraints

The minimization of truss weight under frequency constraints is defined as follows:

$$\begin{array}{lll} \text{Minimizing} & W = \sum_{i=1}^{N} \rho_i L_i A_i & \\ \text{Subject to:} & \omega_i \ge \omega_{i,\min} & i = 1,\ldots,K \\ & A_{i,\min} \le A_i \le A_{i,\max} & i = 1,\ldots,N \\ & \boldsymbol{x}_{i,\min} \le \boldsymbol{x}_i \le \boldsymbol{x}_{i,\max} & i = 1,\ldots,M \end{array} \quad (9)$$

where W is the truss weight; ρ_i, L_i, and A_i are the density, length, and cross-section area of the ith element, respectively; ω_i and $\omega_{i,\min}$ are the i^{th} natural frequency and corresponding frequency limit; $A_{i,\min}$ and $A_{i,\max}$ are the lower bound and upper bound of the i^{th} cross-section area; $\boldsymbol{x}_{i,\min}$ and $\boldsymbol{x}_{i,\max}$ are the lower bound and upper bound of the i^{th} coordinates; N, K, and M are the number of elements in the truss, number of frequency constraints and number of nodes coordinate constraints, respectively.

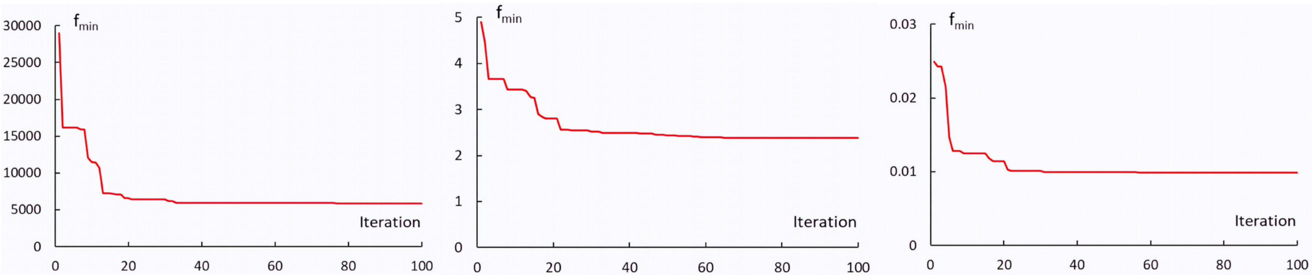
In this study, three planar trusses (10-bar, 37-bar, and 200-bar) and three spatial trusses (52-bar, 72-bar, and 120-bar) are tested. The truss parameters are listed briefly in Table 7 [20–29]. Details of these problems can be found in [20]. Their shapes are illustrated in Fig. 5. With structural characteristics of the trusses, for simplification and reduction of the search space, elements of some trusses are grouped into the same cross-section areas. Also, thanks to the symmetry, the coordinates of nodes are grouped into the same values.

Table 5 Optimal results of engineering design problems by *sDE* strategies

Problem	Iterations	*sDE/rand/1*	*sDE/ best/1*	*sDE/rand/2*	***sDE/best/2***	*sDE/target-to-best/1*	Other methods
Pressure vessel	200	5896.2802	7434.9354	5891.9362	**5885.3216**	6105.4634	7198.0428 [15] 6288.7445 [18] 5889.911 [19]
	100	5903.2898	6707.5505	5947.5025	5887.6393	6364.8340	
Welded cantilever beam	200	2.3810	2.7373	2.3811	**2.3809**	2.5164	2.3859 [16]
	100	2.3858	2.8574	2.3937	2.3812	2.9246	
Spring	200	0.0099	0.0104	0.0099	**0.0099**	0.0099	0.0127 [17, 18] 0.01267 [19]
	100	0.0099	0.0109	0.0099	0.0099	0.0100	

