THE GLOBAL AND LOCAL SCALE DEVIATIONS OF GEODETIC NETWORKS OBSERVED BY ELECTROMAGNETIC WAVE PROPAGATION IN REAL ATMOSPHERE

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Abstract

Since the last adoption in 1963 by IUGG of basic formulas for velocity index of air elapsed more than thirty years. An overview of thermodynamic investigations is given in development of determination and the most prospective computational formulas are enlisted for velocity index. Requirements for evaluation of principally new formulas are introduced. Suggestions are proposed for improvement of distance measurements' precision, based on electrodynamic approach.

Keywords: velocity index, electronic distance measurement.

Introduction

Measurement of length using propagation of electromagnetic waves has become a standard procedure for scientific and engineering purposes. Such kind of measurement requires accurate definition of time and length units. The second as a unit of time was defined by the XIIIth General Conference on Weights and Measures in 1967 as follows:

'The second is the duration of 9192631770 periods of the transition between two hyperfine levels of the ground state of the caesium-133 atom.'

This atomic standard of the second can be reproduced to an accuracy of 1 in $10^{-12}$ (or $10^{-15}$ for a short time). The time standard is maintained by hydrogen masers, rubidium and caesium atomic clocks.

The new definition of the metre was adopted by the XVIIth General Conference for Weights and Measures 20th October 1983 in Paris. It reads as follows:

'The metre is the length of the path travelled by light in vacuum during a time interval of $1/299792458$ of a second.'
Wavelengths from frequency stabilized laser sources can be reproduced to 1 part in $10^{-12}$ or better. The new definition effectively fixes the value of the velocity of light, thus making $c_0 = 299792458 \pm 1.4 \text{ m/s}$ in vacuum as a physical constant (Bay, 1974).\footnote{The unified Time – Length Unit System has been suggested by Hungarian origin American physicist Zoltán Bay in 1965, and accepted in 1983 as international standard.}

The above definitions of basic units and their repeatability make possible the absolute distance measurement in vacuum with precision to 1 in $10^{-12}$ or better. However, the overwhelming majority of length measurements is performed in real atmosphere, under ambient conditions. It requires the determination of velocity index $n_{\text{air}} = c_0 / c_{\text{air}}$ with the same accuracy to achieve a comparable precision in electromagnetic (light, microwave) distance measurement. Since it is well known that the velocity index of air is far not a physical constant but variable in time and space along the wave path, then for a precise distance measurement a certain representative (integral) value of velocity index has to be found simultaneously for any wavelength applied.

Neglecting the influence of velocity index in terrestrial electromagnetic distance measurement misleads to severe intolerances, hence a distance of 1000 m in vacuum will be measured by HeNe laser light as much as cca +300 mm longer in conventional normal atmosphere. Determination of equatorial length by such neglection would lead to a systematic error cca +12000 m in 40000 km, and cca +1800 m error in the radius of the Earth. The overall relative bias of normal atmospheric delay is therefore cca +300 ppm. The variability of the real atmosphere under ambient conditions causes an error cca $\pm$100 ppm in electronic distance measurement.

National and continental geodetic networks are under measurement or remeasurement using different electromagnetic distance measurement technique, such as: Very Long Baseline Interferometry (VLBI), Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR), Global Positioning System (GPS), Electronic Distance Measurement (EDM), etc.

Notwithstanding the high resolution of measuring instrumentation the absolute scale of the determined length as well as of the geodetic network is still dependent on uncontrolled atmospheric influences. Uncertainties and deficiencies are recovered by numerous experiments. Can these systematic errors be removed by the renewal of velocity index models, it was and remains of the scientific and engineering interest.
The Velocity Index of Dry Air and its Dispersity
at Ground Station of Observation for
Standard Composition: the Thermodynamic Approach

