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USE OF APPROXIMATE ENGINEERING METHODS FOR SEMI-RIGID FRAME ANALYSIS

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Abstract

This paper gives a short summary about the possibilities of constructing more precise, but not too difficult connection models for frame analysis.

It is dealing with simple methods, which can be useful tools for

- either to construct envelope curves for the elastic and for the plastic behaviour separately,
- or to determine the elastic – plastic response of the frame by concentrating all of non-linearities into real or pseudo connection springs, which are connecting the elastic members.

Keywords: steel frames, semi-rigid, beam-to-column connection, column base.

1. Introduction

The purpose of frame analysis is to determine the limits of structural usefulness and to compare the predicted behaviour with the required one. Such an analysis is always part of a design process, wherein adjustments and new analyses are carried out until the design requirements are met.

Frame behaviour could be best characterised by the relationship between the applied loading and characteristic deformations, the structural response.

It would be desirable to construct load-deflection curves for each structure because they would contain all the information that is necessary for checking the structural behaviour. But such curves are more or less non-linear from the very beginning because of second order geometrical effects and material non-linearities. Later the slope of the curve is further reduced because of local plastification or of some instability phenomena.

The situation becomes more complicated, if structures contain such joints, which are non-negligibly different from the ideal ones, as hinged and rigid.

In case of frames connections have special importance, as their load carrying and deformation characteristics can modify the overall behaviour of the frame in a very wide range. Between the theoretically clear, two extreme cases, as hinged and rigid, the palette of practically used connections is very wide and allowance should be taken for it during the design process (CHEN and FIELDING (1972); CHEN (1987); CHEN and LUI (1991); LUI (1985)).

In case of semi-rigid joints the importance of using approximate engineering methods is increasing, as they can produce upper bound envelope curves for the real behaviour in a relatively simple way. The accuracy of the results is depending on the simplifications involved.

The most general moment – relative rotation relation of a cross-section includes elastic, plastic and strain-hardening zones. If the phenomenon of plate buckling is included into this relationship, the curve may contain a softening branch, when the softening effect of plate buckling is greater than the effect of strain-hardening (IVÁNYI (1992)).

Similar statements are valid for the joints of bar systems, however, the presence of a softening zone is more typical because of the occurrence of plastification and stability phenomena in joint elements.

For plastic analysis of steel frames one of the most well-known approximate engineering methods is the Mechanism Curve Method (HORNE and MERCHANT (1965); HORNE and MORRIS (1981)), which is suitable to determine the collapse mechanism load factor of steel frames.

This method, in an extended form, is also able to take the effect of geometrical changes into account with a reasonable accuracy. The rigid-hardening material model can also be introduced into this method and it can handle together the effects of geometrical changes and of strain-hardening phenomenon.

It can also be shown that the effect of plate buckling can also be involved in a rather similar way (IVÁNYI (1992)).

In case of steel frames the cross-sections of potential plastic hinges are including those ones, where the joints are situated, e.g. the supports and the intersection zones of the vertical and horizontal members.

In these areas, if the joints are of partial strength ones (MAZZOLANI (1990)), they can initiate a redistribution of internal forces before expansion of real plastic hinges in the beam or column sections. In this case all members can be elastic, when the collapse load is achieved because of joint plastification.

If the sections of real plastic hinges in members are characterised in a similar way, as the semi-rigid joints themselves, and they are concentrating the plastic, hardening or softening characteristics of the frame members, the bars themselves can be dealt with as fully elastic ones and all deviations from this are included in the behaviour of real or 'pseudo' joints.

2. Modelling Joints in Frames

In Annex JJ of Eurocode 3 joints are built up from two main parts, as sheared column web and components of connection (endplate, bolt, etc.) (*Fig. 1*). In certain cases it would be evident to divide the flexibilities into two springs, that is for a web panel spring and a connection one.

The web panel spring would have the task to take into account the shear deformation of the web panel, this way it could be a translational spring, as it has been treated in several papers before. This translational spring could be taken into the diagonal line connecting the tension flanges of the beam and column.