Table 6 Optimal solutions by the strategy *sDE/best/2* with 200 iterations

Problem	$\boldsymbol{x}_1$	$\boldsymbol{x}_2$	$\boldsymbol{x}_3$	$\boldsymbol{x}_4$	Optimal results
Pressure vessel	0.7782	0.3846	40.3196	199.9995	5885.3216
Welded cantilever beam	0.2444	6.2175	8.2915	0.2444	2.3809
Spring	0.0500	0.3744	8.5466	n/a	0.0099



(a) (b) (c)

Fig. 4 Convergence of the strategy *sDE/best/2* in engineering problems: (a) Pressure vessel, (b) Welded cantilever beam, (c) Coil spring

Table 7 Parameters of trusses under frequency constraints, from [20]

Parameters	Unit	Data of problems 10-bar	37-bar	200-bar	52-bar	72-bar	120-bar
Modulus of elasticity, E	N/m^2	6.98×10^{10}	2.1×10^{11}	2.1×10^{11}	2.1×10^{11}	6.98×10^{10}	2.1×10^{11}
Material density, ρ	kg/m^3	2770	7800	7860	7800	2770	7971.81
Frequencies constraints	Hz	$\omega_1 \geq 7$ $\omega_2 \geq 15$ $\omega_3 \geq 20$	$\omega_1 \geq 20$ $\omega_2 \geq 40$ $\omega_3 \geq 60$	$\omega_1 \geq 5$ $\omega_2 \geq 10$ $\omega_3 \geq 15$	$\omega_1 \geq 15.916$ $\omega_2 \geq 28.648$	$\omega_1 = 4$ $\omega_3 \geq 6$	$\omega_1 \geq 9$ $\omega_2 \geq 11$
Cross-section bounds	m^2	$[0.645 \times 10^{-4}, 40 \times 10^{-4}]$	$[10^{-4}, 10^{-3}]$	$[0.1 \times 10^{-4}, 30 \times 10^{-4}]$	$[10^{-4}, 10^{-3}]$	$[0.645 \times 10^{-4}, 30 \times 10^{-4}]$	$[10^{-4}, 129.3 \times 10^{-4}]$
Nodes coordinate bounds	m	n/a	upper nodes y-coord. [0.5, 2.5]	n/a	free nodes, x and y coord. ±2 m	n/a	n/a
Added masses	kg	454 kg at nodes 3–6	10 kg at lower nodes	100 kg at nodes 1–5	50 kg at free nodes 1–13	2270 kg at nodes 17–20	3000 kg at node 1, 500 kg at nodes 2–13, and 100 kg at nodes 14–37
Optimal problem		Cross-section size	Cross-section sizes and y-coord. of upper nodes (symmetry is reserved)	Cross-section size	Cross-section sizes and free nodes' coordinates (symmetry is reserved)	Cross-section size	Cross-section size
Number of element groups		10	14	19	9	16	7
Minimum weight W_{min} (*sDE/best/2*)	kg	524.6188	359.3328	2295.3961	190.3183	324.3183	8707.9846
W_{min} from [20]	kg	524.56	359.45	2296.38	191.28	324.36	8710.90
W_{min} from [21]	kg	n/a	n/a	2122.29	n/a	324.704	8889.439
W_{min} from [22]	kg	529.09	n/a	2298.61	197.31	327.51	9046.34
W_{min} from [23]	kg	535.61	n/a	n/a	n/a	326.67	n/a
W_{min} from [24]	kg	524.88	364.72	n/a	193.36	324.50	n/a
W_{min} from [25]	kg	535.14	363.03	n/a	207.27	n/a	n/a
W_{min} from [26]	kg	532.34	360.56	n/a	195.62	334.66	8886.92
W_{min} from [27]	kg	524.49	359.25	n/a	195.19	324.32	n/a
W_{min} from [28]	kg	530.77	359.94	n/a	n/a	327.670	8888.74
W_{min} from [29]	kg	532.12	358.01	n/a	193.13	328.21	n/a

For comparison with the study in [20], parameters of *sDE* are set similarly, i.e. population $N = 50$ (except $N = 150$ for 200-bar truss), number of iterations $I = 150$. Each problem is performed in 50 independent runs (except 100 runs for the 200-bar truss).