The standard composition of atmosphere contains 78.09% nitrogen, 20.95% oxygen, 0.93% argon and 0.03% carbon-dioxide, defined to be dry 0% of humidity, at 15°C temperature and 1013.25 mbar (760 mmHg) atmospheric pressure. For this very strict condition, the velocity index was found to be an empirical function of wavelength of light (Barrel and Sears, 1939) depending only on partial refractivity and density of air components. Standard velocity index dispersity vs. wavelength is

\[(n_g - 1)10^6 = 287.604 + 3 \cdot \left( \frac{1.6288}{\lambda^2} \right) + 5 \cdot \left( \frac{0.0136}{\lambda^4} \right) \quad (1)\]

The formula is valid for effective wavelength: 0.44 µm < \( \lambda < 0.65 \) µm, and it was adopted by International Union of Geodesy and Geophysics (IUGG) XIIIth General Assembly in Berkley, 1963, for the reference velocity index of EDM instruments. (For a given value e.g. HeNe laser's \( \lambda = 0.6328 \) µm wavelength \( n_g = 1.0003002 \).)

Eq. (1) doesn't take into consideration the influence of the concentrations of some minor constituents like: neon 18 ppm, helium 5 ppm, methane 1.5 ppm, krypton 1 ppm, hydrogen 0.5 ppm. Some of constituents appear due to air pollution in industrial and urban areas, such as ozone 0.005 - 0.5 ppm, nitrogen-dioxide, 0.001 - 0.1 ppm with numerous solid particles and aerosols. The carbon-dioxide content is assumed 0.03% or 316 ppm in 1974 increased to 360 ppm level in 1994 and expected to increase to 600 ppm in 2070 (Iribarne and Cho, 1980).

Group Refractive Index of Light and Near Infrared
in Real Atmosphere under Ambient Conditions

For terrestrial reference station at one or both ends of measured distance the group velocity index can be modelled by separate influences of dry and wet atmospheric components of air:

\[(n_L - 1)10^6 = f_x(\lambda) \cdot D_s(p, T) + f_w(\lambda) \cdot D_w(e, T) \quad (2)\]

where:
\[n_L = \text{the ambient velocity index}\]
\[f_x(\lambda) = \text{dry dispersity}\]
\[D_s(p, T) = \text{density factor of dry air}\]
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\[ f_w(\lambda) = \text{wet dispersity} \]
\[ D_w(e, T) = \text{density factor of wet air} \]
\[ t, T = \text{temperature of air in } ^\circ\text{C}, ^\circ\text{K} \]
\[ p = \text{total atmospheric pressure in mbar} \]
\[ e = \text{partial water vapour pressure in mbar} \]

The International Union of Geodesy and Geophysics (IUGG) resolved in 1963 at its XIIIth General Assembly in Berkeley that the velocity index can be reduced to ambient conditions by:

\[
(n_L - 1) = (n_g - 1) \frac{273.15 \cdot p}{(273.15 + t) \cdot 1013.25} - \frac{11.27 \cdot 10^{-6} \cdot e}{(273.15 + t)}. \tag{3}
\]

BARRELS and SEARS (1939) state that their equation is valid in temperature and pressure ranges of +10 to +30 °C and 720 to 800 mmHg, respectively no range of partial water vapour pressure is stated. However, EDLÉN’S (1966) statement ‘not deviating too much from 10 mmHg’ may be taken as a guide.

The numerous deficiencies of Eqs. (1) and (3) were evaluated by DEICHL (1984). The humidity term is clearly a weak element in the above equations.

Whatsoever, the humidity (or \( e \)) has little influence for electro-optical distance measurement of precision not better than \( 5 \cdot 10^{-6} \) D or 5 ppm, therefore the humidity term in Eq. (3) can be omitted only under conditions close to normal atmosphere.

In case of higher precision distance measurements, such as satellite laser ranging or baseline EDM measurement, the following procedure of ground velocity index determination is strongly recommended:

1. Careful measurement of basic atmospheric parameters, the dry temperature \( t \) °C, wet bulb temperature \( t' \) °C and atmospheric pressure \( p \) in mbar (in the line of sight, unaffected by heat sources, such as instrument, observer, Sun, etc.)

