7. Parametric Study on the Influence of Semi-Rigid Frame Knees and Support Conditions

The relative moment – relative rotation curves, according to Eurocode 3, of frame knees '$KS1$' and '$KS2$' of the frame (*Fig. 7*) are shown in *Fig. 9*. '$KS1$' is the non-stiffened version, which is really semi-rigid. '$KS2$', containing horizontal stiffeners at flange levels, has a moment–rotation curve just beneath the limit curve in EC3 for sway frames.

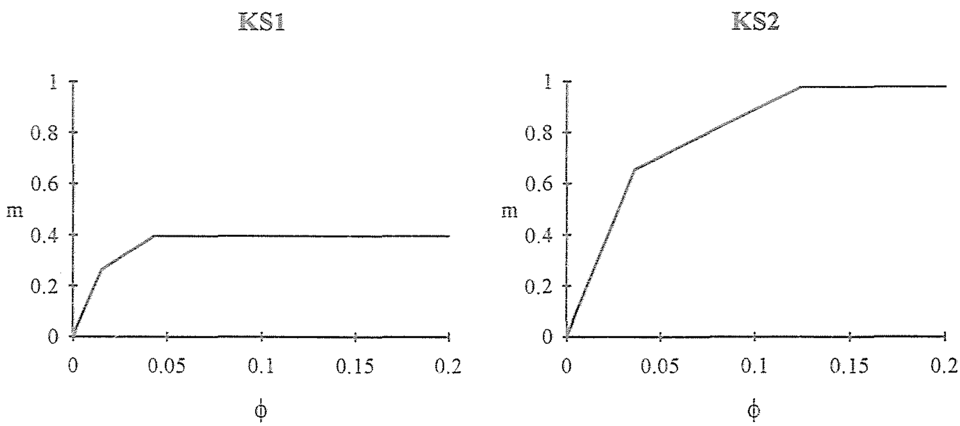

Fig. 9. Moment–rotation curves by EC 3

The horizontally and diagonally stiffened '$KS3$' is a rigid and full strength joint in EC3.

The column base was structurally solved as pinned. Neither calculations nor measurements have been made to check the reality of this supposition.

Separated tests have been carried out on knee models (*Fig. 10*).

During the experimental work (IVÁNYI Jr. (1993, 1994)) special emphasis was due to the measurement of relative rotation of joints. The absolute rotation of sections A and B in the figure was measured by means of optical methods. From the difference of the absolute rotations at A and B, the relative rotation of joint has been calculated. This way the moment–rotation curves for the joints can be constructed.

Similar measurements have been carried out during the tests on the frames themselves. In those cases the load–rotation diagrams may be determined.

From the numerical point of view there are differences among the $M-\phi$ curve properties, if those by EC3, by separated knee models and by

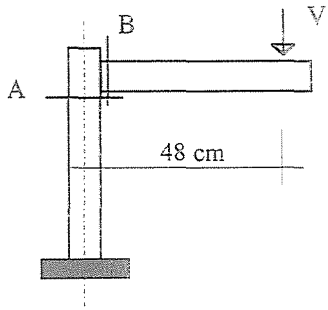

Fig. 10. Frame knee models

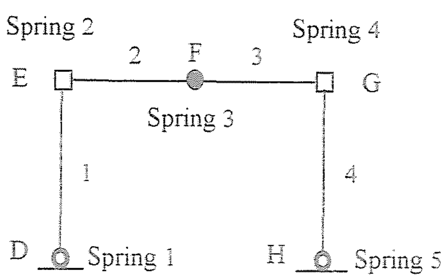

Fig. 11. Frame model built up from springs and elastic members

frame measurements are compared. Therefore, more serious and systematic analysis is necessary.

For illustration purposes the spring properties are supposed in the parametric study, as follows:

Supports (spring 1 and spring 5):

HINGED	$S_{sec,\,o} = 450$ kNm	$M_{max} = M_{pl}/100 = 0.069$ kNm	$\phi(M_{max}) = 0.0050$
SEMI-RIGID1	$S_{sec,\,o} = 650$ kNm	$M_{max} = M_{pl}/10 = 0.69$ kNm	$\phi(M_{max}) = 0.0050$
SEMI-RIGID2	$S_{sec,\,o} = 850$ kNm	$M_{max} = M_{pl}/3 = 2.3$ kNm	$\phi(M_{max}) = 0.0050$
SEMI-RIGID3	$S_{sec,\,o} = 1050$ kNm	$M_{max} = 2M_{pl}/3 = 4.6$ kNm	$\phi(M_{max}) = 0.0075$
RIGID	$S_{sec,\,o} = 1250$ kNm	$M_{max} = M_{pl} = 6.9$ kNm	$\phi(M_{max}) = 0.0100$

Beam midspan (spring 3):

as the calculated moment capacity of the section is $M_{pl} = 6.9$ kNm, plastic hinge properties are approximated as

$$S_{sec,\,o} = 3000 \text{ kNm} \quad M_{max} = M_{pl} = 6.9 \text{ kNm} \quad \phi(M_{max}) = 0.0040$$

Frame knees (spring 2 and spring 4)

/1	$S_{sec,\,o} = 850$ kNm	$M_{max} = M_{pl}/3 = 2.3$ kNm	$\phi(M_{max}) = 0.0050$
/2	$S_{sec,\,o} = 1050$ kNm	$M_{max} = 2M_{pl}/3 = 4.6$ kNm	$\phi(M_{max}) = 0.0075$
/3	$S_{sec,\,o} = 1250$ kNm	$M_{max} = M_{pl} = 6.9$ kNm	$\phi(M_{max}) = 0.0100$

In the present study the $M - \phi$ curve above $\phi(M_{\max})$ is constant, beneath this value the curve is approximated as:

$$M = S_{\text{sec}} \cdot \phi = S_{\text{sec, o}} \cdot (1 - a \cdot \phi^{n}) \ , \tag{5}$$

$$\frac{S_{\text{sec}}(M_{\max})}{S_{\text{sec, o}}} = \frac{n}{(n+1)} \ , \tag{6}$$

$$a = \frac{1}{(n+1) \cdot \phi(M_{\max})^{n}} \ . \tag{7}$$

If the mentioned expressions remain valid above $\phi(M_{\max})$, instead of unlimited plastic behaviour the influence of descending moment–rotation characteristics can be analysed.

