

STRESS-STRAIN CONDITION AROUND A CYLINDER EXPANDING IN THE SOIL

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Abstract

The problem of expansion of an underground cylindrical cavity emerges in a number of geotechnical problems for example when the bearing capacity and load-settlement diagram of deep foundations determined, pressuremeter tests are interpreted, or the tension-strain diagram of grouted soil anchors is calculated and so on. This paper presents a new method to calculate stress/displacement conditions around an expanding cylinder, and the pulling resistance of an axially loaded cylinder after its expansion.

The cylindrical expansion in the soil examined as a process, to respect the interaction of the surrounding soil mass, determined the change in soil density. The paper shows example of calculation and simple diagrams to determine the radius of the compaction zone and the distribution of stresses, and volume changes in the soil.

Keywords: expanding cylinder, pressuremeter, grouted soil anchor, volume change of soil, bearing capacity of pile, bearing capacity of anchor, plastic stress state, pull-out resistance.

1. Introduction

Determination of soil stresses, deformations and displacements developing around a vertical, cylindrical hole expanded in the soil correlate to:

- evaluate pressuremetric measurement results and determine in-situ soil characteristics;
- investigate soil stresses and displacements around deep foundations;
- shell resistance determination for different kinds of piles;
- determinate tension-elongation diagrams of grouted soil anchorages and to dimensioning anchorages, etc.

Given is in the infinite soil semi-space a cylinder (pile, anchor, or pressuremeter) which will be expanded in the soil axisymmetrically by some volume increasing method (mortar grouting, concreting, water jet).

This study, based on the method elaborated by the author deals with the following problems in relation to the expansion of a cylinder in the soil.¹

¹The literature connected with the topic and detailed examination can be found in the author's dissertation (1992) and the author's earlier work in this theme.

Let us see what problems will emerge in connection with having the cylinder expanded in the soil:

- What will be the stresses acting on the cylinder wall when different expansion rates applied?
- How will the soil stress-strain condition vary at a given distance from the axis of the expanded cylinder and what will be the strain in the soil? (*Fig. 1*).
- What will be the tearing resistance of the soil when an expanded cylinder is pulled?

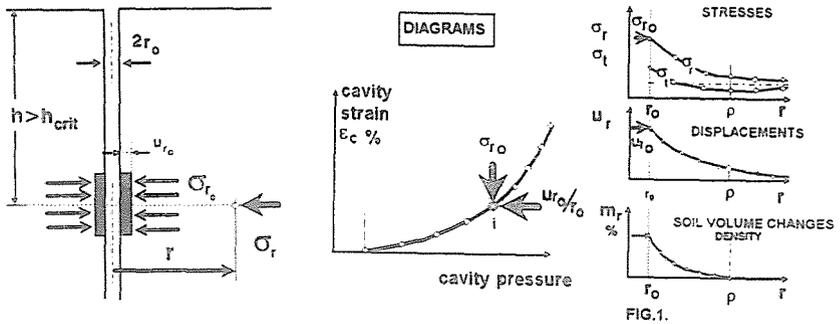


Fig. 1. Basic diagrams for the problems of an expanded cylindrical cavity in the underground

2. Theoretical Investigations

The problem of a cylindrical hole being expanded in the soil semi-space has to be considered *as a process*, whereby also the trend of stress variation and soil deformation is examined.

Around the expanded cylindrical hole in the soil, which is considered to have infinity, an axisymmetrical strain condition is assumed in the plane.

In the initial condition, the soil is subject to *compressive stresses due to normal geostatic pressure*, and during expansion of the cylinder, these stresses do change.

When the expansion of the cylinder begins, radial compressive stresses increase in the soil, while tangential compressive stresses start to decrease from the initial level of compressive stress at rest in place. (*Fig. 2*).

Shear stresses develop in the soil; stresses are transformed and a process of shear strength mobilization takes place. Expansive forces cause the soil to compress radially, what provokes primarily an annular displacement

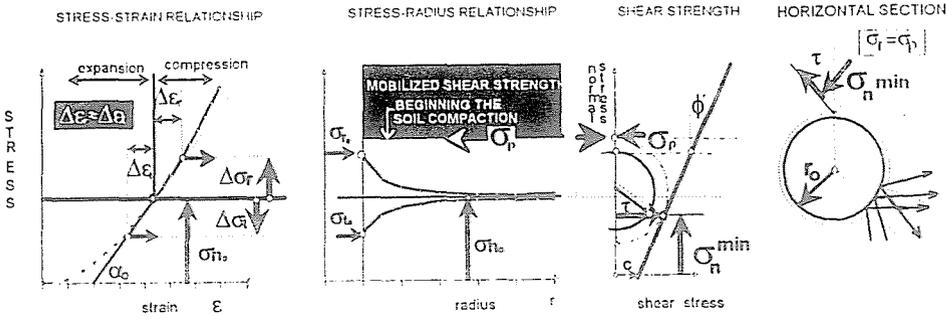


Fig. 2. Fundamental relationships between soil stresses and strains at the beginning of expansion

of the soil grains to certain limits. The fictitious soil ring around the cylinder changes into a soil ring of greater arc length but of less thickness.

In this 'initial stress state' only a slight expansional volume change (dilatation) occurs due to the mobilization of the shear strength in the soil, which does not yet reach the shear strength; so, no residual deformation takes yet place, but the elementary soil prism will be distorted and soil particles start to rearrange themselves.

As shear strength has not been mobilized yet, *no significant variation occurs in soil density.*

In this initial stress zone the soil stress distribution may be determined, according to the system conditions of permanent density.