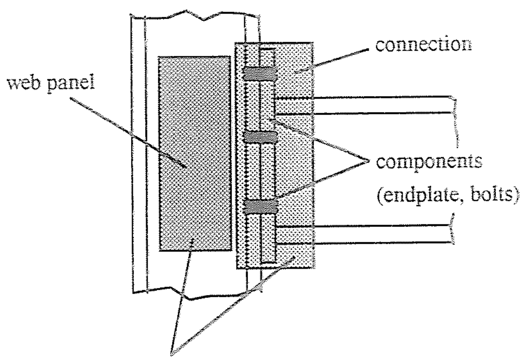


Fig. 1. Joint model of Annex JJ

The connection spring (including all other flexibilities) could be represented typically by a rotational spring, which could be situated to the end section of the beam.

Calculation results have shown that using these two kinds of spring models for characterising the behaviour of the joint has advantages.

These two springs are more suitable to simulate the joint characteristics and, which can be more important, they produce lower load carrying capacities and lower elastic critical loads for the numerically tested frame, than the calculation, which strictly follows the connection model of Eurocode 3.

Taking into account the second order effects and using these in separated way springs produce a relatively complicated calculation. Later on another possibility for handling these influences will be summarised in this paper.

3. Influences at Column Bases

In case of column bases there are some similarities. The typically bolted connection between the base plate and the foundation can have flexibilities, which are comparable with those of the bolted beam-to-column connections.

If some attention is paid for the 'connection' between the soil and the foundation, rather similar displacements to those of the frame joints can be found.

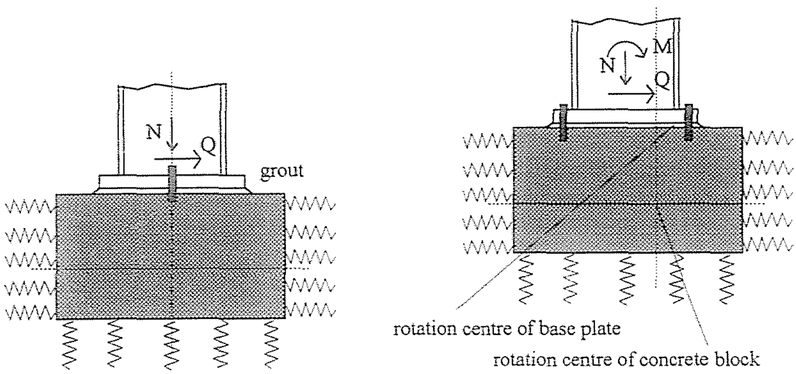


Fig. 2. Column bases

The position of the rotation centre of the concrete block is depending on the soil characteristics. The relatively rigid concrete block is situated between this point and the lower surface of the base plate which acts as a continuation of the column. But between the baseplate and the top surface of the concrete block in most practical cases a grout is situated, having different behaviour as the concrete foundation block.

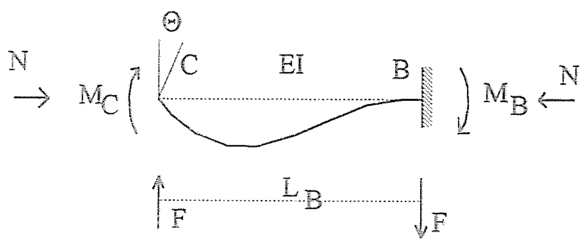
These similarities and differences gave the idea to develop a generalised bar model, which can take 'connection' and 'joint' flexibilities into account.

4. Use of Stability Functions

Stability functions can be practical tools to carry out an elastic second order analysis (HORNE and MERCHANT (1965); MAJID (1972)). These functions have been developed in different forms. In this paper those of Horne will be used.