According to the second expression, choosing a value for n, the ratio of secant stiffnesses can be suitably changed.

Above $\phi(M_{\max})$ the curve is constant, that is

$$M = M_{\max} \quad \text{and} \quad S_{\text{sec}} = M_{\max}/\phi \ . \tag{8}$$

To illustrate the influence of different parameters for the calculated, second-order load carrying capacity of the frame, the maximal vertical loads are compared to the first order collapse load, calculated on fixed frame, having rigid connections and rigid-plastic material. This first order collapse load from combined mechanism is

$$V_{pl} = \frac{4M_{pl}}{L} = \frac{4 \cdot 6.9}{1} = 27.6 \text{ kN} \ . \tag{9}$$

Fig. 12 clearly shows that column base strength increase has the most remarkable influence if both groups of connections (beam-to-column and column bases) are similar, that is all of them are nearly hinged or nearly fixed ones.

As the program prints complete structural response curves for characteristic internal forces and moments, for absolute and relative displacements, for the change of spring rigidities; the internal redistributions and their influence can be analysed. To illustrate these outputs, *Fig. 13* shows the load – joint rigidity curves for hinged column base and different frame knee connections.

8. Simple Plastic Analysis

As the extended use of the stability functions is too complex for designers' everyday practice, it is worth seeing, what kind of uncomplicated techniques are available in the plastic field, which could be coupled a simple elastic check to give an upper bound estimate for the behaviour.

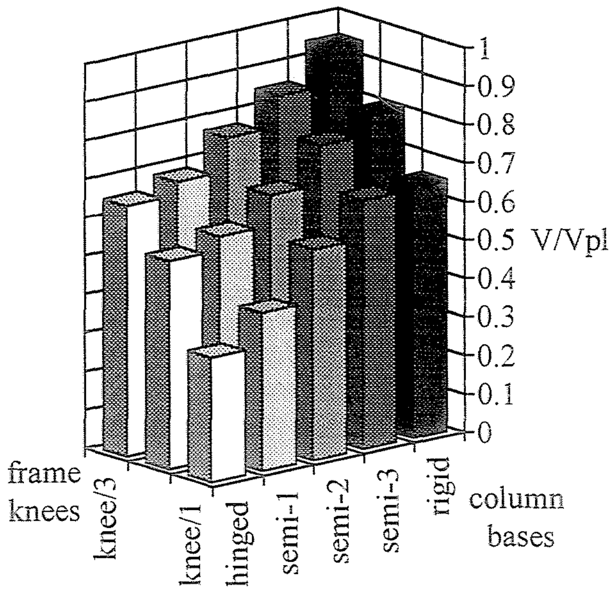

Fig. 12. Influence of parameters for load capacity

Plastic analysis of structures can be carried out using different, less or more simplified material models (*Fig. 14*). Numerous approximate engineering methods are introduced in the literature. The simplest one is the rigid–ideally plastic supposition, which neglects the effect of both elastic and strain-hardening states of steel. Because of their opposite effects the rigid–plastic material model produces a relatively good agreement with the plastic load carrying capacity, if a certain plastic deformation is achieved.

A relatively exact approximation is the rigid–plastic-hardening model, which can be simplified to the rigid-hardening one.

In case of increasing loading plastification is progressively developing in parts of member sections and along the member length. To take this phenomenon into account is a relatively difficult process and concentration of plastification into designated sections, into the plastic hinges, is a general way of treatment.

Activity of HORNE (HORNE, M. R. – MORRIS, L. J. (1981)) in using different material models is well-known. The Mechanism Curve Method can be applied to take the effect of finite deformations and strain hardening of steel into consideration.

Plastic collapse loads are idealised failure loads of elastic–plastic structures and they correspond to infinitely small structural deformations and

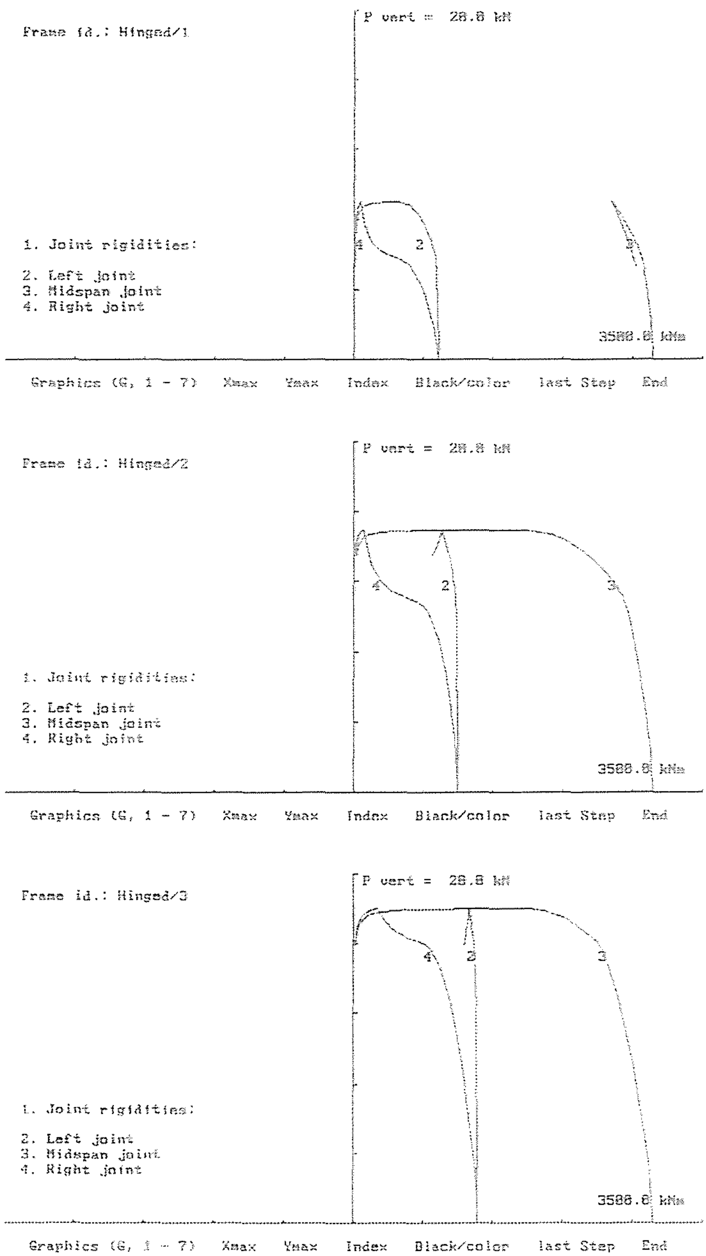

Fig. 13. Change of joint rigidities

to infinitely small plastic hinge rotations. Choosing an adequate pattern of plastic hinges (a so-called yield mechanism) the virtual work equation can be written as:

$$\lambda_p \sum_i Q_i u_i = \sum_j M_{pj} \Theta_j \ , \tag{10}$$

whereλ is the load parameter, Q_i are the external loads, u_i are the displacements of external loads, M_p are the moment capacities of plastic hinge sections, Θ_j are hinge rotations.

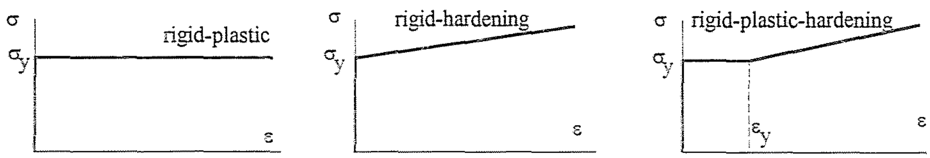

Fig. 14. Material models for structural steel

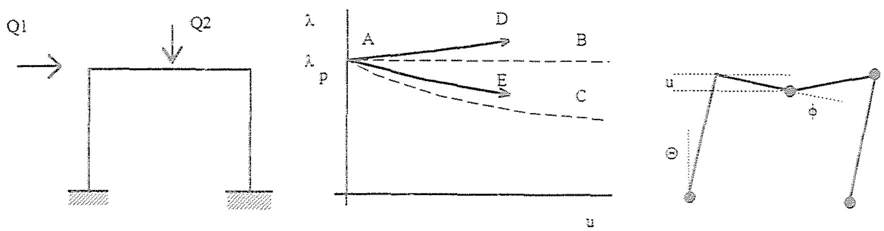

Fig. 15. Variation of plastic collapse load parameter

From this equation the value of λ_p can be determined, which corresponds to (practically) zero deformations (*Fig. 15* – line $A - B$). If we want to know the influence of finite deformations, we have to follow the variation of the load parameter at increasing finite values of the rotation ϕ. The $\lambda - \phi$ curve is, of course, non-linear and its exact determination is quite difficult.

HORNE (HORNE, M. R. – MORRIS, L. J. (1981)) has shown that a simple treatment can give a value for the load parameter, which is correct for the first power of ϕ. The virtual work equation for the incremental deformations may be written as:

$$\lambda \sum_i Q_i u_i + \sum_k N_k L_k \phi_k d\phi_k = \sum_j M_{pj} d\Theta_j \ . \tag{11}$$

The new second term on the left hand side (where N_k are the normal forces, L_k are the lengths) is the additional external work of finite deformations. Supposing, that

$$N_k = \lambda/\lambda_p \cdot N_{kp} \quad \text{and} \quad du_i/u_i = d\phi_k/\phi_k = d\Theta_j/\Theta_j \ , \tag{12}$$

the above expression can be transformed to:

$$\lambda \left(\sum_i Q_i u_i + \sum_k \frac{N_{kp}}{\lambda_p} L_k \phi_k^2 \right) = \sum_j M_{pj} \Theta_j \ . \tag{13}$$

This expression gives the relationship $A - C$ in *Fig. 15.*

Because of the influence of strain hardening the plastic moment capacity of a section is increased from M_p to $M_p + m$. Using the rigid-hardening model and the principle of 'equivalent cantilever' this m increase can be written as:

$$m = M_p \sqrt{\frac{b \cdot E \cdot \Theta}{f \cdot h \cdot K \cdot \sigma_y}} \ , \tag{14}$$

where E/K is the slope of the strain hardening line, f is the shape factor of the symmetrical I-section, b is the section depth and h is the length of the equivalent cantilever.

As this new term increases the internal work, it could be included on the right hand side and the governing equation becomes:

$$\lambda \left(\sum_i Q_i u_i + \sum_k \frac{N_{kp}}{\lambda_p} L_k \phi_k^2 \right) = \sum_j (M_{pj} + m_j) \Theta_j \ . \tag{15}$$

If this result is compared with the original equation, which was not affected either by finite deformations or by strain hardening, the relation of the two new terms determine the final shape of the $\lambda - u$ curve. If strain hardening is dominant compared to the influence of deformations, line $A - D$ is the result, while in the opposite case it is line $A - E$.

If the 'stability ratio' is defined as:

$$R = \frac{\sum_j m_j \Theta_j}{\lambda_p \sum_k N_k L_k \phi_k^2} \ , \tag{16}$$

the above statement can be transformed to the following:

- if $R > 1$, then the tendency of the $\lambda - u$ curve is increasing, while
- in case of $R < 1$ the deformations will reduce the load parameter.

In the first case the plastic hinge can be called as a plastic-hardening one, while in the second as a plastic-softening one.

The above discussed idea can be used to take other special effects into consideration, as well.

9. Special Phenomena in Frames

In plastic design of steel structures special care should be taken for the instability problems, among others, for plate buckling. To carry out this, an approximate engineering method is proposed by IVÁNYI (IVÁNYI, M. (1992)).

If the plate slendernesses are chosen in that they prevent development of plate buckling before achieving the plastic load carrying capacity (before forming of the yield mechanism), plate buckling is taken into account in the so-called indirect way.

If direct analysis of plate buckling in the plastic range is carried out, a similar curve is the result, as line $A-C$ in *Fig. 15.* Of course, a different (generally greater) load parameter is belonging to plate buckling in the undeformed state, than to the plastic mechanism.