Volume change of an elementary soil cube:

$$m_r = \frac{V_r - V_0}{V_0} = (1 + \Delta\epsilon_r) \cdot (1 + \Delta\epsilon_t) - 1 \approx 0. \tag{1}$$

Equilibrium of forces acting on the elementary soil cube in the formula, in terms of stresses:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_t}{r} = 0. \tag{2}$$

Soil stress distribution can then be calculated by substituting the differences of specific deformations related to the initial condition, into the equation defining the volume constancy.

Distribution of radial soil stresses: approximately $\sigma_r \leq \sigma_p$

$$\sigma_r = \sigma_{h_0} + (\sigma_{r_0} - \sigma_{h_0}) \left[\frac{r_0}{r} \right]^2 \frac{1}{1 - \frac{a}{4} \frac{\sigma_{r_0} - \sigma_{h_0}}{\sigma_{h_0}} \left[1 - \left(\frac{r_0}{r} \right)^2 \right]}. \tag{3}$$

Correlation of radial and tangential stresses:

$$\sigma_t = \sigma_{h_0} - \frac{\sigma_{h_0}^a}{1-a} (\sigma_r^{1-a} - \sigma_{h_0}^{1-a}), \quad (4)$$

where a – power function parameter (see later)

By expansion radial soil stresses are gradually increased, at last the *difference of principal stresses attains a threshold value which is in correlation with the soil shear strength, and there a maximal proportion, to be considered as constant, develops between the characteristic principal stresses.* With increasing radial principal stresses, the difference between principal stresses will vary according to the relationship in conformity to the shear strength.

In this condition, permanent deformations develop in the soil, hence a plastic stress state ensues.

Being *the mobility in the soil kinematically restricted*, there is no possibility for lateral dislocation, or further annular loosening, *because of the interaction of the surrounding soil mass*; so, after having attained the shear strength, a volume change arises *resulting in an increased density.*

Because of this compactive effect, the domain of shear strength mobilization is aptly *called as the zone of compaction.*

Thus, in this case, a plastic stress state develops at a definite maximal proportion or ratio of principal stresses, to *which state different values of radial soil stresses may belong.*

Stress state of the soil in the tested range is conventionally described by the Mohr–Coulomb relationship.

Accordingly, the following correlation can be written for the principal stresses in the soil for $\varphi \geq 0$

$$\sigma_t = \xi(\sigma_r - \sigma_u), \quad (5)$$

where

$$\xi = \frac{1 - \sin \varphi}{1 + \sin \varphi} \quad \text{and} \quad \sigma_u = \frac{2}{\sqrt{\xi}} c \quad \text{unconfined compressive strength.} \quad (6, 7)$$

Taking randomly a small, elementary soil prism from the surroundings of the cylinder, and examining the force equilibrium condition of the soil prism in the terms of stresses, it will permit to write the following formula for the principal stresses and their variation vs. radius:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_t}{r} = 0. \quad (8)$$

We are allowed to conclude therefore that inside the zone of compaction stress relations are statically unambiguously determined and the distribution of stresses may be computed in the function of the radius for

$$\varphi > 0, \quad \sigma_r = (\sigma_\rho + c \cot \varphi) \left(\frac{\rho}{r} \right)^{\frac{2 \cdot \sin \varphi}{1 + \sin \varphi}} - c \cot \varphi, \quad (9)$$

where σ_ρ - radial soil stress at the boundary of the compaction zone.

Knowing the regularity of stress distribution would not reveal, however, the magnitude of the stress values which vary continuously with expansion, because we do not know of the spatial extension of the continuously increasing plastic stress zone, that is, of the radius of the zone of compaction (ρ).

Its determination requires to examine the soil deformations inside the compaction zone in the knowledge of determined stress distribution regularities. (*Fig. 3*).

In conformity with known stress distribution relations, let us assume that the limits of plastic extension will be defined by radius ρ_1 and stress σ_{r01} .

Distribution of radial and tangential soil stresses inside the compaction zone is presented by diagram ①.

Upon increasing the expansion of the cylinder, stress distribution curve ① gets over into curve ② and of course, also the radius of the compaction zone grows to ρ_2 .

With any increment of the expansion stress on the cylinder wall, also the initial *compressive stresses on an elementary soil prism* continue to increase (σ_r and σ_t), *the soil becomes harder and less compressible*.

Since continuous increase of effective stresses enforces continuous deformation, the relation between specific volume deformations and stress can be described properly by a power function.

Being the radial and annular stresses in a linear correlation in the compaction zone, the correlation between average soil stress and volume deformation is in a similar correlation to that of radial stress and radial specific deformation.

Also from the figure it is obvious that with increasing expansion pressure, also the radius of the compaction zone increases continuously.

In course of expansion, soil stresses are increasing in an ever greater soil mass and as soil stresses occupy an increasing interface, the forces in an elementary soil prism are also increasing.

