CONVERSIONS BETWEEN HUNGARIAN MAP PROJECTION SYSTEMS

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Received: January 12, 1995

Abstract

When different map projection systems are applied simultaneously over a given area, the need for conversion permanently arises in the overlapping areas of these systems. Conversions, however, cannot always be made by closed mathematical formulas and in these cases it frequently raises serious problems to apply a correct transformation method. Hence such an algorithm and program package was developed for all combination conversion between Hungarian map projection systems and their reference surfaces which does not cause any difficulties even for users having no deeper knowledge in map projections.

Keywords: map projection, conversion between map projections.

1. Introduction

A multitude of map projection systems has resulted in Hungary due to many subsequent (and mostly justified) changes in geodetic reference systems. Three different stereographic map projection systems were used for geodetic purposes and three conformable tangent cylindrical systems were required as well. Two 6° zones of Gauss–Krüger and UTM projection cover over the area of the country, thus, more than one system is used even within one single kind of projection. Besides, the Unified National Projection system (EOV) was introduced in the whole area of Hungary as well. The former Hungarian Gaussian sphere, tangent to the Bessel ellipsoid is the common reference surface of Hungarian stereographic and conformable tangent cylindrical systems, the new Hungarian Gaussian sphere, tangent to the IUGG-67 ellipsoid, is the reference for EOV system and the Krassovsky ellipsoid is the reference surface for Gauss–Krüger projections in Hungary. Furthermore, WGS-84 ellipsoidal or geocentric Cartesian co-ordinates have resulted from GPS measurements more recently, and even it is required in international relations to use the UTM system in more recent times. This picture is complicated further on by the fact that besides the above mentioned systems also military stereographic, in the area of Budapest city stereographic and in some villages of the country even systems without projection are used.

When different map projection systems are used simultaneously over the same area, the need for conversion frequently arises inside overlapping fields. Circumstances are the same when there are different zones within a single projection (e.g. Gauss–Krüger or UTM projections), in this case, inside the periphery of neighbouring zones co-ordinates are to be converted usually. More generally speaking: when the projection system of our maps differs from that of our control points available, our measurement results have to be transformed into the projection system of our map so that they could be represented on it.

Conversions may take place either by the so-called co-ordinate method (with closed mathematical expressions) or through transformation equations (polynomials) provided by using so-called common points that have known co-ordinates in both systems.

It is only possible to make exact conversions with closed mathematical expressions in cases when both projection systems have the same reference surface and points of the same triangulation network coming from the same adjustment represented in both projection systems. If a point belonging to a different triangulation network is converted from one system into the other, then transformed co-ordinates will not fit suitably into points of the triangulation network presented on the projection plane in question. It is true because one should consider differences that may arise from the different position and orientation of the two triangulation networks, and also base extension networks and angle observations are quite different. A refinement of any triangulation network with recent measurements or with a readjustment alters co-ordinates of horizontal control points with respect to the reference surface and hence co-ordinate on the projection plane as well. The effects are the same when some parameters of the reference surface are modified even if otherwise our triangulation network remains the same. Any re-orientation of the network (a rotation of the reference surface) does not hinder exact conversion. When the co- ordinate method is applied, conversion may be made by rigorous mathematical expressions found in some reference works listed.

In each case when any of the above mentioned requirements has not been met, the conversion is not possible by closed mathematical formulas. The conversion therefore can be performed only by transformation equations were deduced as polynomials from so-called common points that have co-ordinates in both projection systems. In this case maximum five-order conformal polynomials can be applied depending on the number of common points. For example, the connection between x, y co-ordinates of the projection system I and x', y' co-ordinates of the projection system II is established by the