2. Computation of the saturation water vapour pressure by formulas Magnus-Tetens (RINNER and BENZ, 1966), modified by Goff and Gratch over water (\( t' > 0 ^\circ\text{C} \))

\[
E_w' = [1.0007 + (3.46 \cdot 10^{-6} \cdot p)] \cdot 6.1121 \cdot \exp \left[ \frac{17.502 \cdot t'}{240.97 + t'} \right]. \tag{4a}
\]

over ice (\( t' < 0 ^\circ\text{C} \))

\[
E_{\text{ice}}' = [1.0003 + (4.18 \cdot 10^{-6} \cdot p)] \cdot 6.1121 \cdot \exp \left[ \frac{22.452 \cdot t'}{272.55 + t'} \right]. \tag{4b}
\]

3. Applying the following formulas for computation of partial water vapour pressure after Sprung (RINNER and BENZ, 1966)
over water

\[ e = E'_w - 0.000662 \cdot p \cdot (t - t') \], \hspace{1cm} (5a)

over ice

\[ e = E'_{\text{ice}} - 0.000583 \cdot p \cdot (t - t') \]. \hspace{1cm} (5b)

4. Compute: \( p = p - p_w \) and \( T = 273.15 + t \).

5. Find the wet and dry density factors after Owens (Rüeger, 1990; Birch and Downs, 1993)

\[
D_w = \frac{p_w}{T} \left\{ 1 + p_w \left[ 1 + \left( 3.7 \cdot 10^{-4} \right) \cdot p_w \right] \left[ -2.37321 \cdot 10^{-3} + \frac{2.23366}{T} - \frac{710.792}{T^2} + \frac{7.75141 \cdot 10^4}{T^3} \right] \right\} \hspace{1cm} (6)
\]

\[
D_s = \frac{p_s}{T} \left[ 1 + p_s \left( 57.90 \cdot 10^{-8} - \frac{9.3250 \cdot 10^{-4}}{T} + \frac{0.25844}{T^2} \right) \right]. \hspace{1cm} (7)
\]

6. Using the following equation find the optical velocity index \( n_L \)

\[
(n_L - 1) \cdot 10^8 = \left[ 1646386.0 \left( \frac{238.0185 + \sigma^2}{(238.0185 - \sigma^2)^2} \right) + 47729.9 \left( \frac{57.362 + \sigma^2}{(57.362 - \sigma^2)^2} \right) \right] D_s + [6487.31 + 174.174 \cdot \sigma^2 - 3.55750 \cdot \sigma^4 + 0.61957 \cdot \sigma^6] D_w , \hspace{1cm} (8)
\]

where \( \sigma = 1/\lambda \) - vacuum wavenumber, \( \lambda \) in \( \mu \text{m} \).

The validity ranges of this formula are: \( 0.23 - 1.69 \, \mu \text{m} \) for the carrier wavelength of EDM, from \(-40 \, ^\circ\text{C}\) to \(+50 \, ^\circ\text{C}\) and from 533 to 1067 mbar at ambient conditions. An error in \( t \) of 1 \( ^\circ\text{C} \) effects velocity index by 1 ppm, an error in \( p \) of 1 mbar – by 0.3 ppm and an error in \( e \) of 1 mbar – by 0.04 ppm. Determination of effective wavelength with an accuracy of 5 \( \mu \text{m} \) yields an error 0.1 ppm.

**The Refractive Index for Microwaves at Ground Station of Observation**

It is believed that the atmosphere at terrestrial level of measurement is not a dispersive medium for wavelength over 8 \( \mu \text{m} \). Therefore \( \lambda \) is not an
argument of microwave velocity index model, adopted by XIIth General Assembly of IUGG, 1963, Berkley. The equation itself is the result of experiment carried out at the National Physical Laboratory (UK) by ESSEN and FROOME (1951), with the aid of microwave cavity resonators and reads as follows:

\[(n_M - 1) \cdot 10^6 = \frac{77.624}{T} \cdot (p - e) + \frac{64.70}{T} \cdot \left(1 + \frac{5748}{T}\right) \cdot e, \quad (9)\]

where:

- \(n_M\) = velocity index for microwaves
- \(t\) = dry temperature of air in °C
- \(p\) = total pressure of air in mbar
- \(e\) = partial water vapour pressure

The partial water vapour pressure \(e\) must be evaluated from psychometric \(Eqs. (4a), (4b), (5a)\) and \((5b)\). The formula itself is accurate to 0.1 ppm under normal conditions and better than 1 ppm under extreme conditions (EDGE, 1960), with sufficiently precise measurement of ambient atmospheric parameters. Whereas an error in \(t\) of 1 °C causes error 1.4 ppm, while 1 mbar in atmospheric pressure – 0.3 ppm, and 1 mbar error in water vapour measurement causes an error 4.6 ppm in distance, respectively.

### Extrapolation of Velocity Index Value between the Endpoints of Distance Measurement

The above \(Eqs. (8)\) and \((9)\) are the best known and recently suggested for precise distance measurement, involving simultaneous determination of tropospheric meteorological parameters.

The value of optical velocity index of air computed by \(Eq. (8)\) serves for basic extrapolations between endpoints of electro-optic EDM measurement and for ground station value for space geodetic measurements, such as SLR, LLR, VLBI.

The velocity index evaluated at endpoints usually does not approximate the integral value along the wavepath between the endpoints. There are some cases when the most variable lower troposphere is close to standard model state. These are the phenomenon of isothermia or the so-called ‘well mixed’ state of air (RAHNEMOON, 1988). If the continuous electro-optical distance measurement is accompanied with simultaneous monitoring for time deduction of existence either of these status, the best error free measurements can be selected on the base of maximum likelihood by post-processing. Furthermore, the deficiency of local scale can be eliminated by
very precise 'local scale parameter ratio' method (Angus-Leppan, 1979) giving maximum weight in adjustment to those of distance with maximum 'simultaneous similarity'. The most sophisticated method of monitoring the distribution of velocity index along the wave path is the 'two-color' dispersion method (Prilepin, 1951) used recently both for terrestrial electro-optical measurements (Georan, Terrameter) and for Satellite Laser Ranging (Fundamental Station Wettzel / DGK Report 1984). Although it removes most of systematic errors under standard conditions, the two-color dispersion method usually cannot fully eliminate the effect of tropospheric irregularities and inhomogeneities of temperature, pressure and water vapour pressure. It is recommendable to pursue the similarity or likelihood test. The velocity indices for two or more wavelengths ($\lambda_i$) must be evaluated by Eq. (8) for real atmosphere using average meteodata from both terminals. Distance differences $\Delta D_{i,i+1}$ have to be evaluated simultaneously to distance measurement (using approximate value of distance at ±0.1 % accuracy level).

The result of measured distance differences shall be compared against the forecasted. If the comparison by a simultaneous similarity test yields high correlation, then the endpoints average value of velocity index should represent the integral velocity index for the total wavepath. If the distance differences are weakly correlating, then it is suspected that the endpoint value does not represent the integral velocity index.

**Tropospheric Velocity Index for Microwaves**

Hence, the dry troposphere are insignificantly dispersive media for microwaves, the multiwavelength method (otherwise applied successfully for ionospheric correction in VLBI-double range technique or GPS/$L_2 - L_1$/ionosphere-free carrier method) is inapplicable for evaluation of velocity correction in troposphere in the range of microwaves.

As it is stated earlier the microwave propagation velocity is rather sensitive to water vapour distribution in troposphere. The most responsible measurements, such as VLBI and precise geodetic GPS observations are accompanied with microwave water vapour radiometry in the ranges of molecular water resonance 21 GHz and 23 GHz. The water vapour distribution along the wavepath is significantly variable in time and again the only simultaneous atmospheric tomography can help to evaluate the precise velocity correction for microwaves.

Based on the radiometric observations of water vapour distribution a new rectification model has been suggested (Rahnemoon, 1988). Numerical ray-tracing postprocessing algorithm is elaborated to block-diagram
level, in which the initial value of velocity index is still based on meteoro-
logical parameters in the vicinity of ground observation station and veloc-
ity index Eq. (9).