The following relationship can be assigned between specific soil deformation and stress:

$$\varepsilon_r = a_1 \left(\frac{\sigma_r}{\sigma_c} \right)^{a_2}, \quad \sigma_c = 1 \text{ kN/m}^2, \quad (10)$$

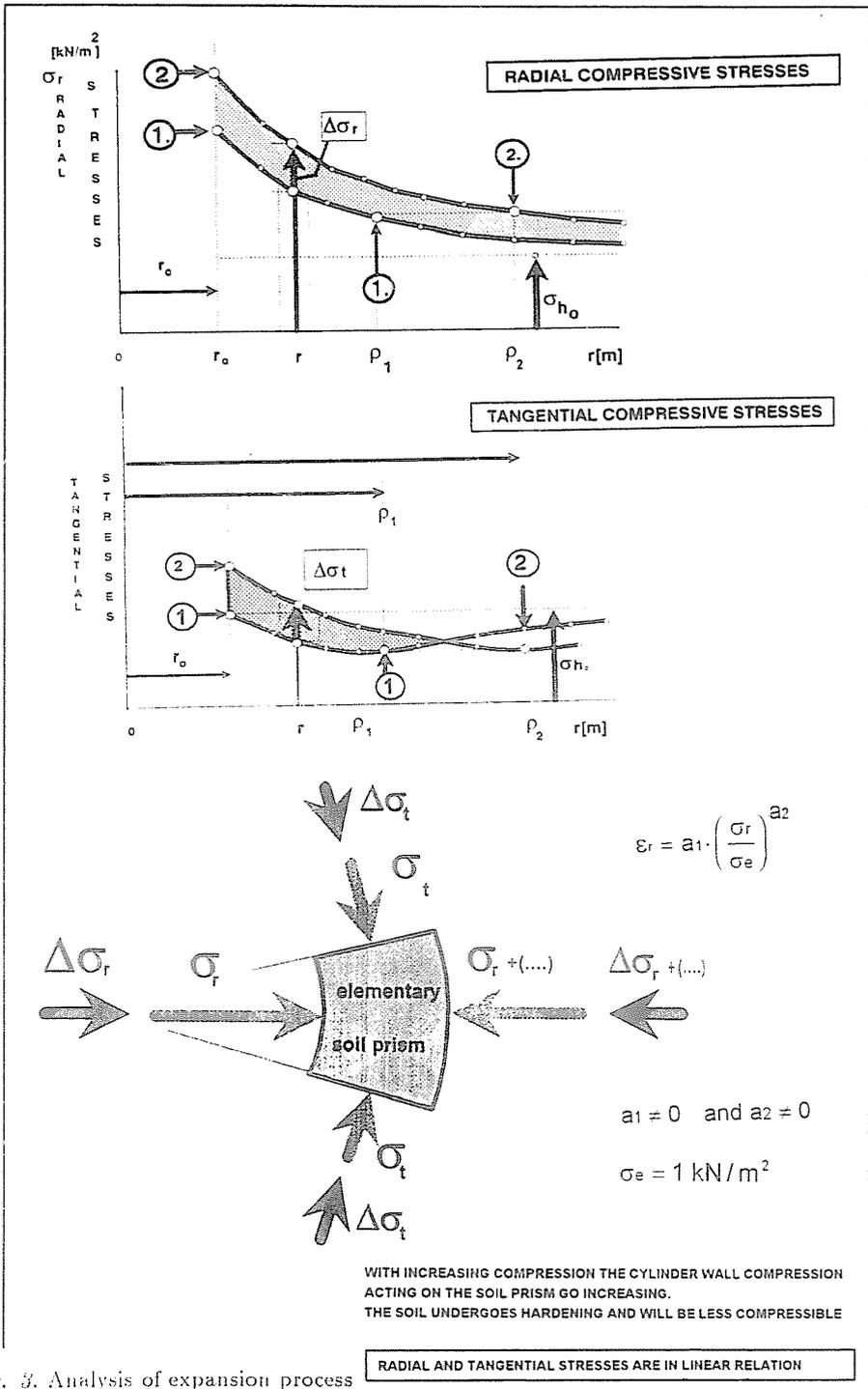


Fig. 3. Analysis of expansion process

where

$$a_1 \neq 0 \quad \text{and} \quad a_2 \neq 0$$

By differentiation:

$$\frac{d\varepsilon_r}{d\sigma_r} = \frac{a_1 a_2}{\sigma_e} \left(\frac{\sigma_r}{\sigma_e} \right)^{a_2-1} \Rightarrow \text{inverted} \quad \underbrace{\frac{d\sigma_r}{d\varepsilon_r}}_{E_r} = \underbrace{\frac{\sigma_e}{a_1 a_2}}_{E_0} \left(\frac{\sigma_r}{\sigma_e} \right)^{1-a_2} \quad (11, 12)$$

denoting

$$\frac{\sigma_e}{a_1 \cdot a_2} \equiv E_0 \quad \text{and} \quad 1 - a_2 \equiv a, \quad a < 1. \quad (13, 14)$$

Thereby we get

$$E_r = E_0 \left(\frac{\sigma_r}{\sigma_e} \right)^a, \quad a \leq 1. \quad (15)$$

The relation between the ability of the soil for deformation to soil stresses helps to determine soil displacements. The following relationship can be assumed between radial specific compression and radial components of displacement:

$$\varepsilon_r = \frac{du_r}{dr}. \quad (16)$$

Knowledge of the two defined relationships, i.e. that between stresses and specific compression, and that between radial displacement component and specific compression is necessary and sufficient to determine the radial distribution of soil displacement within the compaction zone.