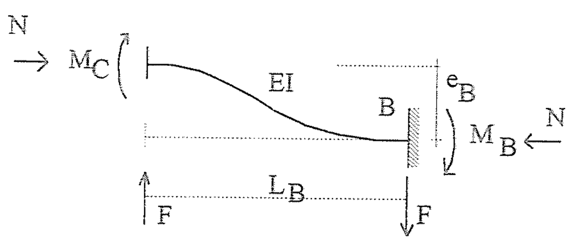
For the elementary case, i.e. when a member having fixed ends is influenced to a rotation on the left hand side end (*Fig. 3*), the end forces and moments can be expressed in the form given under the figure.

From this case all of the others (sway of fixed end – *Fig. 4*, hinged cases, etc.), when there is no distributed load along the length of the member, can be developed with superposition, supposing the same normal force for the added cases.



$$M_A = s.k.\Theta = S.\Theta$$
$$M_B = s.c.k.\Theta = T.\Theta$$
$$F = -s(1+c).k.\Theta/L = -\,U.\Theta\,/L$$

Fig. 3. Rotation at left end



$$k = EI/L$$
$$M_A = M_B = -u.k.e_B/L = -\,U.e_B/L$$
$$F = (2.u-\pi^2\rho).k.e_B/L^2 = V.e_B/L^2$$

Fig. 4. Sway at right end

The s (stiffness) and c (carry over) functions are depending on the relative intensity of the normal force, $\rho = N/N_E$, where $N_E = \pi^2 k/L$ (and N is positive in compression).

These functions let to take into account the influence of shear deformations of the web panel (Ermopoulos and Vayas (1991); Fielding and

HUANG (1971)). If we consider that the left end of the member is connected to the joint of a frame, at the joint there is a γ shear deformation (*Fig. 5*) and the centre of the panel remains in the same place, then the shear deformation $\gamma/2$ causes at the left end of the member: (i) a Θ rotation equal to $\gamma/2$ and (ii) an upraise vertical sway, which is equal to $L_A \cdot \gamma/2$.

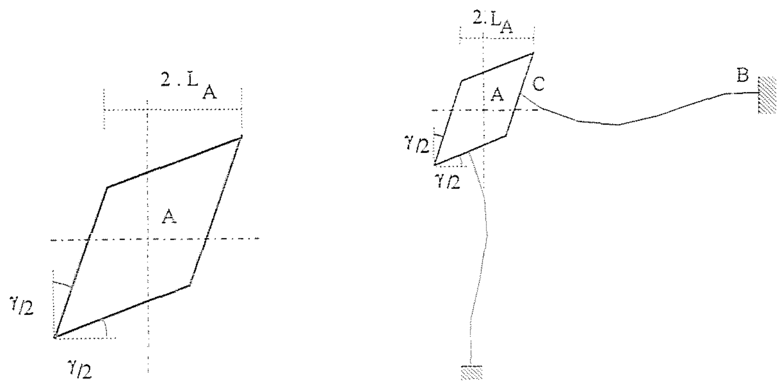


Fig. 5. Shear deformation of web panel and its influence for deformation of members

Combining the cases in *Figs. 3* and *4* the influence of web shear deformation can be derived.

From the right part of *Fig. 5* it can also be clearly seen that shear deformation of the web panel causes opposite effects for beam and column, respectively. This fact should be taken into account when compiling the stiffness matrix for the construction.

Before carrying out this combination process in details, because of its practical advantages, the bar model of *Figs. 3* and *4* is extended. A rigid part of the length of L_A is joined to its left hand side, replacing the web panel between the theoretical centres of the joint (point A) and the beam ends (point C).

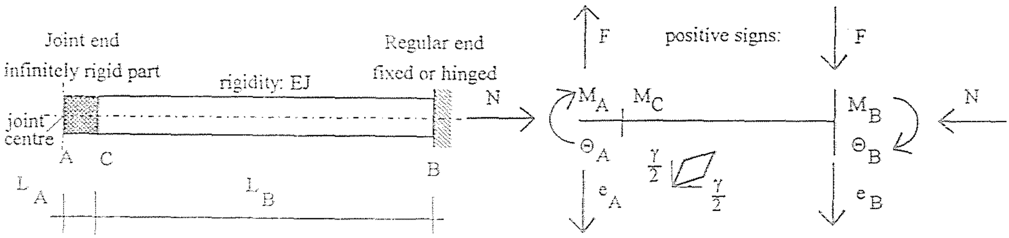
From simple equilibrium equations, written for $A-C$, $C-B$ and $A-B$ members, some new functions can be compiled, describing the relation among end forces and displacements.