Results of five strategies of *sDE* for truss optimization problems in Table 8 show the ***sDE/best/2*** again generally gives the best solutions which are as good as other results in the literature listed in Table 7. Details of the best solutions by *sDE/best/2* are given in Table 9 and Table 10.

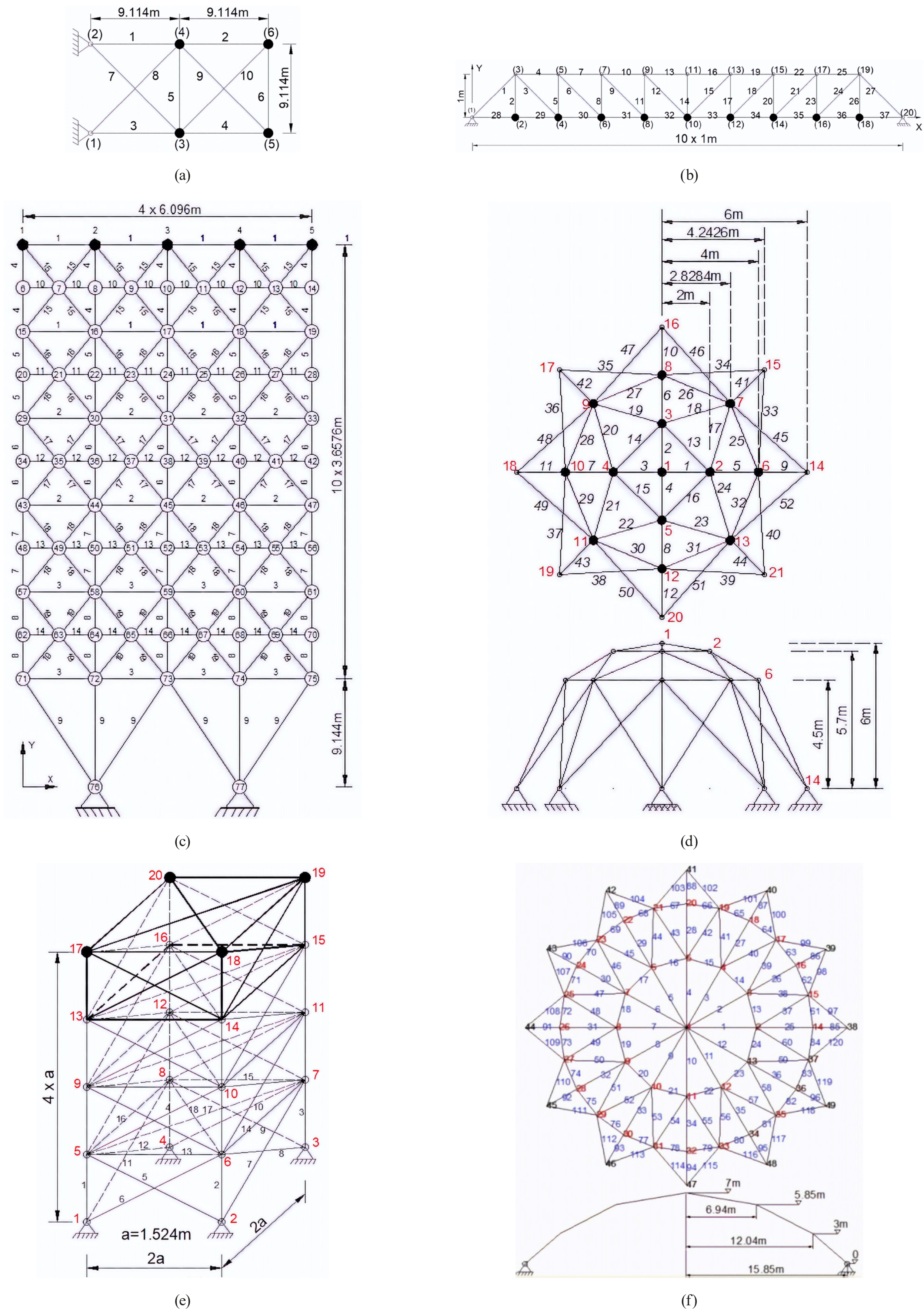


Fig. 5 Truss optimization problems, from [20]: (a) planar 10-bar truss, (b) planar 37-bar truss, (c) planar 200-bar truss, (d) spatial 52-bar truss, (e) spatial 72-bar truss, (f) spatial 120-bar truss

Table 8 Comparison of five strategies of *sDE* for truss optimization problems

Problem	*sDE/rand/1*	*sDE/ best/1*	*sDE/rand/2*	***sDE/best/2***	*sDE/target-to-best/1*
10-bar truss	525.8188	524.7329	527.9732	**524.6188**	539.0347
37-bar truss	360.9799	359.3189	365.8674	**359.3328**	394.2362
200-bar truss	2367.2908	2297.2156	5815.2890	**2295.3961**	fail
52-bar truss	199.2699	190.7274	202.5675	**190.3183**	298.9483
72-bar truss	325.6014	324.3488	327.3215	**324.3183**	436.2554
120-bar truss	8715.9833	8707.9942	8733.1571	**8707.9846**	fail

Table 9 Optimal results of planar trusses

10-bar truss		37-bar truss		200-bar truss	
Parameters	Results	Parameters	Results	Parameters	Results
A_1 (cm^2)	34.5048	Y_3, Y_{19} (m)	0.9406	A_1 (cm^2)	0.2640