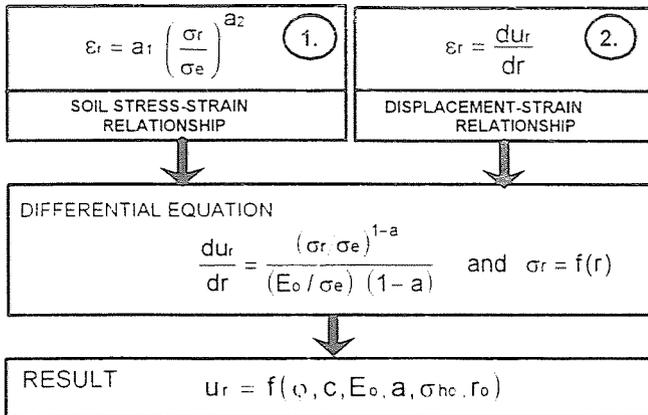


Fig. 4. Determination of displacement inside the compacted zone. The scheme of the solution

The scheme of the solution is presented in *Fig. 4*. Denoting:

$$B = \frac{2 \sin \varphi}{1 + \sin \varphi} \quad \text{and} \quad \sigma_0 = (\sigma_\rho + c \cot \varphi) \quad \text{and} \quad n = \frac{\rho}{r_0}. \quad (17, 18, 19)$$

General formula of the solution is:

$$r = r_0, \quad (20)$$

$$u_{r_0} = \frac{\sigma_0^{1-a}}{E_0} r_0 \left[\frac{n - n^{B \cdot (1-a)}}{(1-a) [1 - B (1-a)]} - \frac{c \cot \varphi}{\sigma_0} \frac{n - n^{-B a}}{1 + B a} - \frac{a}{2} \left(\frac{c \cdot \cot \varphi}{\sigma_0} \right)^2 \frac{n - n^{-B(1+a)}}{1 + B (1+a)} \right] + u_\rho,$$

while the radius of the compaction zone may be obtained from the stress function:

$$\sigma_{r_0} = \sigma_0 \left(\frac{\rho}{r_0} \right)^B - c \cot \varphi \Rightarrow n = \frac{\rho}{r_0} = \left(\frac{\sigma_{r_0} + c \cot \varphi}{\sigma_0} \right)^{\frac{1}{B}}. \quad (21, 22)$$

As a matter of fact, these relationships have led us to know the stress distribution within the compaction zone, and the distribution of displacements, but the magnitude of the stress is not yet known at the boundary of the compaction zone and therefore also the radius of the compaction zone may only be given as a function of this stress.

Two fundamental condition equations may be written for the boundaries of the compaction zone: The Mohr-Coulomb condition:

$$\sigma_t = \xi (\sigma_r - \sigma_u). \quad (23)$$

The condition expressing the volumetric constancy of the soil:

$$m_\rho = 0. \quad (24)$$

Hence

$$m_\rho = \frac{\sigma_\rho^{1-a} - \sigma_{h_0}^{1-a}}{E_0 (1-a)} - \frac{\sigma_{h_0} - \overbrace{\xi (\sigma_\rho - \sigma_u)}^{\sigma_t}}{E_0 \sigma_{h_0}^a} = 0. \quad (25)$$

Soil stresses may be computed from these two fundamental equations with $r = \rho$

$$\sigma_\rho = \frac{\sigma_{h_0}}{a} \left[1 + \xi - \sqrt{(1 + \xi)^2 - 2 a (1 - \xi) - 2 \xi a \frac{\sigma_u}{\sigma_{h_0}}} \right] + \sigma_{h_0}. \quad (26)$$

If the process of expansion is considered, it can be envisaged that during expansion the surrounding soil is subject to stresses in a continuously extending soil mass compared to the initial state, while also the zone of increasing soil stresses outside the compaction zone extends.

This phenomenon is similar to what we have investigated at the beginning of expansion, with the exception that now we have attained the value of the shearing resistance at the boundaries of the compaction zone and the stresses cause permanent deformation, (i.e. compaction), in the surroundings of the cylinder.

The yield stress to attain the plastic state is at the limiting value of ultimate volume change in the soil, its value is independent from the extension of the plastic zone.

Thus, during expansion, the beginning of the plastic stress state is set to the stress that is relevant to the beginning of volume change (σ_ρ).

This does not mean, however, that soil stresses cannot continue to increase inside the plastic stress zone, since, due to the interaction within the entire soil mass, increased stresses are transferred to increasing surfaces.

The stress distribution outside the plastic stress zone is similar to what was the distribution at the beginning of the expansion:

$$\sigma_r = \sigma_{h_0} + (\sigma_\rho - \sigma_{h_0}) \left[\frac{\rho}{r} \right]^2 \frac{1}{1 - \frac{a}{5} \frac{\sigma_\rho - \sigma_{h_0}}{\sigma_{h_0}} \left[1 - \left(\frac{\rho}{r} \right)^2 \right]} \quad (27)$$

and

$$\sigma_t = \sigma_{h_0} - \frac{\sigma_{h_0}^a}{1-a} (\sigma_r^{1-a} - \sigma_{h_0}^{1-a}). \quad (28)$$

Soil displacements may be examined in the view of known relationships between specific deformations.

$$u_r = \int_\rho^{\tau_h} \frac{\sigma_r^{1-a}}{E_0 (1-a)} \sigma_e^a dr, \quad \text{and} \quad \sigma_r = f(r), \quad (29)$$

where τ_h identifies that ultimate distance to which displacement may develop in the soil. (Its convenient value is 8 to 10 times the initial radius).

Approximate distribution of radial displacements:

$r \geq \rho$

$$u_r = \frac{\sigma_\rho - \sigma_{h_0}}{E_0 \sigma_{h_0}^a} \sigma_e^a \left[1 - \frac{a}{5} \frac{\sigma_\rho - \sigma_{h_0}}{\sigma_{h_0}} \left(\frac{\rho}{r} \right)^2 \right] \frac{\rho^2}{r}. \quad (30)$$

Variation of soil volume (density) develops within the zone, where shear strength for the given stress level has been mobilized.

Soil volume change may be examined by knowing the relationships between specific deformations.

