PERIODICA POLYTECHNICA SER. CIVIL ENG. VOL. 38, NO. 1, PP. 99–108 (1994)

SETTLEMENT ANALYSIS BY THE 'MOVING LIMIT DEPTH' MODEL

Peter SCHARLE

Department of Civil Engineering,
Technical University of Budapest
H–1521, Budapest, Hungary

Received: March 25, 1994

Abstract

Limit depth theories are often used for settlement computation to simplify computational difficulties. Elementary stress distribution assumptions refer to the final stress state while the sequential evolution of the stress and strain fields is neglected.

In the age of PCs there is no reason to maintain old and oversimplified assumptions of this kind. The computational model presented follows the load increase sequence and describes the physical fact of the limit depth change. Field measurements prove the adequacy of this approach.

Keywords: settlement, limit dept, granular material, fabrics rearrangement, constitutive model, computation.

1. Appreciation of 'Limit Depth' Models

In the conventional unaxial model of settlement computation (*Fig. 1*) all variables depend on the coordinate z only. As an effect of the vertical stress σ_z^q caused by the surface load q the vertical strain ε_z is defined as

$$\varepsilon_z = \frac{dz - dz'}{dz} = \frac{de}{1+e} \tag{1}$$

where $e = e(z)$ denotes the void ratio. Total surface settlement of the semi-infinite continuum is obtained as

$$\varepsilon = \int_0^\infty \varepsilon_z dz = \int_0^\infty \frac{de}{1+e} dz \tag{2}$$

For computational purposes, reasonable constitutive assumptions are to be made to establish the nonlinear relationship between ε_z and σ_z^q (FEDA, 1982, p. 250).

Expression (2) seems to be exact since it reflects the infinite extension of the domain considered. In case of granular materials, however, the strains are caused mostly by particle rearrangement. Deformation of solid particles can be neglected. Moreover, site experiments and theoretical considerations give the evidence that a small variation $\delta\sigma_z$ of the initial stress state σ_z^0 does not result in rearrangement. Granular mass has a 'structural resistance', its fabrics (configuration of grains) proves to be quasi-rigid until the stress increment exceeds this resistance (or strength).

'Limit depth' theories of soil mechanics are based on these considerations. Under any finite surface load q one can determine the depth where the geostatic pressure σ_z^0 exceeds by magnitudes the stress σ_z^q. Under this level (out of the active zone) the strains can be neglected, even if they were integrated 'ad infinitum'. Extended site experiments and theoretical considerations (SEYCEK, (B) 1982, HAVLICEK, 1982) prove that these theories not only hold their own against classic, elastic or inelastic continuum models of infinite halfspace but are more adequate from the aspect of physics, as well.

Of course, wording and determination of limit depth is not a subject of deliberation only but a field of arbitrariness, too. As a consequence, frequently used expressions can be criticised (like 'modulus of compression depends on depth' since not the depth but the stress level is of primary interest).

Standards and recommendations for limit depth criteria usually given for stresses in form of

$$\sigma_z^q \leq \lambda\sigma_z^0$$

with $\lambda = 0,1 \ldots 0.5$ can be questioned since the compressibility of fabrics should be taken into account more directly. Nevertheless, in any case where the unaxial model can be accepted as an approximation the limit depth theories are well worth using. This is the reason for presenting a model where (*Fig. 1*)

- sequential character of the surface load evolvement is taken into account, and
- the 'active zone' is bounded from below (and, in actual cases, even from above) with moving boundaries.

It is worth noting that other physical problems like melting have been described similarly.

2. Data Field for the 'Moving Limit Depth' Model

Instead of using classical continuum variables like strain or constitutive moduli the settlement will be expressed with the void ratio and the compression relationship

$$e = e(p) \qquad \text{and its inverse} \qquad p = p(e) \qquad (3a,b)$$

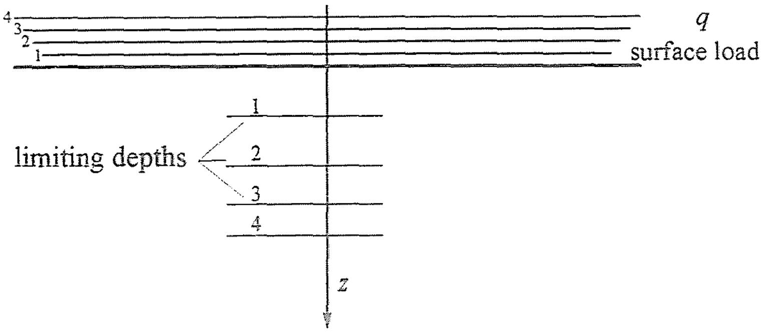

Fig. 1.

will be used as a constitutive law. The compression curve has a paramount role in this model (*Fig. 2*). In principle, the curve has an asymptote at $e = e_{\min}$, but in accordance with the physical considerations mentioned it can be assumed that a finite compression stress p_r belongs to the minimum value of e. Stresses $p \equiv p_r$ do not result in change of configuration (grain crushing has been excluded by this assumption, evidently.

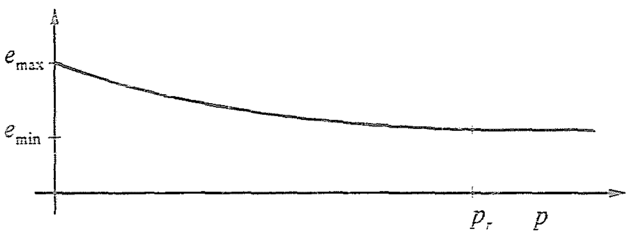

Fig. 2.