A_2 (cm^2)	14.8601	Y_5, Y_{17} (m)	1.3359	A_2 (cm^2)	0.1861
A_3 (cm^2)	35.2763	Y_7, Y_{15} (m)	1.5113	A_3 (cm^2)	5.7968
A_4 (cm^2)	14.4002	Y_9, Y_{13} (m)	1.6557	A_4 (cm^2)	0.5882
A_5 (cm^2)	0.6485	Y_{11} (m)	1.7428	A_5 (cm^2)	1.4846
A_6 (cm^2)	4.5936	A_1, A_{27} (cm^2)	2.8908	A_6 (cm^2)	3.0090
A_7 (cm^2)	24.2256	A_2, A_{26} (cm^2)	1.1504	A_7 (cm^2)	5.3682
A_8 (cm^2)	23.2799	A_3, A_{24} (cm^2)	1.0231	A_8 (cm^2)	8.0812
A_9 (cm^2)	12.5059	A_4, A_{25} (cm^2)	2.8836	A_9 (cm^2)	18.6087
A_{10} (cm^2)	12.7069	A_5, A_{23} (cm^2)	1.0000	A_{10} (cm^2)	0.1002
$\boldsymbol{W_{\min}}$ **(kg)**	**524.6188**	A_6, A_{21} (cm^2)	1.2895	A_{11} (cm^2)	0.1002
		A_7, A_{22} (cm^2)	2.3845	A_{12} (cm^2)	0.1000
		A_8, A_{20} (cm^2)	1.3443	A_{13} (cm^2)	0.1000
		A_9, A_{18} (cm^2)	1.5275	A_{14} (cm^2)	0.1444
		A_{10}, A_{19} (cm^2)	2.4881	A_{15} (cm^2)	0.8303
		A_{11}, A_{17} (cm^2)	1.1960	A_{16} (cm^2)	1.1969
		A_{12}, A_{15} (cm^2)	1.3622	A_{17} (cm^2)	1.5827
		A_{13}, A_{16} (cm^2)	2.0690	A_{18} (cm^2)	2.1200
		A_{14} (cm^2)	1.0000	A_{19} (cm^2)	4.3291
		$\boldsymbol{W_{\min}}$ **(kg)**	**359.3328**	$\boldsymbol{W_{\min}}$ **(kg)**	**2295.3961**

The next good strategies for these problems are *sDE/rand/1* and *sDE/best/1*. On the other hand, *sDE/rand/2* is not effective and *sDE/target-to-best/1* is the worst.