Increment of radial specific compressional deformation related to the original condition:

$$\Delta\varepsilon_r = \frac{\sigma_r^{1-a} - \sigma_{h_0}^{1-a}}{E_0(1-a)} \sigma_e^a \quad \text{compression.} \quad (31)$$

Difference between annular and tangential specific deformations, respectively

$$\text{for } \sigma_t \geq \sigma_{h_0}, \quad \Delta\varepsilon_t = \Delta\varepsilon_r \frac{\sigma_t^{1-a} - \sigma_{h_0}^{1-a}}{E_0(1-a)} \sigma_e^a \quad \text{compression,} \quad (32)$$

$$\text{for } \sigma_t < \sigma_{h_0}, \quad \Delta\varepsilon_t = -\frac{\sigma_{h_0} - \sigma_t}{E_0} \frac{\sigma_e^a}{\sigma_{h_0}^a} \quad \text{expansion.} \quad (33)$$

Volume change of an elementary soil prism

$$m_r = \frac{d\bar{V}_i - d\bar{V}_0}{d\bar{V}_0} \approx \Delta\varepsilon_r + \Delta\varepsilon_t + \Delta\varepsilon_r \Delta\varepsilon_t, \quad m_r > 0. \quad (34)$$

Its knowledge permits to determine the changes in soil density:

$$\rho_s = \frac{\rho_0}{1 - m_r}. \quad (35)$$

Possibilities to determine soil stresses, displacements, volume changes around the cylinder expanded in the soil are illustrated by numerical examples in *Figs. 5, 6*.

The problem will be to examine the effects of expanding a vertical pile cylinder \varnothing 40 cm.

Out of the expansion process, a range of 30 mm expansion will be taken, and variations occurring in the soil in this condition will be examined in detail.

To demonstrate the development of expansion along the depth, tests were made at three different initial static pressure values, assuming that the earth pressure at rest varies linearly with depth.

This numerical example offers the following statements concerning the soil shear strength and the mobility activation of the shear strength:

– For a displacement $u < u_\rho$

This is *the shear stress section*. In this section, shear stresses go increasing without reaching the shear strength.

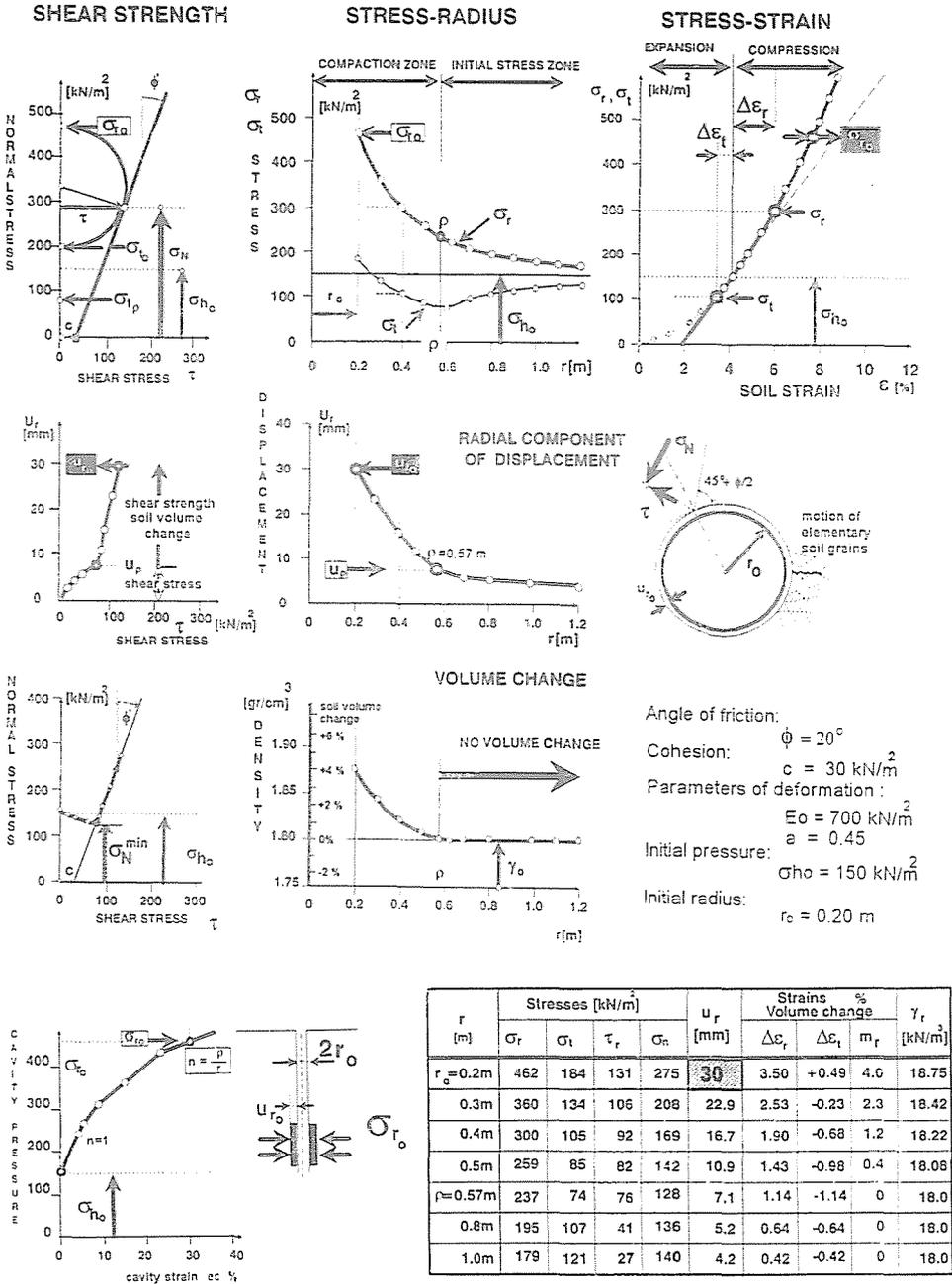


Fig. 5. Determination of stresses, displacements, volume changes developing around a large-diameter cylinder