Without dealing with details of the determination of this curve, we should refer to the fact that instability of plates (local) or elements does not mean the instability of the complete construction. For this that simple frame can be used, which is shown in *Fig. 16.* If the rigid body, which is supported by two columns of different load carrying character, is let to move only vertically, the overall load-deformation diagram is simply the sum of the individual ones.

In the range, where column 1 is unstable and column 2 is stable, the structure can be stable, while the sum of rigidities is positive.

Plate buckling is typically a phenomenon, which produces a softening behaviour, but its influence can be counterbalanced by some other effects, e.g. by strain hardening. As these effects in certain sense are similar, their influence can be analysed also in a similar way.

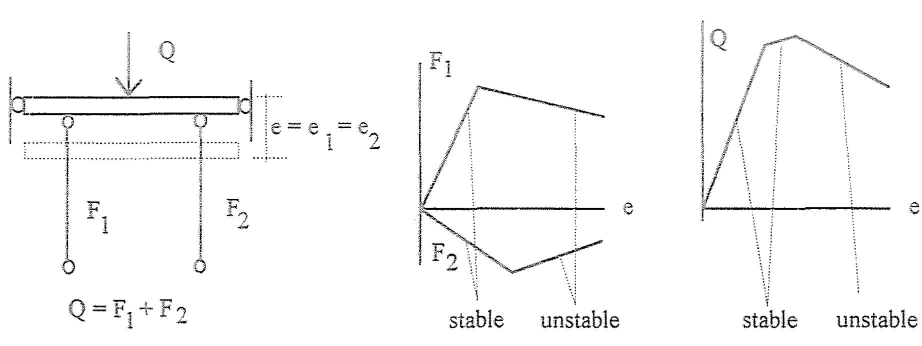

Fig. 16. Unstable state of elements and of the structure

The category of strain hardening plastic hinge can be extended to handle the plate buckling phenomenon, as well. The essence of the Approximate Engineering Method is that these two effects are separated and the interactive plastic hinge is compiled from these components.

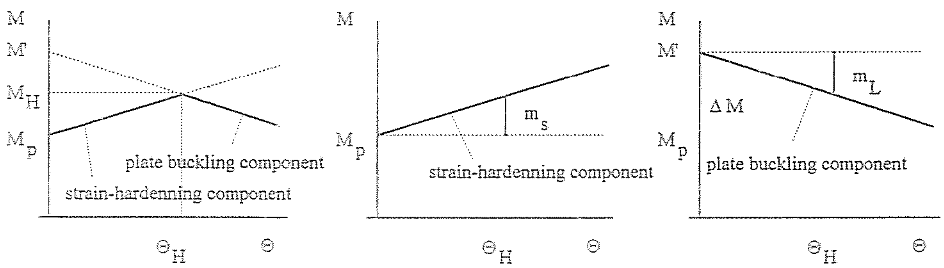

Fig. 17. Interaction of strain hardening and plate buckling

For the plate buckling component it is assumed, that M' buckling moment in *Fig. 17* can be achieved in the rigid state and the curve is descending because of deformations. The intersection point of the two (strain hardening and plate buckling) curves can give generally the maximum of load carrying capacity.

The relating formulae can be expressed using those ones on the previous pages. This time the expression for strain hardening is:

$$\lambda_s \left(\sum Qu + \sum NL\phi^2\right) = \sum (M_p + m_s)\Theta \ , \tag{17}$$

$$\lambda_s = \frac{\sum M_p\theta + \sum m_s\theta}{\sum Qu + \sum NL\phi^2} \ , \tag{18}$$

while for plate buckling it is:

$$\lambda_L \left(\sum Qu + \sum NL\phi^2\right) = \sum (M' + m_L)\Theta \ , \tag{19}$$

$$\lambda_L = \frac{\sum M'\theta + \sum m_L\theta}{\sum Qu + \sum NL\phi^2} \ . \tag{20}$$

These formulae are suitable to draw an upper limit curve for the load carrying capacity with respect to deformations. The axial forces in the members are assumed to be proportional to the intensity of the loading.

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AN ADVANCED DESIGN OF STEEL BEAM-COLUMNS

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Abstract

In this paper an advanced analysis is proposed for computer aided design of steel beam columns. The analysis is based on a finite element method using the thin-walled beam-column element published by Rajasekaran. In the advanced design a design factor is introduced that can be derived from the test based buckling curves.

Keywords: steel beam-column, advanced analysis, computer aided design.

1. Introduction

The up-to-date design standards allow one to apply advanced analysis and design of structures when the structural members are of compact cross-sections with full lateral restraint [1]. The aim of advanced design using advanced analysis is to predict accurately the behaviour of in-plane structures. By advanced analysis the maximum load carrying capacity as well as the full-range load-displacement response should be predicted taking the relevant material properties, residual stresses, geometrical imperfections, connection behaviour, erection procedures and interaction with the foundations into consideration. Advanced analysis was used first to predict the column buckling capacity. The procedure was based on the finite difference method by BEER and SCHULZE [2]. Later this method was used by others [3] to predict the load carrying capacity of columns by taking the probability density function of each variable from the data base of the E.C.C.S. tests. TARNAI [4] applied advanced analysis to predict the capacity of partially restraint columns. Later the finite element method has replaced the final difference method. Including the large displacements some computer codes have been developed for advanced analysis of in-plane frames [5, 6]. Influence coefficient matrix method has been used by CAI et. al. [7] for verifications of the AISC LRFD design interaction equations of beam-columns. They have found that the experimental results for the strength were close to the numerical results. BILD and TRAHAIR [8] used more accurate finite element code to analyse the in-plane strengths of steel columns and beam-columns. However, most of the computer methods based on distributed plasticity suffer from the disadvantage of high computational time and as such are not currently practical for routine design of structures.

In this paper a practical finite element analysis is used in an advanced design of out-of-plane beam-columns. The PASBC code has been developed applying the thin walled finite element which has been published by RAJASEKARAN [9]. The described and proposed procedure can be integrated in a computer aided design method for beam-column structures. The method involves the buckling curves of Eurocode 3 and Hungarian Standard 15024 [13].