Despite the random values of *CR* and *F*, the convergences of the strategy *sDE/best/2* depicted in Fig. 6 show stability and fast convergence in about 50 first iterations. Again, it is noted that the original DE always needs the best tuning values of *CR* and *F*, whereas the proposed technique does not need it. Therefore, the *sDE* is very useful for engineering problems that need a large computation time.

5 Conclusions

The very simple version of the differential evolution algorithm (*sDE*) with the random mutation factor and crossover constant shows promising results in constrained optimizations, engineering design problems, and trusses under frequency constraints. The best advantage of the technique is its simplicity and saving computation time as it is not necessary to tune the mutation and crossover constants for each problem. In the study, the strategy *sDE/best/2* proves its superiority whereas the other strategies *sDE* of the differential evolution do not show a clear advantage in tested problems. The use of the best individual in the strategy *sDE/best/2* can be considered as a balancing role against the exploration characteristic of random values *CR* and *F*. The study also shows a good and quite rapid convergence of the strategy *sDE/best/2*. Therefore, the proposed technique can search for optimal solutions much simpler and faster in engineering problems, which normally consumes a large computation resource. However,

Table 10 Optimal results of spatial trusses

52-bar truss		72-bar truss		120-bar truss	
Parameters	Results	Parameters	Results	Parameters	Results
$A_1 - A_4$ (cm^2)	1.0024	$A_1 - A_4$ (cm^2)	16.8900	$A_1 - A_{12}$ (cm^2)	19.5976
$A_5 - A_8$ (cm^2)	1.5896	$A_5 - A_{12}$ (cm^2)	8.1823	$A_{13} - A_{24}$ (cm^2)	40.0585
$A_9 - A_{12}$ (cm^2)	1.0034	$A_{13} - A_{16}$ (cm^2)	0.6450	$A_{25} - A_{36}$ (cm^2)	10.7010
$A_{13} - A_{16}$ (cm^2)	1.2898	$A_{17} - A_{18}$ (cm^2)	0.6459	$A_{37} - A_{60}$ (cm^2)	21.1484
$A_{17} - A_{24}$ (cm^2)	1.1601	$A_{19} - A_{22}$ (cm^2)	12.9812	$A_{61} - A_{84}$ (cm^2)	9.7990
$A_{25} - A_{32}$ (cm^2)	1.2977	$A_{23} - A_{30}$ (cm^2)	7.7909	$A_{85} - A_{96}$ (cm^2)	11.6332
$A_{33} - A_{40}$ (cm^2)	1.1435	$A_{31} - A_{34}$ (cm^2)	0.6450	$A_{97} - A_{120}$ (cm^2)	14.8813
$A_{41} - A_{44}$ (cm^2)	1.0208	$A_{35} - A_{36}$ (cm^2)	0.6450	$\boldsymbol{W_{min}}$ **(kg)**	**8707.9846**
$A_{45} - A_{52}$ (cm^2)	1.6931	$A_{37} - A_{40}$ (cm^2)	7.8641		
Z_1 (m)	5.9154	$A_{41} - A_{48}$ (cm^2)	7.9666		
X_2 (m)	2.6180	$A_{49} - A_{52}$ (cm^2)	0.6450		
Z_2 (m)	3.7000	$A_{53} - A_{54}$ (cm^2)	0.6450		
X_6 (m)	4.1842	$A_{55} - A_{58}$ (cm^2)	3.4786		
Z_6 (m)	2.5139	$A_{59} - A_{66}$ (cm^2)	7.8214		
$\boldsymbol{W_{min}}$ **(kg)**	**190.3183**	$A_{67} - A_{70}$ (cm^2)	0.6451		
		$A_{71} - A_{72}$ (cm^2)	0.6450		
		$\boldsymbol{W_{min}}$ **(kg)**	**324.3183**		