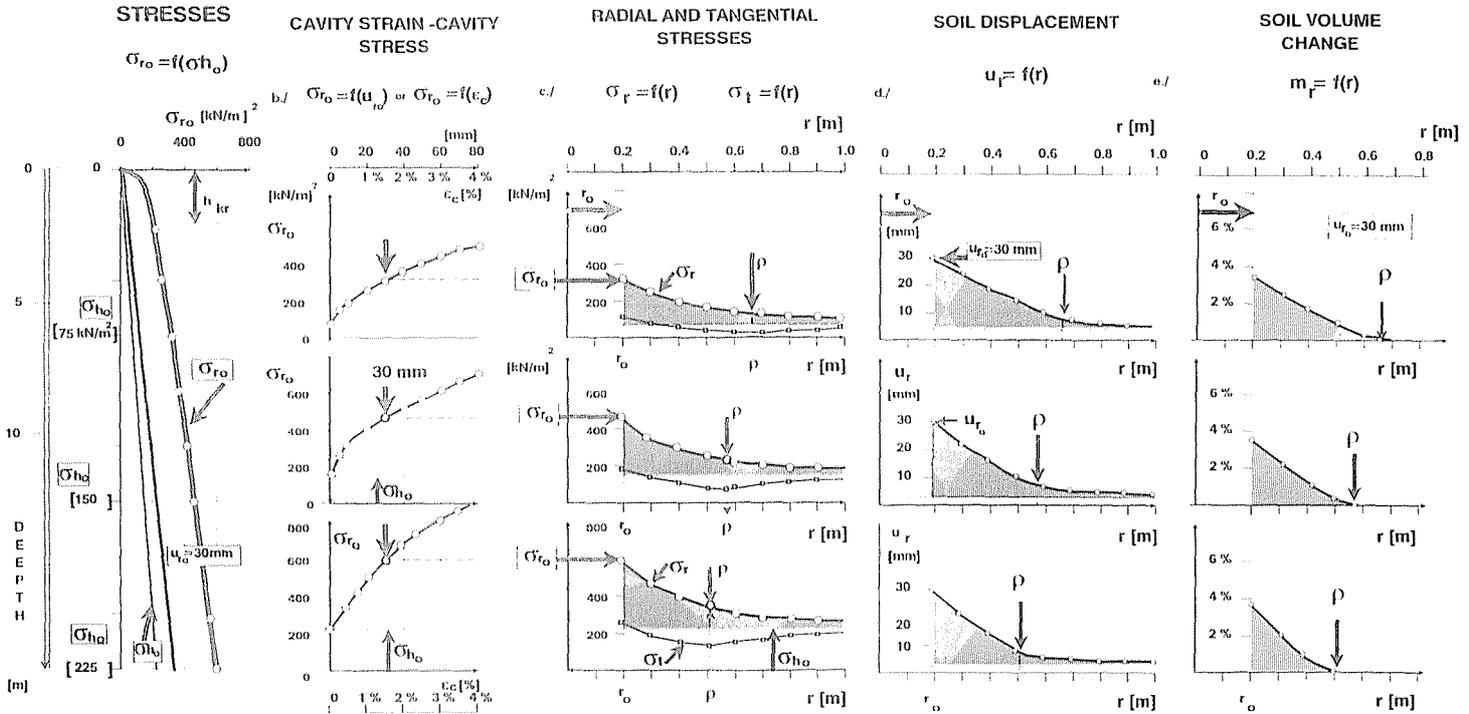


Fig. 6. Determination of stresses displacements and soil volume changes around a cylinder at 30 mm displacement at the cylinder wall

- For a displacement $u \geq u_p$
This is the shear strength section. In this section, the shear strength was already mobilized, with a maximum proportion between principal stresses varying continuously with the radius, in conformity with shear strength regularities.
- For a displacement $u = u_p$
Displacement vs. shear stress diagram shows a definite break point. This is the ultimate displacement where shear strength is mobilized.

Analysis of the correlations between shear stresses and normal stresses leads to interesting statements.

- In knowledge of principal stresses the normal stresses acting on an infinity of sliding planes are not constant but vary; starting from the static earth pressure at rest, normal stresses go decreasing. Here begins the process of shear strength mobilization, shear stresses develop and increase.
- *Reduction of normal stresses has a minimum, where we attain the plastic stress state. Simultaneously this is the point of the highest proportional rate between the principal stresses in the soil.*

3. Diagrams for the Determination of Stress and Strain Conditions in Soils around an Expanded Cylindrical Hole

Soil stresses developing in the compaction zone and acting on the mantle of an expanded cylindrical hole, as well as the progression of the compaction zone, together with the pertinent various strains and displacements can be visualised in a simple dimensionless manner by using only conventional soil parameters. It is reasonable to represent stresses as ratios to the pressures in rest and to use the diagram to determine the radius of the compaction zone and the distribution of stresses.

Advantage of the use of diagrams is to get general and comparative information about the changes occurring in the soil around the cylindrical hole during expansion; i.e. the materialistic rules of the expansion process become displayed by looking on the diagrams.