2. Concept for Advanced Design

In the past decades, mostly in the seventies, the E.C.C.S. has carried out an extensive experimental program on buckling of centrically loaded, hinged columns as well as in-plane loaded beams. The test program has been designed in such a way that the *buckling curves* with a certain probability of failure should be determined. A statistical analysis of the buckling stresses of columns proved that the buckling strengths are Gaussian distributed, therefore the buckling curves were derived as $\chi = \chi_m - 2\chi_s$ where χ_m was the mean value, χ_s was the standard deviation of the reduction factors of more than 1065 column and 235 beam tests. The implicit reduction function based on the Ayrton-Perry formulation has been proposed by MAQUOI and RONDAL [11]. The current design standards use this function. On the other hand, due to the great number of tests to be involved in the above approach, it cannot be extended easily to all the various restraints and load sets. Some investigations have gone into numerical test program [3] based on probability function of variables which determine the load carrying capacity. In case of a simple column the variables are: yield strength, residual strain, eccentricity, amplitude of initial curvature, area, modules of elasticity and length of member. However, the results have justified the E.C.C.S. curves, but have also highlighted the inaccuracy of the theorem of elastic extrapolation used to take the different end-conditions and load sets into consideration. In the case of beam-columns the set of variables should be extended to other parameters which are related to the flexural-torsional deflection as well as the combination of the axial and transverse loads. Because of the considerable expense of both experimental and numerical test programs, there is little chance of an overall investigation for a probability based formulation of the beam-column reduction factor.

Normally there is not enough information about the probability functions of parameters for use statistical analysis to predict the buckling strength of beam-columns. Moreover, the advanced analysis with semi-probability models would be time consuming, consequently too expensive for the practice. Using deterministic model the prediction of the buckling strength depends on the deterministic value of the model parameters, especially of the yield stress, the residual stress and the initial out-of-straightness. *Fig. 1* shows a possible deterministic beam-column model where the model properties can be written as

$$\sigma_y = \beta f_y\,; \qquad \sigma_r = \alpha\sigma_y\,; \qquad u_0 = \delta L, \tag{1}$$

where σ_y is the yield stress, σ_r is the maximum compressive stress in the residual stress model, L is the length of the structural member, and α, β and δ are the model parameters. It is clear that each set of model parameters defines a different buckling strength. Therefore, these model parameters should be calibrated by test results. There are two ways for the calibration:

(1) Two of the model parameters are fixed and the third one should be calibrated.
(2) All the model parameters are fixed and a *design factor* (φ_{bc}) should be introduced and calibrated:

$$\varphi_{bc} = \frac{S_{model}}{S_{b.Rd}}, \tag{2}$$

where S_{model} is the buckling strength given by the deterministic model using fixed model parameters, $S_{b.Rd}$ is the strength known by test result.

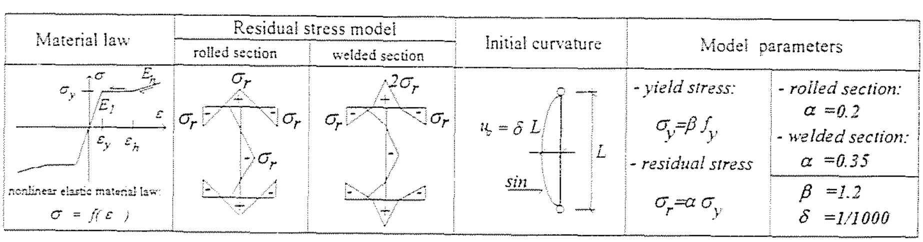


Fig. 1. A deterministic model for beam-columns

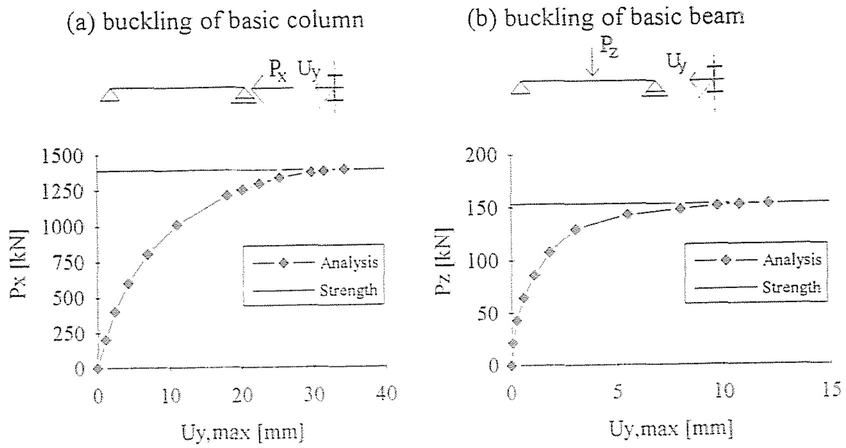
For some practical advantages, the second way is preferred in this paper. Assuming double symmetrical I section, the model parameters can be fixed close to the mean values: according to the European convention, the model parameter for initial out-of-straightness can be fixed as $\delta = 0.001$; the model parameter for yield stress can be fixed as $\beta = 1.2$; the maximum compressive stress in the residual stress model can be fixed as $\alpha = 0.2$ for rolled sections and $\alpha = 0.35$ for welded sections.

3. Analysis

The finite element analysis uses the thin walled beam-column element published by RAJASEKARAN [9]. The details of the computer code have been described in [12]. The solution technique uses a modified Newton–Raphson iteration:

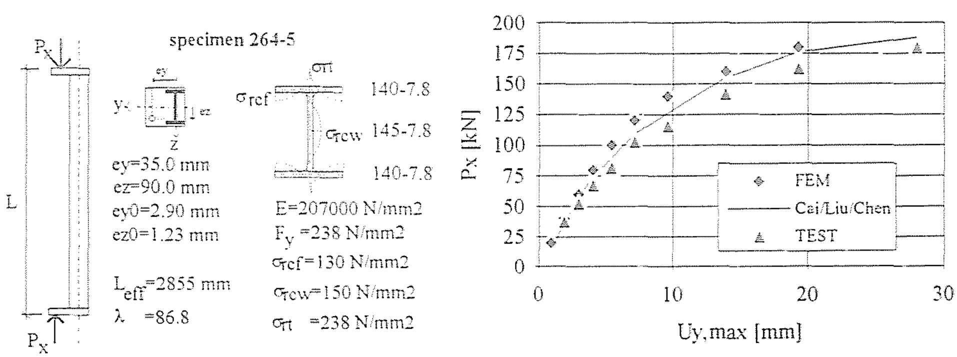
- step 1: second order linear elastic solution for the initial load set using direct iteration,
- step 2: computation of the unbalanced load vector where the balanced load vector is the sum of the geometric and the static load vectors calculated from the normal stress field of the elements,
- step 3: computation of the increment in displacements in the n-th iterative step using direct iteration (second order solution),
- step 4: compute the total displacements in the n-th iterative step as the sum of the increments in displacements,
- step 5: repeat steps 2 to 4 until the ratio of norm of unbalanced load vector and norm of total load vector is less than the allowable tolerance value ε_p (primary convergence condition), or until the ratio of norm of increments in displacements and norm of total displacements is less than the allowable tolerance value ε_s (secondary convergence condition).

According to the convergence test the proper tolerance value for the primary convergence condition is $\varepsilon_p = 10^{-7}$ and for the secondary convergence condition is $\varepsilon_s = 10^{-3}$. Using these tolerance values the iteration is normally stable and the limit load can be available without divergence. *Fig. 2* shows a typical column buckling and a beam buckling (lateral torsional buckling) analysis, where the member reduced slenderness (about the minor axis with the member length) is $\overline{\lambda} = 1.2$. The method has been compared with the result of the influence coefficient matrix method and with the experimental test that were published by CAI, LIU and CHEN [7]. *Fig. 3* shows the specimen and the results.



(section: HE300 A ; $E = 210000\ N/mm^2$, $E_h = 7000\ N/mm^2$; f_y=235 N/mm²)

Fig. 2. Buckling analysis



(FEM=PASBC code based on finite element analysis; Cai/Liu/Chen=influence coefficient matrix method: TEST=specimen test by Cai, Liu and Chen 1991)

Fig. 3. Comparison of the analysis with the coefficient matrix method and test

No.	nomination	dimensions
1	HE 300 A	Eunorm (h/b<1.2)
2	IPE 300	DIN (h/b>1.2)
3	W-1	flange: 300-12 web: 300-8
4	W-2	flange: 200-12 web: 400-8

Fig. 4. Rolled and welded I sections used in the present paper

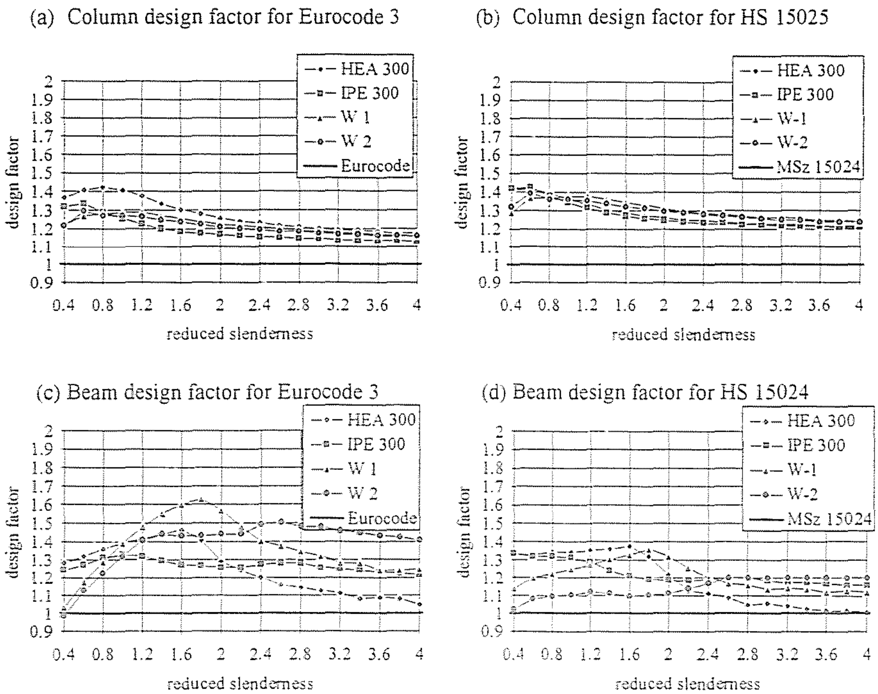


Fig. 5. Design factor for basic column and beam buckling

4. Calibration of the Design Factor

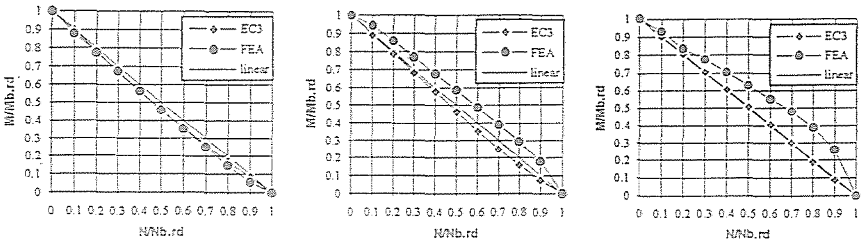
The design factor can be calibrated by the test based buckling curves which have been adapted in the up-to-date standards. The calibration has been carried out for Eurocode 3 and for Hungarian Standard 15024 assuming the rolled and welded sections shown in *Fig. 4*. Mild steel material was assumed where the design strength is $f_y = 235\,\mathrm{N/mm^2}$, the initial elastic modulus is $E_1 = 210000\,\mathrm{N/mm^2}$ ($E_1 = 210000\,\mathrm{N/mm^2}$ for HS 15024), the hardening modulus is $E_h = 7000\,\mathrm{N/mm^2}$ and $\varepsilon_h = 10\varepsilon_y$. The proper design factors for the basic buckling problems are shown in *Fig. 5*.