**Requirement for Renewal and Validation of Velocity Index Formula of Air for Optical and Microwave Spectrum**

Since the time (1963) of determination of velocity indices elapsed more
than thirty years. During the use of the standard formulas considerable
experience has been collected and numerous recommendations have been
implemented to precise electronic distance measurement. The worldwide
experience has to be evaluated and the proposal for new determination of
working formulas has to be developed. By the resolution of International
Association of Geodesy in 1993, a decision about the elaboration of new
velocity index formulas has been accepted.

The working plan of the Special Commission for fundamental con-
stants in Geodesy includes the establishment of a Special Study Group
‘Velocity index of air for visible, near infrared and microwaves’, chaired by
J. M. Reger, UNSW Kensington, Australia. According to the commission’s
working plan the importance of renewal became evident by following rea-
sons:

- there is no guarantee for conventional use of appropriate formulas for
  velocity index of air;
- designers and users of EDM instruments are not confident in application
  of adequate working equations;
- the best of EDM instruments are providing internal resolution
  $\pm(0.1 \text{mm} + 0.1 \cdot 10^{-6} D)$;
- numerous deficiencies of IUGG 1963 formulas are discovered at recent
  level of measurements precision;
- new instruments are designed using the near and medium infrared
  spectrum, uncovered by validity ranges of existing standard formulas:
- the influence of water vapour, carbon-dioxide and other minor and
  non-ideal constituents of air are not yet investigated satisfactorily;
- ground station velocity index is applied as initial value for space
  geodetic extrapolations;
- dispersion anomalies are discovered in visible and near infrared radi-
  ations;
- the correction for influence of electrodynamic state in real atmosphere
  is not yet attempted.
Further Outlook: the Electrodynamic Approach

Originally the Lorentz–Lorenz equation – based on the quantum theory of atomic and molecular polarisability and the classical electromagnetic theory – relating to the polarisability of the refractive index:

\[ \frac{n^2 - 1}{n^2 + 1} = \sum_i R_i \zeta_i, \]  

(10)
in which \( R_i \) is the specific refraction and \( \zeta_i \) the partial density of \( i \)th component of the atmospheric mixture. The density of atmospheric gas mixture is found empirically by thermodynamic parameters:

\[ \zeta_i = D_i(p, T) \]  

(11)
separately for dry and wet components. (Note Eqs. (6) and (7) for \( D_s \) and \( D_w \)). The specific refraction can be expressed by (Owens, 1966):

\[ R_i = \frac{4}{3} \pi (N_A/M_i) \alpha_i, \]  

(12)
where \( N_a \) is Avogadro's number, \( M_i \) is the molecular weight and \( \alpha_i \) is the polarisability of \( i \)th component.

The specific refraction as well as the polarisability, in classical thermodynamic approach, are believed to be invariant under changes of density to a high degree of approximation for the components of air at atmospheric pressures. Dependence of refractivity and polarisability upon wavelength for optical ranges is expressed by dispersity formula (1), (and more precisely in (8), separately for dry and wet components), while refractivity is believed to be absolute constant at standard thermodynamic state and composition for microwaves.

Although the standard IUGG and the recently suggested precise formulas are in general use, there are strong evidences, that pure thermodynamic treatment could not yield an adequate phenomenological model for atmospheric velocity index, on that those formulas are fundamented.

Analysing the assumptions made in practical application of Lorentz–Lorenz Eq. (10) one can conclude:
- the value of polarisability factor \( \alpha_i \) has never been measured, and its invariance has never been proved;
- the formula is valid for the mixture of ideal nonpolar gases (such as \( O_2, N_2, Ar \)), but empirically applied also for water vapour and carbon dioxide which are neither of those;
- empirical coefficients for model (2) have been evaluated in thermodynamic equilibrium, neglecting ambient electrodynamic influences; under laboratory circumstances;
The Influence of Electrodynamic State of Molecules: the Polarisation Dispersity