These functions are given in *Table 1* for fixed and hinged bars.

Therefore, more serious and systematic analysis is necessary.

If a relative rotation should be taken into account at section C between the column flange and the beam end, it can be done by adding a new degree of freedom there and using functions in *Fig. 3*.

Table 1



Basic notation:	Fixed at B:		Hinged at B:
$k = EJ/L_B$	$s \cdot k = S$	$s \cdot (1+c) \cdot k = U$	$s \cdot (1-c^2) \cdot k = S$
$\rho = NL_B/(\pi^2 k)$	$sck = T$	$(2s(1+c) - \pi^2\rho)k = V$	$S" - \pi^2 \rho k = V"$

	M_A	M_B	F
	regular end at B is fixed		
Θ_A	$\left[S + \frac{L_A}{L_B}(1 + \frac{L_A}{L_B})V\right] = S_k$	T_k	$-U_k \frac{1}{L_B}$
Θ_B	$\left[T + \frac{L_A}{L_B}U\right] = T_k$	S	$-U \frac{1}{L_B}$
e_A	$\left[U + \frac{L_A}{L_B}\right] V \frac{1}{L_B} = U_k \frac{1}{L_B}$	$U \frac{1}{L_B}$	$-V L_{B^2}$
e_B	$-\left[U + \frac{L_A}{L_B}\right] V \frac{1}{L_B} = -U_k \frac{1}{L_B}$	$-U \frac{1}{L_B}$	$V L_{B^2}$
$\frac{\gamma}{2}$	$\left[S - 2U(\frac{L_A}{L_B})^2 + \pi^2 \rho k \frac{L_A}{L_B} 1 + \frac{L_A}{L_B})\right] =$ S_γ	$\left[T - \frac{L_A}{L_B}U\right] =$ T_γ	$-\left[U - \frac{L_A}{L_B}V\right] \frac{1}{L_B} = T_\gamma$ $-U_\gamma \frac{1}{L_B}$
	regular end at B is hinged		
Θ_A	$\left[S_k - \frac{T_k^2}{S}\right] = S_k"$	0	$-U_k" \frac{1}{L_B}$
e_A	$\left[U_k - \frac{U}{S}T_k\right] \frac{1}{L_B} = U_k" \frac{1}{L_B}$	0	$-\left[V - \frac{U^2}{S}\right] \frac{1}{L_{B^2}} =$ $-V" \frac{1}{L_{B^2}}$
e_B	$-U_k" \frac{1}{L_B}$	0	$V" \frac{1}{L_{B^2}}$
$\frac{\gamma}{2}$	$\left[S_\gamma - \frac{T_k}{S}T_\gamma\right] = S_\gamma"$	0	$-\left[U(1 - \frac{T_\gamma}{S}) - \frac{L_A}{L_B}V\right] \frac{1}{L_B} =$ $-U_\gamma" \frac{1}{L_B}$

In case of displacement method the equilibrium condition can be written as

$$[\underline{q}^*] = [\underline{\underline{K}}^*][\underline{u}^*] \,, \tag{1}$$

in which q^* is the load vector,
K^* is the stiffness matrix and
u^* is the displacement vector.

The displacement vector is built up from two main parts, from the group of regular nodal displacements (absolute rotations and deflections) and from the group of relative ones, caused by local flexibilities as shear deformations, connection components deformations, etc. According to the regular nodal displacements the corresponding equilibrium equations can be compiled. The second part of equilibrium equations are simply expressing the moment – relative rotation correlation at the local flexibilities, that is the $M - \phi$ functions.