5. Design of Basic Beam-Columns

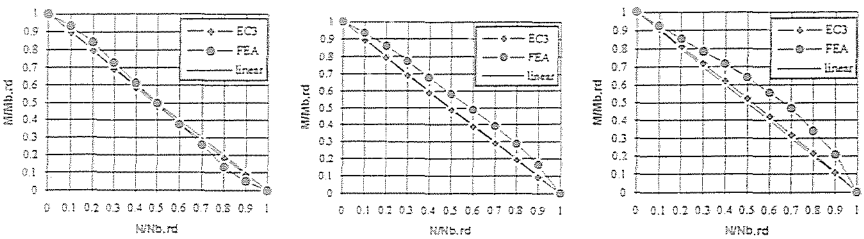
For beam-columns the design factor can be interpolated linearly between the basic beam and column problem. The interpolation is governed by the axial force ratio $r_N = N_{Sd}/N_{b.rd}$. This method extends the hypothesis of

(a) predicted strength for Eurocode 3

Section HEA 300 $[\bar{\lambda} = 0.4 \; - \; 1.2 \; - \; 2.0]$

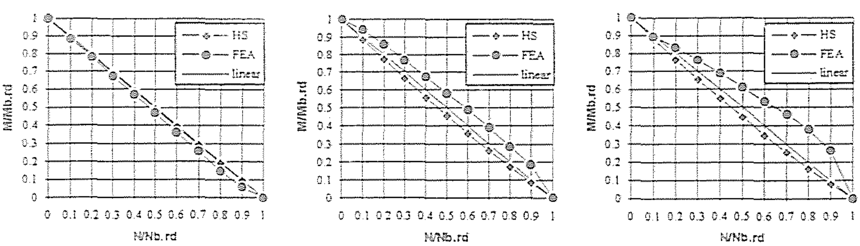


Section W-1 $[\bar{\lambda} = 0.4 \; - \; 1.2 \; - \; 2.0]$



(b) predicted strength for HS 15024

Section HEA 300 $[\bar{\lambda} = 0.4 \; - \; 1.2 \; - \; 2.0]$



Section W-1 $[\bar{\lambda} = 0.4 \; - \; 1.2 \; - \; 2.0]$

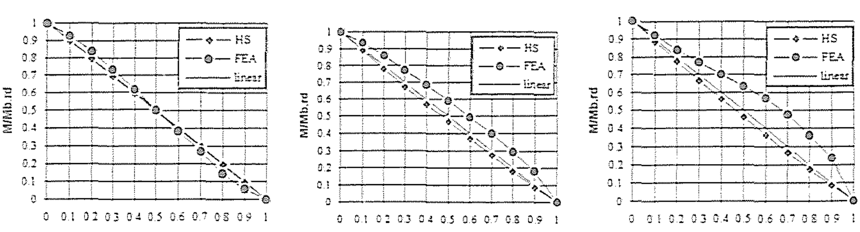


Fig. 6. Predicted strength for basic beam-columns

elastic extrapolation to beam-columns. The procedure excludes the application of the design interaction equations. *Fig. 6* shows some results of this method in comparison with the Eurocode 3 and the Hungarian Standard 15024.

6. Design of Basic Beam-Columns with Non-Basic Loading

A design factor has been introduced for the basic buckling problems such as column, beam and beam-column buckling. This factor reduces the strength that is given by the finite element analysis to the relevant strength specified by the actual standard. For any beam-column problem where the loading differs from the basic load set the design factor can be interpolated linearly by the actual normal force ratio $N_{Sd}/N_{b.rd}$. *Fig. 7* shows an example where the compressed member is loaded by single end-moment. The resulting strengths are compared with the Eurocode 3 and the Hungarian Standard 15024 specifications in *Fig. 7a* and *7b* for a wide flanged rolled and a welded section.

7. Design of Restraint Beam-Columns

The design factor depends on the reduced slenderness of the member. In case of non-basic beam-column where the ends are restrained the reduced slenderness should be calculated by elastic stability analysis. In an advance design this analysis is not evaluated in natural. If we use the member slenderness (slenderness of the basic column model), considerable difference may be given in the design factor. *Fig. 8a(c)* shows the predicted strength of fixed columns when the design factor has been calculated from the member slenderness, *Fig. 8b(d)* shows the results when the elastic extrapolated slenderness has been used. *Fig. 9* shows the results for fixed beams.

8. Conclusions

(1) The procedure that is based on the thin-walled finite element published by Rajasekaran is a sufficient tool for the finite element analysis of deterministic beam-column models if the primary convergence condition which controls the norm of the unbalanced loads is complemented with a secondary convergence condition which controls the norm of the increments in displacements due to the unbalanced loads. The secondary convergence condition stops the iteration when the