$$
\begin{aligned}
x' =& A_0 + A_1x + A_2y + A_3x^2 + A_4xy + A_5y^2 + A_6x^3 + A_7x^2y + A_8xy^2 + \\
& A_9y^3 + A_{10}x^4 + A_{11}x^3y + A_{12}x^2y^2 + A_{13}xy^3 + A_{14}y^4 + A_{15}x^5 + \\
& A_{16}x^4y + A_{17}x^3y^2 + A_{18}x^2y^3 + A_{19}xy^4 + A_{20}y^5 , \\
y' =& B_0 + B_1x + B_2y + B_3x^2 + B_4xy + B_5y^2 + B_6x^3 + B_7x^2y + B_8xy^2 + \\
& B_9y^3 + B_{10}x^4 + B_{11}x^3y + B_{12}x^2y^2 + B_{13}xy^3 + B_{14}y^4 + B_{15}x^5 + \\
& B_{16}x^4y + B_{17}x^3y^2 + B_{18}x^2y^3 + B_{19}xy^4 + B_{20}y^5
\end{aligned}
\tag{1}
$$

polynomials. Coefficients $A_0 - A_{20}$ and $B_0 - B_{20}$ (altogether 42 coefficients) can be determined by using common points suitably through an adjustment process. In this case slightly different co-ordinates will result after the conversion process depending on the position and number of selected common points and the applied method.

2. Computer Software Development

Since it may cause problems even for experts to apply correct methods of conversion between a multitude of map projection systems, such a program package has been worked out by which conversions can be made between Hungarian map projection systems and their reference co-ordinates in all combinations, the usage of which can cause no problem even for users having no deeper knowledge in map projections.

Conversion between co-ordinates

VTN = System without projection
BES = Bessel's Ellipsoidal
SZT = Budapest Stereographic Projection
KST = Military Stereographic Projection
HER = North Cylindrical System
HKR = Middle Cylindrical System
HDR = South Cylindrical System
VST = Stereographic System of the City Budapest
IUG = IUGG-67 Ellipsoidal
EOV = Unified National Projection
KRA = Krassovsky's Ellipsoidal
GAK = Gauss–Krüger Projection
WGS = WGS-84 Ellipsoidal
XYZ = Spatial Cartesian Geocentric (GPS)
UTM = Universal Transverse Mercator

is performed by the conversion program in the area of Hungary in 212 combinations as it is listed in *Table 1.*

This table conveys us information on the possibility and accuracy of conversions very simply.

Double lines in this table separate map projection systems belonging to different reference surfaces. (By reference surface the ellipsoid is meant, though the fact should be acknowledged that the approximating (Gaussian) sphere serves also as a reference surface for those map projection systems where a double projection is applied and an intermediate sphere is the reference surface at the second step of the projection to get co-ordinates on a plane or on a plane developable surface. Co-ordinates on this approximating sphere have no practical role for users.)

Plus '+' signs at the intersection fields of rows and columns indicate that an exact conversion between the two map projection systems is possible using closed mathematical formulas found in reference works of (HAZAY, 1964), (VARGA, 1981, 1986) for transformation. In this case the accuracy of transformed co-ordinates is the same as the accuracy of co-ordinates to be transformed.

Cross '×' signs of this table indicate the impossibility of transformation between the two map projection systems with closed mathematical formulas and the conversion — according to rules found in (Rules for Map Projection's Use, 1975) is performed using e.g. polynomials as in *Eq.* (1) of a finite (maximum five) degree. In these cases theoretically there is only a possibility of conversion with limited accuracy (e.g. the accuracy of converted plane co-ordinates is generally about only $\approx \pm 10\,\text{cm} \div \pm 20\,\text{cm}$).

Parenthetic plus '(+)' and cross '(×)' signs remind us of the fact that a conversion is possible and it can be done by our program but there is no practical need — except of scientific reasons — to do so. (E.g. between map projection systems with no overlapping areas or if they are not very close to each other there can be no practical need to make conversion).