A purely approximative information about soil characteristics (in accordance with practical demand, but without detailed calculations) would be enough to determine those domains of stresses, strains and volumetric changes which may develop in the soil.

Practical use of the diagrams is demonstrated in a numerical example (see *Fig. 7*) where known are: the shear strength, the initial stress condition and the deformation properties of the soil:

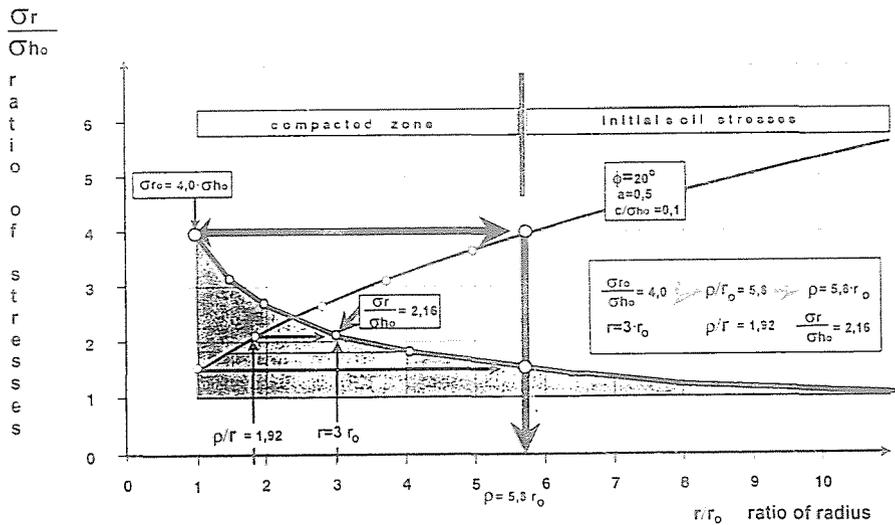
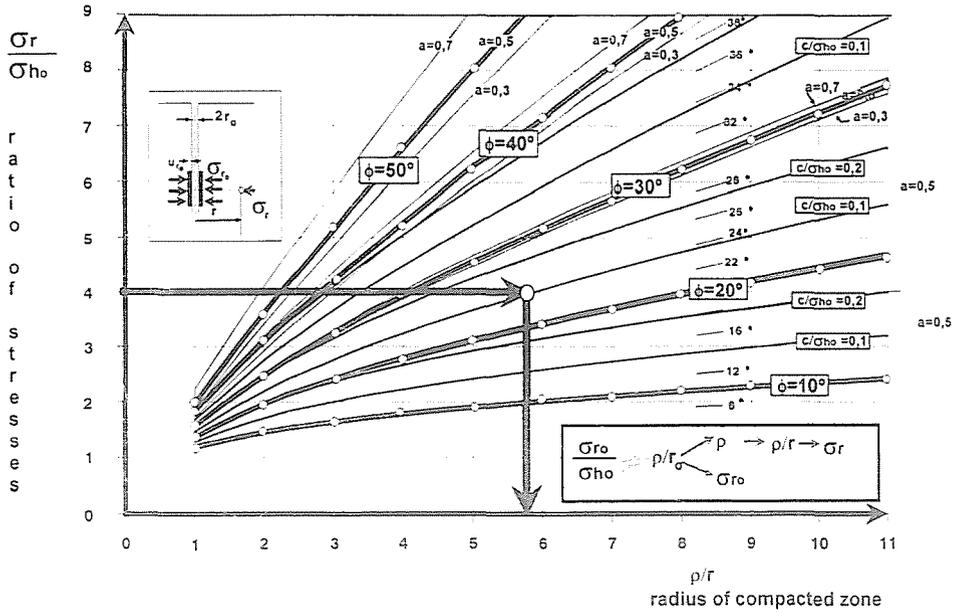


Fig. 7. Determination of the radius of compacted zone and the distribution of soil stresses inside the compacted zone

internal friction	$\varphi = 20^\circ$
cohesion	$c = 20 \text{ kN/m}^2$
modulus of deformation	$E_s = 700 \sigma^{0.45}$
soil pressure at rest	$\sigma_{h_0} = 100 \text{ kN/m}^2$
initial radius of hole	$r_0 = 0.05 \text{ m}$

Say, we would like to know what will happen, if the initial pressure is increased to four times the initial value?

From the chart 7/a we can read that the extension of the compression zone is 5.7 times the initial radius, i.e. the radius of the compression zone will be 28.5 cm.

Knowing the radius of the compaction zone, the distribution of stresses in the zone can be determined by simply taking the ratio of the limiting radius of the compacting zone to the actually chosen radius and there we read off the ratio between the stresses; the result can be plotted to the chosen radius (see *Fig. 7/b*).

Fig. 8 gives information about the radial displacement of the cylinder wall at the given stress level. From the diagram, that characterizes the soil, we can calculate the displacement in relation to the given stress level for any arbitrary taken radius inside the compression zone; i.e. we can calculate the distribution of soil displacement in the function of the radius. Explanation of terms can be learned from the figures.

$$u_r = \frac{\sigma_{h_0}^{1-a}}{E_0} \sigma_\epsilon^a r K_u, \quad (36)$$

4. Determination of Pulling (Tearing, Breaking) Resistance of an Axially Loaded Cylinder after its Expansion

We want to know the limiting value of resistance that can be exercised by an axially loaded (pulled or pressed) cylinder after its expansion.

Application of an axial force increases (as an external constraint), shear stresses in the surrounding of the cylinder.

Due to the surrounding soil mass, horizontal stresses will not decrease, but vertical stresses do increase in the compacted zone.