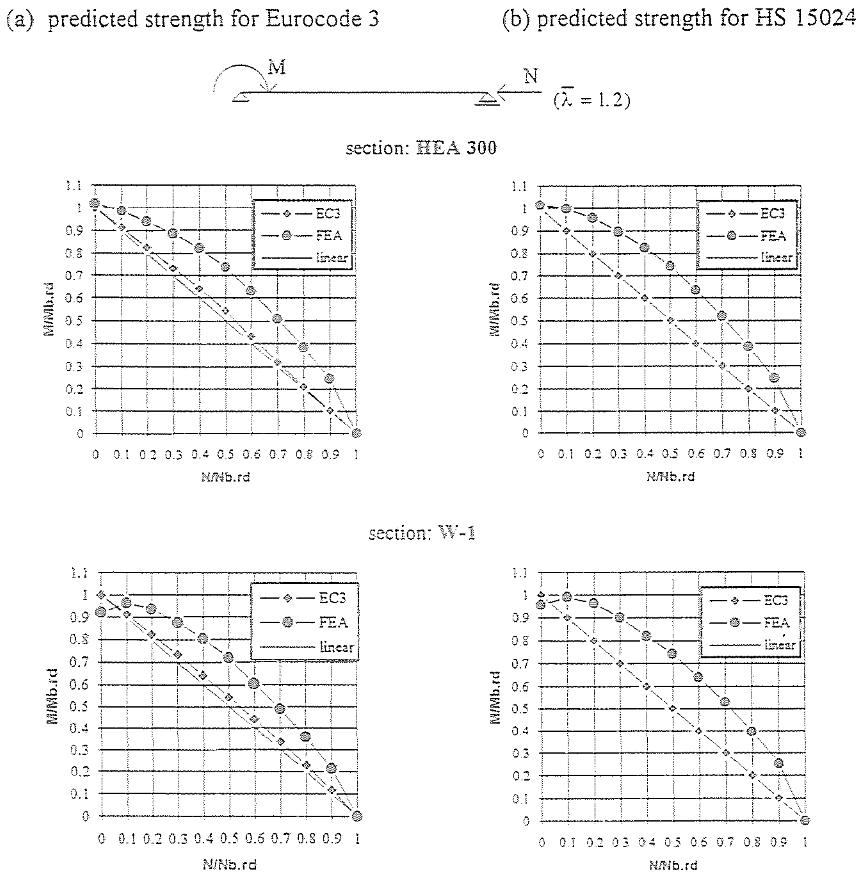


Fig. 7. Predicted strength for beam-column with non-basic loading

norms of the increments in displacements becomes insignificant even if the primary convergence condition is not satisfied.

(2) The design factors of the basic problems depend on the member slenderness. In case of columns with different sections the design factors are close to each other for both specifications. The only exception is the wide flanged HEA 300 section in case of Eurocode 3. In case of beams the design factors show significant spreading even in the higher range of member slenderness.

(3) For basic beam-columns of the intermediate and higher range of member slenderness the predicted strengths using linearly interpolated de-

sign factors between the basic column and beam buckling are higher than those given by the design interaction equations of the standards. The difference in higher range ($r_N > 0.7$) of axial compression is significant. The method gives similar result for basic beam-columns with non-basic load sets.

(4) The generalisation of the method for restrained (built-up) beam-columns meets difficulties: the design factors vary significantly with the member slenderness, therefore it should be determined from the effective slenderness that can be calculated by elastic stability analysis. Using the member slenderness (which is independent of the end-restriction) the advanced design of restraint columns results in maximum 13% overestimation and maximum 6% underestimation of strength in case of Eurocode and 9% (5%) in case of HS 15024. The advanced design of restraint beams results in maximum 28% overestimation and maximum 19% underestimation of strength in case of Eurocode and 27% (9%) in case of HS 15024.

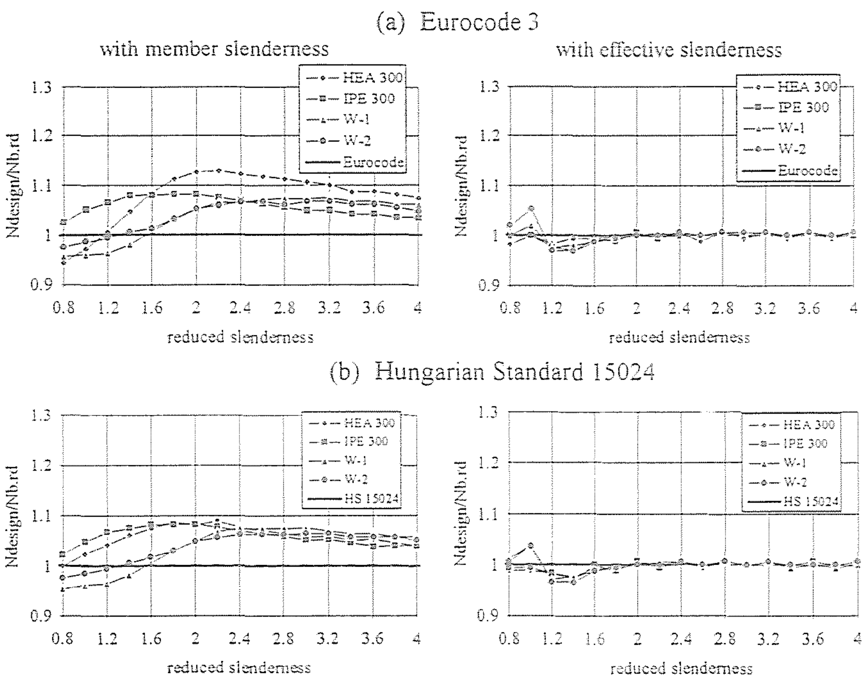


Fig. 8. Predicted strengths for fixed columns

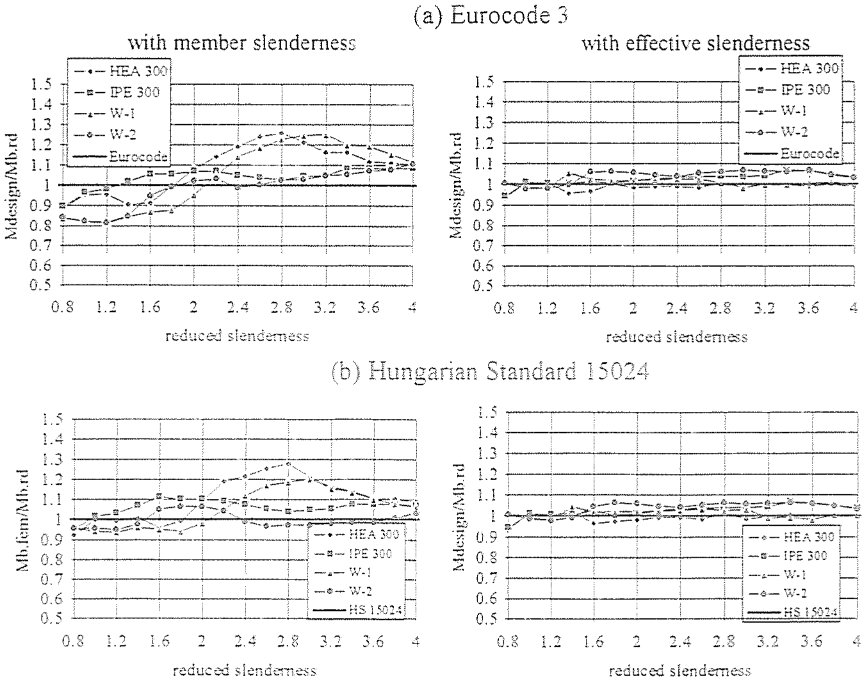


Fig. 9. Predicted strengths for fixed beams

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