Minus '–' signs in the table are reminders of the fact that an identical (transformation into itself) conversion has no meaning except for the Gauss–Krüger and UTM projection systems where the need of conversion between different zones frequently arises. Hence a '!+!' sign indicates that it is possible to make exact conversions between different zones of the Gauss–Krüger and UTM map projection systems.

It has to be noted that only an approximate conversion using common points is possible from the Stereographic projection system of the city Budapest into some other (e.g. into Budapest Stereographic) projection systems that have even the same reference surface (Bessel ellipsoid) because the triangulation networks are different.

Table 1

	VTN	BES	SZT	KST	HER	HKR	HDR	VST	IUG	EOV	KRA	GAK	WGS	XYZ	UTM
VTN	–	×	×	×	×	×	×	(×)	×	×	×	×	×	×	×
BES	×	–	+	+	+	+	+	×	×	×	×	×	×	×	×
SZT	×	+	–	+	+	+	+	×	×	×	×	×	×	×	×
KST	×	+	+	–	+	+	+	×	×	×	×	×	×	×	×
HER	×	+	+	+	–	+	(+)	(×)	×	×	×	×	×	×	×
HKR	×	+	+	+	+	–	+	(×)	×	×	×	×	×	×	×
HDR	×	+	+	+	(+)	+	–	(×)	×	×	×	×	×	×	×
VST	(×)	×	×	×	(×)	(×)	(×)	–	×	×	×	×	×	×	×
IUG	×	×	×	×	×	×	×	×	–	+	×	×	×	×	×
EOV	×	×	×	×	×	×	×	×	+	–	×	×	×	×	×
KRA	×	×	×	×	×	×	×	×	×	×	–	+	×	×	×
GAK	×	×	×	×	×	×	×	×	×	×	+	!+!	×	×	×
WGS	×	×	×	×	×	×	×	×	×	×	×	×	–	+	+
XYZ	×	×	×	×	×	×	×	×	×	×	×	×	+	–	+
UTM	×	×	×	×	×	×	×	×	×	×	×	×	+	+	!+!

Since our recent information shows that there are some villages not only in the southern part of Transdanubia but also in the county Szabolcs-Szatmár-Bereg that have maps without projection, hence conversion between North Cylindrical System (HER) and System Without Projection (VTN) is allowed and the sign '×' appears instead of '(×)' sign in the corresponding field of the table.

The logical frame of our map projection conversion software can be grasped in *Fig. 1* and *Fig. 2*.