Unlike to laboratory circumstances electromagnetic disturbances are affecting the velocity index of air under ambient conditions. Magnetic field of Earth, tropospheric electric field, electromagnetic waves and radiations are modifying the electrodynamic symmetry of polar or even nonpolar molecules, generating 'artificial' anisotropy in propagation of measuring beam for EDM. Due to this fact the propagation of measuring beam can be characterized by two velocity indices, one for ordinary and another for extraordinary directions. The difference between velocity indices are usually described as:

$$\Delta n_{oe} = (n_o - n_e) = v \cdot d \cdot \xi,$$  \hspace{1cm} (13)

where: $n_o$, $n_e$ – velocity indices for ordinary and extraordinary beam
$d$ – measured distance,
$v$ – materials constant,
$\xi$ – resulting external electrodynamic field.

Existence of Nonsymmetrical Electrodynamic State in the Troposphere

a. The influence of Earth’s magnetic field is quasistationary, and has an asymuthal character, with influence maximum in N-S direction. The real value and its local variation are precisely measurable by magnetometers. The effect is undetectable in laboratory circumstances.
b. The solar radiation has its influence on electromagnetic state of troposphere according to local daytime, cloud coverage and seasonal variations. Its direction varies in E-W direction from horizontal to vertical.
c. According the atmospheric physics a certain vertical electrostatic gradient is always present in the troposphere (WILSON, 1920). Even under fair weather conditions there is a high electric potential between the conductive ionosphere and Earth surface, insulated by troposphere (atmospheric capacitor model: KASEMIR, 1977). The fair weather gradient – in the lower troposphere – varies daily between 30 V/m and 160 V/m and can increase to 300 KV/m during the thunderstorm, but always affected by cloud coverage. [A summary of atmospheric electricity is given by BENČZE et al. 1982.)

Existence of these electrodynamic phenomenae under ambient atmospheric conditions proves that the electrodynamic state in real atmosphere is never to be neutral, therefore:

$$\xi \neq 0$$  \hspace{1cm} (14)
and due to this fact, the atmospheric anisotropy is always existing:

\[ (n_o - n_e) \neq 0 \]  

(15)

Possible of Determination of Anisotropic Delay

**Case 1. Nonpolarised amplitude modulated light beam of conventional EDM, in anisotropic air**

Due to distortion of originally planar wavefront during the propagation in anisotropic media, the signal can be separated into ordinary \( I_o \) and extraordinary \( I_e \) components. The classic formula of distance measurement containing thermodynamic velocity index \( n_L \):

\[ 2D = \frac{c \cdot \tau}{n_L} \]  

(16)

now should contain the weighted average of refracted components, detected by photodetector:

\[ n_p = \frac{n_o \cdot I_o^2 + n_e \cdot I_e^2}{I_o^2 + I_e^2} \]  

(17)

Supposing the existence of anisotropy \( n_o \neq n_e \) and the different attenuation of wavefront \( I_o \neq I_e \), there would be difference between velocity indices:

\[ n_p \neq n_L \]  

(18)

which will further lead to systematic error in distance measurement for all conventional EDM.

**Case 2. Linearly polarised amplitude modulated light beam in anisotropic air**

Hence, the air anisotropy may have arbitrary orientation, the asnhuth of ordinary plane must be found by rotating of polarisation of transmitted beam, as the first step. Minimum propagation time of measuring signal will indicate that the polarisation of transmitted beam is matching with ordinary axis of air anisotropy. As a second step, measurement of maximum propagation will be determined by further 90° rotation of transmitted beam. The difference if any between ordinary and extraordinary ‘distances’ will indicate the existence and significance of air anisotropy.
Conclusion

In fact, the theoretical base of recent velocity index formulas - the Lorentz-Lorenz equation - itself - has also several assumptions (OWENS 1967, BÖTTCHER, 1952). The numerical coefficients of working formulas are determined in laboratory circumstances (in vitro). Measurements at recent precision level discovered numerous discrepancies. Further investigations in diagnostics of atmosphere (in vivo) would lead to more precise and complex theoretical model of atmospheric refraction. Practical investigations discovering the existence and significance of atmospheric anisotropy's influence are inevitable.

References