Maximum of shear stresses [that is, the shearing resistance] can be reckoned on the basis of Coulomb's equation:

$$\tau_r = \sigma_r \operatorname{tg} \varphi + c. \quad (37)$$

Though we do not know which will be the surface where the cylinder breaks off, but, with view to stress distribution, we can assume an axial symmetric round surface.

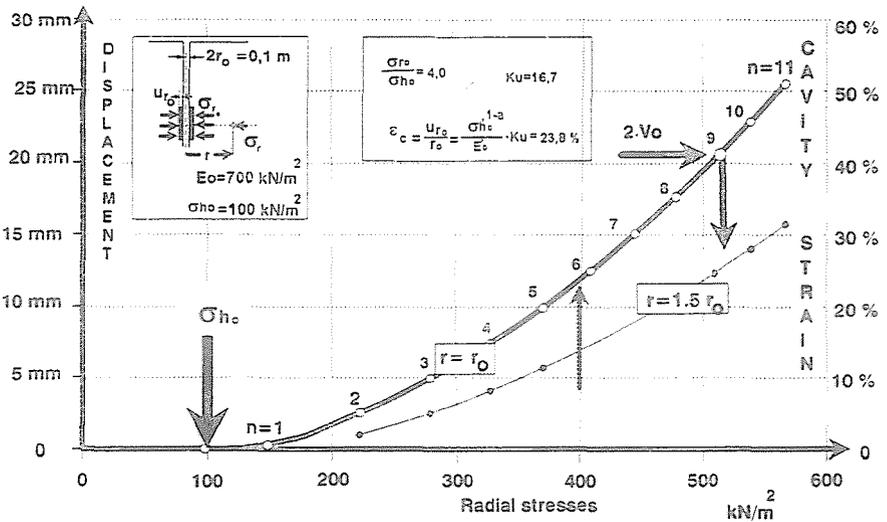
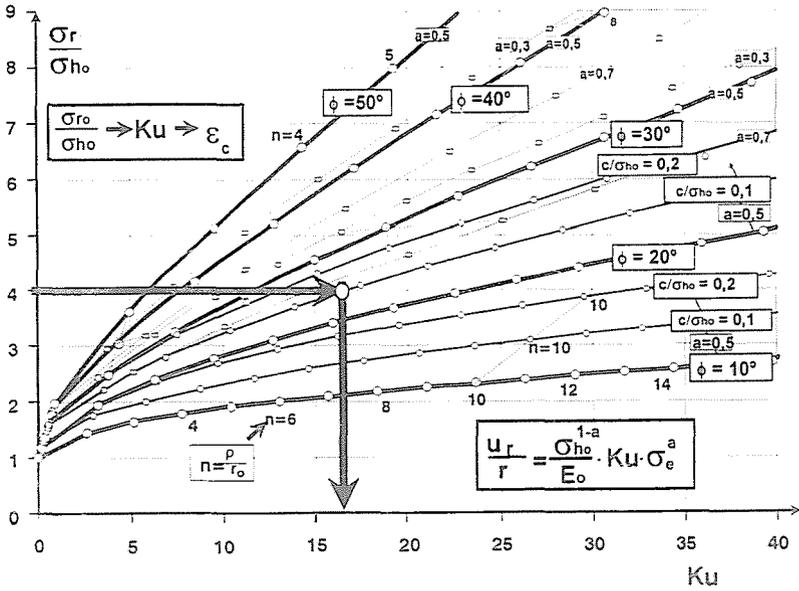


Fig. 8. Diagram for the determination of the displacement and the distribution of displacements inside the compacted zone.

A potential shearing surface increases linearly with the radius

$$A = 2r_k \pi . \quad (38)$$

We are looking for the least breaking off resistance of the cylinder what can be reckoned as the minimum of a product of the surface area and the probable shearing strength:

$$tm_{\min} = |\tau_r 2r_k \pi|_{\min} . \quad (39)$$

Unknown is the radius of this body at the least value of the product. Coming nearer to the shell of the cylinder the probable shearing strength is increasing, but simultaneously the break off surface gets smaller and the product of these two factors will have a little change at the cylinder wall. In practice this will mean that a little error in the size of the radius would hardly influence the result.

5. Validity Limit of the Model

The developed theoretical solution is based on the investigation of spherical stress conditions. In this respect it has been supposed, however, that the third principal stress is the tangential stress.

Near to the ground surface, beyond a certain degree of expansion, soil particles have the freedom to move towards the surface, because the geostatical pressure is not enough to produce the necessary balance. So, in this region, stress condition becomes more complicated. In this elaboration the downward extension of this zone below the surface is designated as h_{crit} .

Complex spherical effects have to be analysed to investigate the balance of forces to find out this critical depth below surface. Dead weight of the soil mass that is presumed to heave towards the surface, shearing resistance forces on the plane of tearing and stresses from the expansion pressure have to be examined simultaneously.

6. Time Effect

Time depending loss of stresses may result from two basic grounds:

- First, such an effect may be the result of shrinkage in the cylinder (example for concrete pile). At high stress levels even a small shrinkage of the cylinder may initiate significant loss in stress. This effect can be calculated by the proposed method when the time dependent process of shrinkage in the cylinder can be inferred.

- Second, similar effect can arise in time when the soil mass undergoes creeping, stress loss originates from some posterior effect initiated by consolidation. A certain time is needed to accelerate the pressure being transferred to the soil. If the speed is more than the capability of the soil to respond with plain displacement, than volume change elapses in time and this initiates consolidation. Displacement of soil grains, is primarily governed by the development of pore-water pressure.

The presented model gives the last values of strains.

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