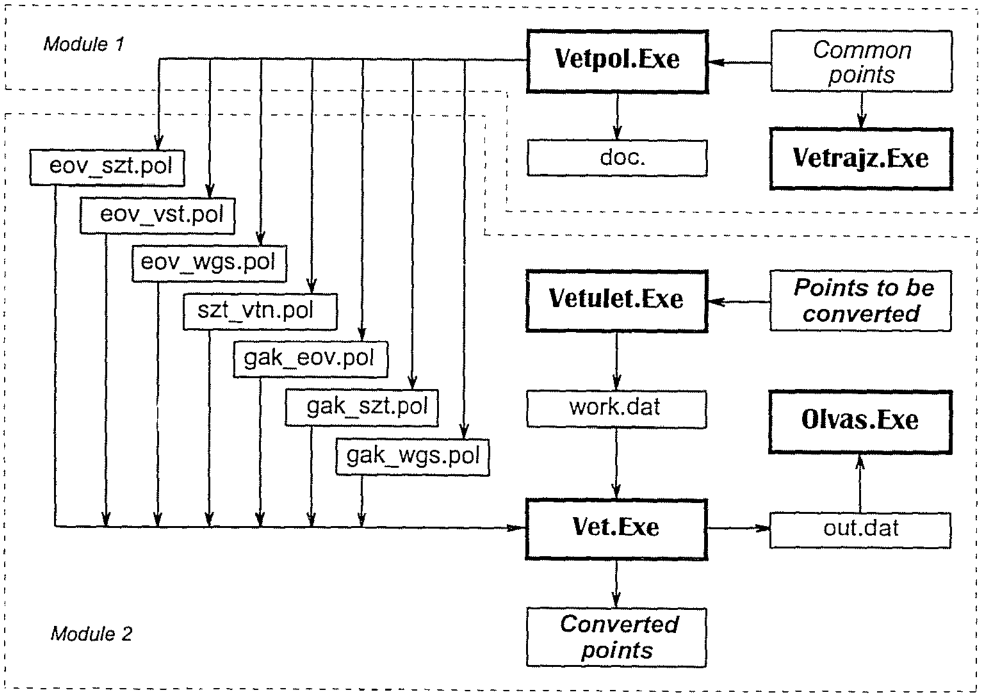

Fig. 1.

Our software has two main parts: a module which yields coefficients of transformation polynomials and another module which performs actual conversions. Broken lines surround these two modules in *Fig. 1*.

Module 1 computes coefficients of transformation polynomials in *Eq.* (1) when it is impossible to convert between the two systems through the co-ordinate method, that is through closed mathematical expressions. Program `Vetpol.Exe` makes it possible to calculate the coefficients of polynomials when some common points are adequately given. Program `Vetpol` creates binary files `eov_szt.pol`, `eov_vst.pol`, `eov_wgs.pol`, `szt_vtn.pol`,

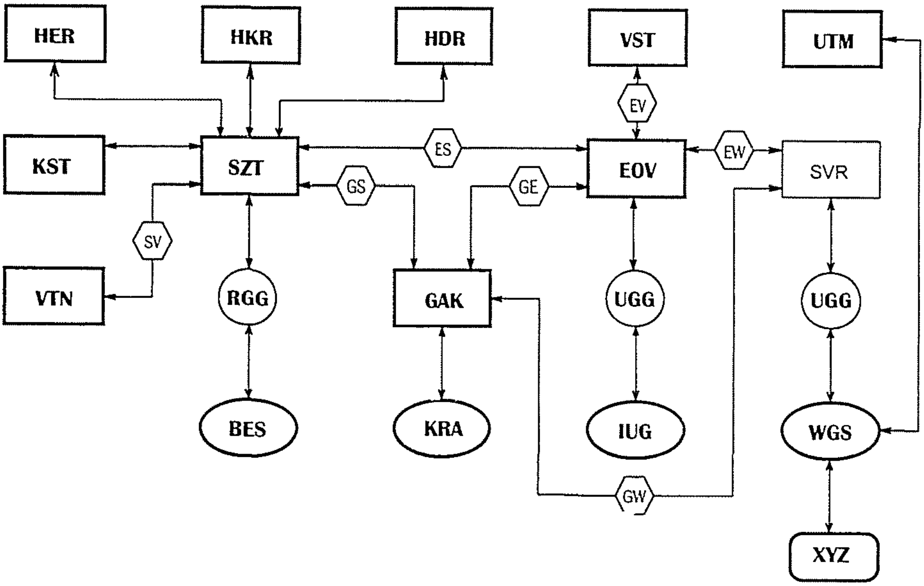

Fig. 2.

gak_eov.pol, gak_szt.pol, and gak_wgs.pol containing coefficients of transformation polynomials, required for conversions between EOV – Budapest Stereographic, EOV – City Stereographic, EOV – WGS-84, Budapest Stereographic – Without Projection, Gauss–Krüger – EOV, a Gauss–Krüger – Budapest Stereographic and Gauss–Krüger – WGS-84. Program Vetpol automatically determines the degree of transformation polynomials as a function of the number of common points. If there are *21* or more common points then all the (namely 42) coefficients of a five-degree polynomial in the expression (1) can be determined. When the number of common points lies between *15* and *20*, then the degree of polynomials is *4*, if the number of common points is between *10* and *14* then the degree is *3*, and with the number of common points between *6* and *9* the degree of polynomials required for transformation is *2*. At least *6* common points are necessary to compute coefficients of the polynomials, however, an effort should be made to use as many common points as possible to determine these polynomial coefficients. If the number n of common points is such as $7 \leq n \leq 9$, $11 \leq n \leq 14$, $16 \leq n \leq 20$ or $n \geq 21$, then the number of equations is greater than necessary (the problem is overdetermined), hence the most reliable values of unknown polynomial coefficients are determined through adjustment by program `Vetpol`.