It should also be taken into consideration that the relative rotations are always of opposite sense as the moments.

The functions for the shear or the rotation rigidity of a joint can be of any such shapes, which are in correlation (given by any mathematical function) with the change of loading.

The corresponding literature (CHEN (1987); CHEN and LUI (1991); LUI (1985)) illustrates that wide palette, which can be compiled either from experimental results or from modelling the above functions in different ways.

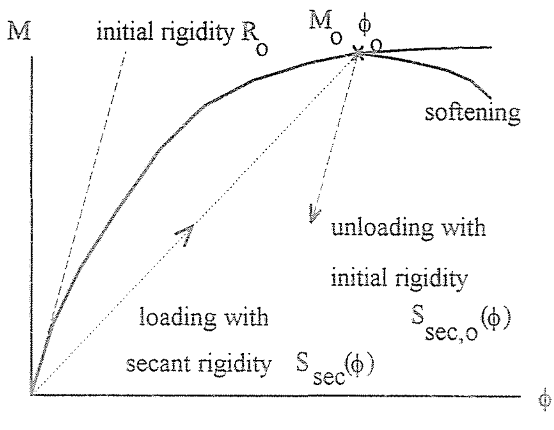


Fig. 6. Spring characteristic

The function of $M - \phi$, for example (*Fig. 6*), has a non-linear shape in case of loading, while in case of unloading it follows the straight line determined by the initial slope of the curve (by the initial rigidity) and the M_0 and ϕ_0 values achieved.

In general form the $M - \phi$ functions can be written as:

$$M = R \cdot \phi + M_1 \ , \tag{2}$$

where in case of loading:

$$R = S_{\mathrm{sec}}(\phi) \ , \qquad M_1 = 0 \ , \tag{3}$$

in case of unloading:

$$R = S_{\mathrm{sec,\,o}} \ , \qquad M_1 = M_0 - S_{\mathrm{sec,\,o}} \cdot \phi_0 \ . \tag{4}$$

If the external load on the structure is increasing in a monotonic way and there are no joint simulating springs, which are unloading, the actual secant stiffness, $M/\phi = S_{\mathrm{sec}}(\phi)$ should be substituted into the stiffness matrix.

5. Extended Use of Stability Functions and Springs

This method detailed above is suitable to analyse elastic members, taking allowance for geometric non-linearities (second order effects). At the same time, as the spring rigidity functions theoretically can be of any shape, with this set of expressions those cases also can be treated, when the spring rigidities involve some plastic, hardening or softening phenomena.

If any of the rotational springs at member ends is handled this way, it can include even the plastic behaviour of that section. The function used for representing the moment–rotation relation can be either bi-linear or piecewise linear or continuous and preferably smooth.

If a spring is inserted in any section of a member, where a real connection does not exist and the spring characteristics are including the properties of a plastic hinge, this so-called pseudo connection section is able to model plastic hinges, as well.

This way the use of the stability functions together with all of those possibilities, which were detailed before, can be extended to carry out an elastic – plastic hinge analysis. In this method all plastification is gathered into the plastic hinge and the complete member between two nearby hinges is supposed to be perfectly elastic.

The more exact plastic zone method, which took into consideration the reality, that plastification is a progressing process both in the fibres of a section and in a portion of a member, is more complicated and generally gives slightly different results.

In the case, when the frame to be analysed is carrying distributed loads, it is preferable to substitute it with concentrated ones, because the sections, in which plastic hinges develop in a member of distributed loading, are generally not known in advance.

This modification has two advantages:

(i) there is no change in shear force between two neighbouring points and
(ii) possible sections of plastic hinges are only these division points of the members.

One very important remark, which should be taken into account in any case, is the following: either non-linear behaviour of connection or plastic behaviour of the material is the reason of the relative deformations, it should always be kept in mind, that loading (increase of deformations accompanied by either increase or decrease of load intensity) follows the mentioned curves, while unloading (decrease of deformation and loading together) is always elastic (follows the initial rigidity). Therefore, if at any increment of the loading process a decrease of a certain displacement is seen, this fact should be noticed and it must be allowed for the compilation of the stiffness matrix.