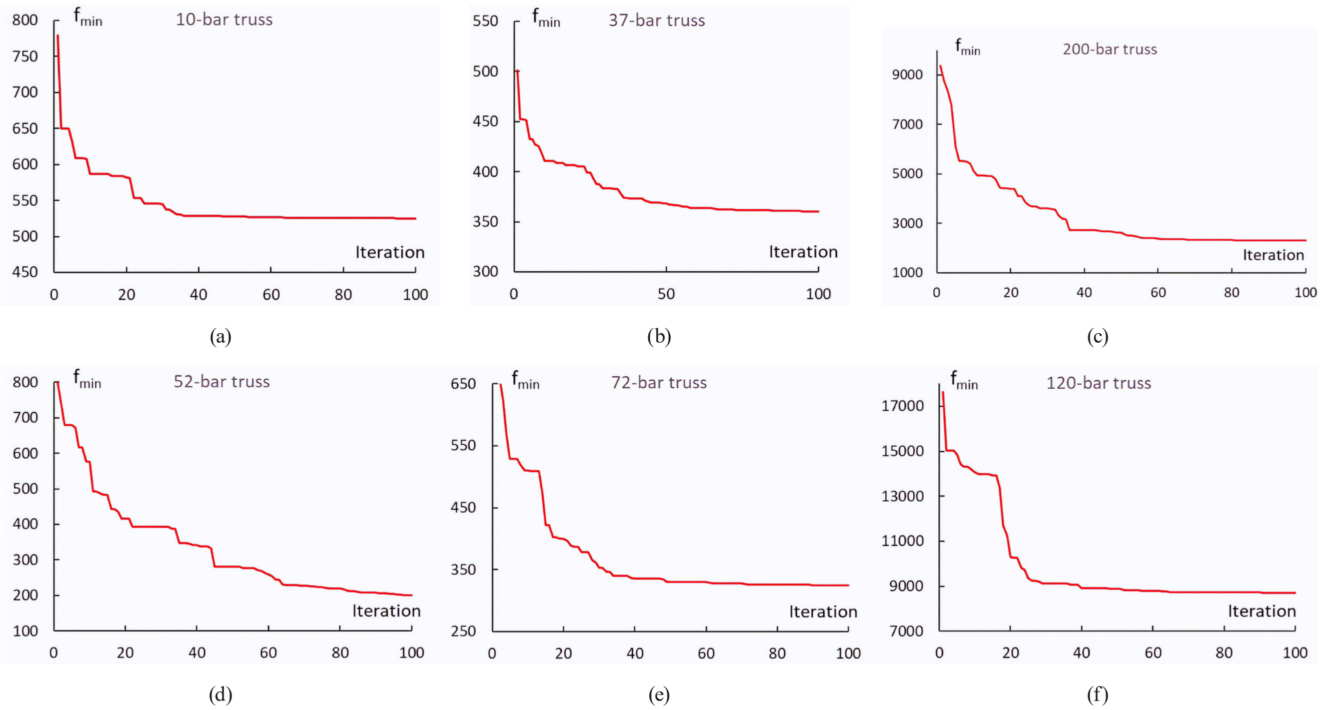


Fig. 6 Convergence of the strategy *sDE/best/2* of six truss problems

it needs more comprehensive analyses to evaluate quantitatively the superiority of the proposed method. For example, the sensitivity analysis to determine the impact of a random selection of mutation and crossover constants, in conjunction with the time-complexity analysis.

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