Program `Vetrajz.Exe` is also a member of *Module 1* by which the geometrical arrangement of common points can be displayed on screen to check the evenness of our point distribution.

Actual conversions can be made by *Module 2* (*Fig. 1*). Three important programs can be found in this module: input-output organizer program of the conversion software, namely `Vetulet.Exe`, main conversion program `Vet.Exe`, and `Olvas.Exe` is a utility program to read and print output files.

Co-ordinates of points to be converted can be input from both keyboard and disk files by the program `Vetulet.Exe`. A built-in special editor helps to handle co-ordinates from the keyboard or to transfer them into a work file work.dat in the required format. This special editor also serves to check input co-ordinates on a high level and, therefore, it is practically impossible to read erroneous co-ordinates. Co-ordinates from disk files will also pass through the above strict trouble shooting process and they will be transferred into a work file `work.dat` as well.

Co-ordinates in the work file `work.dat` are transformed into the required system by the main conversion program `Vet.Exe`. The operation of this main program and the conversion logic between the 15 different map projection systems is summed up in *Fig. 2*. Transformation paths — and their directions — between different systems are pictured by arrows. It can be seen that in most cases it is possible to convert between two arbitrary systems only through other intermediate systems (e.g. if a conversion between UTM and EOV systems is needed, then UTM co-ordinates first have to be converted into WGS-84 ellipsoid, then into the new Gaussian sphere and then into a so-called auxiliary system and finally they should be converted from this SVR system into EOV). If any two systems are connected in *Fig. 2* by a continuous line, then an exact conversion by the co-ordinate method, i.e. through closed mathematical expressions can be made; when the path, however, passes through a hexagonal block, then between the two systems, pointed by arrows, only an approximately accurate conversion could be made by transformation polynomials. Two-letter abbreviations in hexagonal blocks show which binary data file, containing transformation polynomials, has to be used to convert between the two neighbouring systems (their meaning in accord with *Fig. 1* is:

ES =`eov_szt.pol`, EV =`eov_vst.pol`, EW =`eov_wgs.pol`,
SV =`szt_vtn.pol`, GE =`gak_eov.pol`, GS =`gak_szt.pol`,
GW =`gak_wgs.pol`).

When there is more than one path possible between any two systems, the path is chosen along which conversion is more accurate. Transformed co-ordinates in different formats are passed into `out.dat` and `trf.dat` files by program **Vet.Exe.**

`Olvas.Exe` is a utility program that serves to display (read) and print output files. The content of the output file `out.dat` can be examined by this program on the screen and it can also be printed optionally.

3. Software Testing and Tests of Accuracy

It was mentioned previously that it is possible to convert through closed mathematical expressions between certain map projection systems. The conclusion could have been drawn as a result of our test computations that in these cases the accuracy of computed plane co-ordinates is 1 mm and of geodetic co-ordinates is 0.0001". These conversions are referred to in *Table 1* with '+', '(+)', and '!+!' signs or these systems are connected by continuous lines (arrows) in *Fig. 2*.