6. Demonstrative Example

The frame in *Fig. 7* was realised from hot-rolled I sections (section height: 80 mm, span of beam: 2 m, height of column: 1 m), having different welded beam-to-column connections, nominally pinned at supports and tested in the framework of a diploma project in the Laboratory of the Department of Steel Structures, TUB.

Three types of frame knees were tested: (i) no stiffeners, (ii) horizontal stiffeners, (iii) horizontal + diagonal stiffeners. The ratio of $H : V = 5 : 1$.

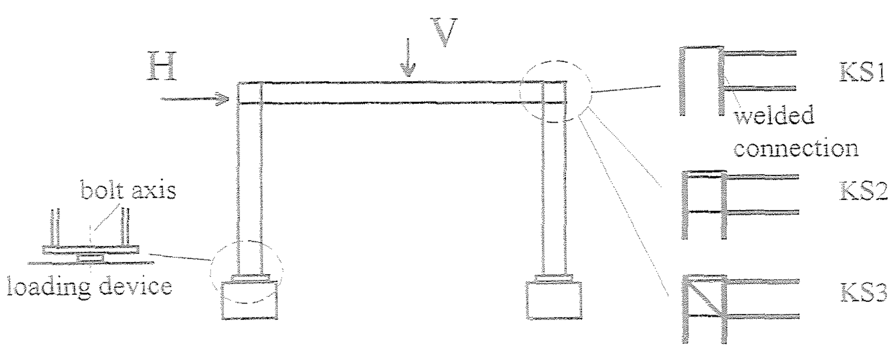


Fig. 7. Test model built up from hot rolled sections

Left part of the *Fig. 8* shows the sections, where springs should be inserted to take account for local flexibilities, plastic hinge rotations, etc. Allowing for the real conditions, some simplifications were made, as (i) a strong steel device was constructed to transfer the loading and to support the test frame – there were no foundation displacements; (ii) flexibility of welded connection on the interface of beam-to-column is small – there is no need

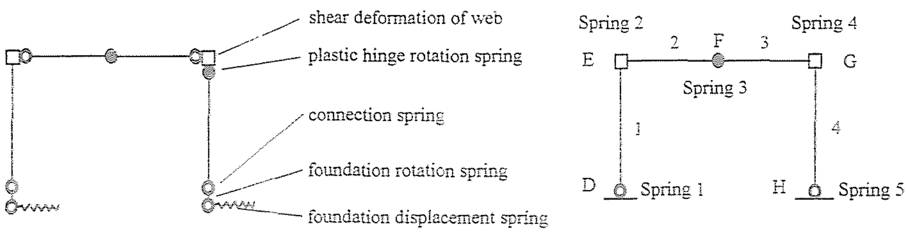


Fig. 8. Frame model built up from springs and elastic members

for connection springs at beam ends, (iii) beam-to-column connections are not stronger than the sections themselves – no need for plastic hinge springs around the joints.

The simplified model of the frame is shown on the right of *Fig. 8.* The model consists of four elastic bars (1–4), five nodes ($D-H$) and springs, Spring 2 and 4 for shear deformation, Spring 1 and 5 for nominally pinned supports, Spring 3 for plastic hinge under the vertical force. Axial deformation of bars is neglected.

Table 2 shows the stiffness matrix and load vector. The first three columns of stiffness matrix correspond to usual absolute rotations of nodes $E-F-G$, the fourth one to horizontal sway at $E-G$, the fifth one to vertical deflection at F. Columns 6–10 are relating to spring rotations.

Rows 1–5 of the stiffness matrix are giving the regular equilibrium equations for moments M_E, M_F and M_G, for forces H_E and V_F, respectively.

Rows 6–10 represent the corresponding relationships between the moment – relative rotation characteristic for the spring and the actual moment at the end of the bar.