In all the other cases when the transformation path between any two systems passes through a hexagonal block (or blocks), the accuracy of transformed co-ordinates depends on, on the one hand, how accurately the control networks of these systems fit into each other; and on the other, how successful the determination of transformation polynomial coefficients was. It follows also from these facts that no matter how accurately these transformation polynomial coefficients were determined, if the triangulation networks of these two systems do not fit into each other accurately — since there were measurement, adjustment and other errors during their establishment — then certainly no conversion of unlimited accuracy can be performed (in other terms, only such an accurate conversion between two map projection systems is possible that the accuracy allowed by the determination errors or discrepancies of these control networks). This fact, of course, does not mean that one should not be very careful when the method of transformation is selected or — when the polynomial method is applied — the coefficients in *Equ.* (1) are determined.

Our first tests aimed at the question to decide which one of the two methods: Helmert transformation or polynomial method is more advantageous to be used. We arrived at the result that although the Helmert transformation is computationally more simple, its accuracy in the majority of cases does not even approximate the accuracy provided by the polynomial method. Since a simple programming can be a motive for only software 'beginners' therefore we took our stand firmly on the side of the use of the polynomial method.

When the polynomial method is chosen, the next important question is to determine the optimal degree of the polynomial. By considering a simple way of reasoning, one could arrive at the conclusion that the higher the degree of the polynomial, the higher the accuracy of map projection conversions will be. On the contrary, it could be proved by our tests that the maximum accuracy was resulted by applying five degree polynomials. No matter whether the degree was decreased or increased, the accuracy of transformed co-ordinates was lessened alike (more considerably by decreasing, less considerably by increasing).

It is true, really, that minimum 21 common points are required to determine coefficients of a five degree polynomial, but our experience revealed that the accuracy of conversions could be increased further on by using a considerably greater amount of common points and the most probable values of these unknown polynomial coefficients were determined through an adjustment.

A documentation file, provided by the program `Vetpol`, conveys some information characteristic of the accuracy of conversions by the polynomial method. Coefficients of transformation polynomials are first provided by the program `Vetpol` based on co-ordinates of common points y_i, x_i and y'_i, x'_i in systems I and II, respectively. Then y_i, x_i co-ordinates in system I are transformed into co-ordinates ty'_i, tx'_i in system II by using these coefficients and finally the standard error characteristic to conversion

$$\mu = \sqrt{\frac{\sum_{i=1}^{n} (ty'_i - y'_i)^2 + \sum_{i=1}^{n} (tx'_i - x'_i)^2}{n}} \tag{2}$$

will be determined.

For your guidance it could be mentioned that, for example, between the Budapest Stereographic and the EOV systems the standard error is ± 0.252 m from the expression (2) for the complete area of Hungary when 134 common points are used and the same figures are ± 0.004 m, ± 0.037 m and ± 0.217 m between Budapest City Stereographic an EOV, EOV and WGS-84, and EOV and Gauss–Krüger systems by using 43, 34 and 50 common points, respectively.

Our experience has shown the fact that although the accuracy can somewhat be enhanced by increasing the number of common points within the polynomial method but the accuracy of conversion cannot be increased beyond a certain limit even with this method since there is a difference between the two triangulation networks. In certain cases, however, an improvement could be gained when transformation polynomial coefficients are not determined for the complete area of the country but for only smaller sub-areas common points are given and transformation polynomial coefficients are determined by program Vetpol. In such cases conversions, of course, must not be made outside the sub-area where the coefficients of transformation polynomials were determined by program Vetpol.

It is worthy of note that also heights of points can be handled, when necessary, by the software. For example, when XYZ geocentric co-ordinates, determined from GPS, are to be transformed into any other system, then besides the transformed y, x projection co-ordinates or φ, λ ellipsoidal (geodetic) co-ordinates, also the

$$h = H + N$$

heights above the WGS-84 ellipsoid will be resulted, — where N denotes geoid–ellipsoid distance i.e. *geoid undulation* above WGS-84 ellipsoid and H is the height above geoid (height above sea level). So if the geoid–ellipsoid distance in a certain point is known, there is also a possibility to determine heights of practical value by the GPS technique.

Finally, we would like to mention that by our software with certain modifications one is able to convert between other map projection systems as well that are used in other countries.

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