Therefore, the construction of the stiffness matrix is non-regular, it is non-symmetric. From the theoretical point of view it is not exact, as in this case the product of load vector by displacement vector is not equal to the external work. Despite this, keeping simplicity and descriptiveness of such construction of the stiffness matrix, the relative displacements are kept further on, although to include absolute displacements everywhere instead of the relative ones could be done relatively easily.

Loading of the computational model can be increased step-by-step. Control of the loading process can be made by force or by displacement. Load control is suitable, while the structural model is stable, in the unstable stable control of the process is made by horizontal deflection at node E. In this latter case column 4 of the stiffness matrix and the load vector

Table 2
Stiffness matrix

	1	2	3	4	5	6	7	8	9	10
1	$(S_k)_1$ $(S_k)_2$	$(T_k)_2$	0	$-(\frac{U_k}{L_B})_1$	$-(\frac{U_k}{L_B})_2$	$(T_k)_1$	$-(S_\gamma)_1$ $(S_\gamma)_2$	0	0	0
2	$(T_k)_2$	$(S_k)_2$ $(S_k)_3$	$(T_k)_3$	0	$-(\frac{U}{L_B})_2$ $(\frac{U}{L_B})_3$	0	$(T_\gamma)_2$	$(S)_3$	$(T_\gamma)_3$	0
3	0	$(T_k)_3$	$(S_k)_3$ $(S_k)_4$	$-(\frac{U_k}{L_B})_4$	$(\frac{U_k}{L_B})_3$	0	0	$(T_k)_3$	$(S_\gamma)_3$ $-(S_\gamma)_4$	$(T_k)_4$
4	$-(\frac{U_k}{L_B})_1$	0	$-(\frac{U_k}{L_B})_4$	$(\frac{V}{L_B^2})_1$ $-(\frac{V}{L_B^2})k_4$	0	$-(\frac{U}{L_B})_1$	$(\frac{U_k}{L_B})_1$	0	$(\frac{U_\gamma}{L_B})_4$	$-(\frac{U_\gamma}{L_B})_4$
5	$-(\frac{U_k}{L_B})_2$	$-(\frac{U}{L_B})_2$ $(\frac{U}{L_B})_3$	$(\frac{U_k}{L_B})_3$	0	$(\frac{V}{L_B^2})_2$ $(\frac{V}{L_B^2})_3$	0	$-(\frac{U_\gamma}{L_B})_2$	$(\frac{U}{L_B})_3$	$(\frac{U_\gamma}{L_B})_3$	0
6	$(T_k)_1$	0	0	$-(\frac{U}{L_B})_1$	0	$(S)_1$ Spr_1	$-(T_\gamma)_1$	0	0	0
7	$(S_k)_2$	$(T_k)_2$	0	0	$-(\frac{U_k}{L_B})_2$	0	$(S_\gamma)_2$ Spr_2	0	0	0
8	0	$(S)_3$	$(T_k)_3$	0	$(\frac{U}{L_B})_3$	0	0	$(S)_3$ Spr_3	$(T_\gamma)_3$	0
9	0	$(T_k)_3$	$(S_k)_3$	0	$(\frac{U_k}{L_B})_3$	0	0	$(T_k)_3$	$(S_\gamma)_3$ Spr_4	0
10	0	0	$(T_k)_4$	$-(\frac{U}{L_B})_4$	0	0	0	0	$-(T_\gamma)_4$	$(S)_4$ Spr_5
	Load vector:									
	0	0	0	H	V	0	0	0	0	0

$\mathrm{Spr}_1 - \mathrm{Spr}_5$ denote the actual secant stiffness of springs.

are exchanged and the system of equations is solved for a given horizontal deflection.

Iteration at a new loading step starts with the previous normal force distribution in the bars, the normal forces are recalculated from the new displacements, while the given tolerance in the compatibility of results is fulfilled.

Some illustrative outputs of the program are given in *Fig. 13* in the next chapter. Systematic comparison of computer simulation results are also published there.