Consequently, two characteristic stress parameters are to be considered:

- $\delta p(p)$ denotes a stress-dependent lower bound — structural resistance or structural strength — for stresses causing rearrangement in fabrics and causing strain by this way;
- p_r denotes the upper bound for stresses causing compression (in case of higher pressures the fabrics remains rigid).

Structural resistance δp can be determined in several ways (it is often considered as a function of depth, for example). We shall define it as a function of δe – a small variation of void ratio. This assumption makes possible to connect structural resistance with a characteristic level of change in fabrics. Having a compression curve $e = e(p)$ a simple calculus makes possible to express δp in form of

$$\delta p = \left\{ \frac{de(p)}{dp} \right\}^{-1} \delta e \tag{4}$$

A simple example of parabolic compression curve is described in detail in the Annex, where $\delta p(p)$ turns to be hyperbolic.

More sophisticated models demand more data. In our case initial void ratio $e^0(z)$ and initial vertical (geostatic) stress $\sigma_z^0(z)$ distributions are to be given (measured directly or computed from other measured data). Using *Eq.* (3) the stress-equivalent void ratio distribution $e(\sigma_z^0)$ can be obtained (*Fig. 3*).

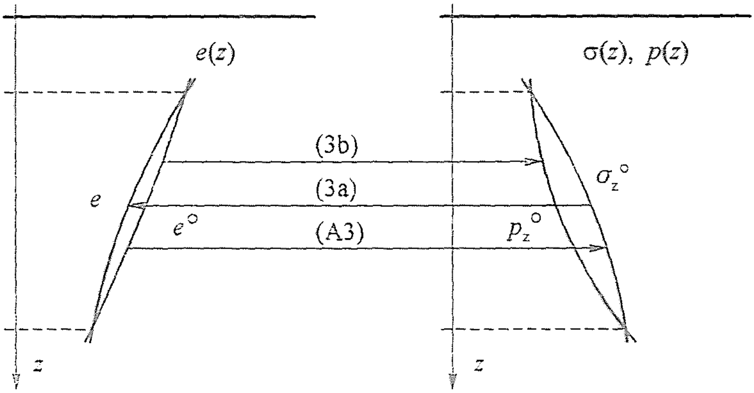

Fig. 3.

In general, the difference $e^0 - e$ does not disappear. Therefore, two cases can be distinguished:

- $e^0 < e$: fabrics is 'overconsolidated' due to some earlier compression;
- $e^0 > e$: fabrics is 'underconsolidated', instable and a small variation of stresses probably results in a mobilization which ends in a void ratio corresponding to the actual stress.

Neither cases must be excluded in the initial state.

Similar comparisons can be made in terms of stresses. Using (3b) we can determine from e^0, the corresponding $p^0(z)$ distribution. Now p^0 and σ_z^0 will differ, in general (*Fig. 3*), over- and underconsolidated regions can be detected again.

These data (together with the compression curve) are sufficient. The computation follows a sequential (cyclic) way. Surface load increment dq

will cause mobilization and compression in several layers where the structural strength $\delta p = dq$. As a consequence, void ratio in these layers decreases. Having updated the void ratio, stress and structural strength distributions the next load increment can be taken.

Actual expression of $e = e(p)$ initial values of the e and σ distributions may differ from case to case. Therefore, further assumptions, simplifying approximations can be (or must be) made. The Annex presents a very practical example.

3. Remarks

In several cases partial surface loads can be considered with plausible stress extension assumptions. It may happen, than, that the active zone becomes bounded from above, too. The model is able to describe this very practical phenomenon.

The model is strongly connected with assumptions valid in the unaxial case only. Therefore, two- and three-dimensional generalizations cannot be hoped. On the other hand, the approach is consistent with the stress-path considerations, where the multidimensional structural resistance $\delta p_{ij}(p_{ij})$ can be interpreted easily. Imagine the demand for initial data and number of cycles...

Title of the paper has been choosen to addresss those experiments accomplished and reported in the early 80-es by SEYCEK. His experimental data and theoretical analysis resulted in a physical description accepted in this paper. Even a short remark referring to the moving boundary can be found in (SEYCEK, (A) 1982).

The method presented reflects the very physical conditions. The settlement is bounded by the integrated 'void ratio reserve' ('compressibility reserve'). In comparison with other methods often resulting in overestimated settlements the 'moving limit depth' approach may prove to be competitive at a reasonable price of some additional initial data.

References

FEDA, J.: Mechanics of Particulate Materials — The principles, Academia, 1982.

HAVLIČEK, J.: Sedani zakladu plosne zakladanych staveb, Inzenyrske stavby, 1982, Vol 7, pp. 327–340.

SEYCEK, J.: Prubeh sedani zakladove pudyi do hloubky, Inzenyrske stavby, (A) 1982, Vol. 8, pp. 368–379.

SEYCEK, J.: Problematika spravneho stanoveni modulu pretvoreni zemin, Inzernyrske stavby, (B) 1982, Vol. 9, pp. 413–421.

ANNEX
Computational Model

To avoid unnecessary computational hocus-pocus the direct discrete formulation going well with personal computers will be presented for a two phase (solid/gas) granular material (*Fig. A1*). Characteristic parameters

- γ_s – unit weight of solid particles;
- $e_{\max}$ – maximum void ratio;
- e_{min} – minimum void ratio;
- p_r – limit compression stress corresponding with $e_{\min}$ (normally consolidated, quasi-rigid state)

are taken as state-independent constants given for the whole region.

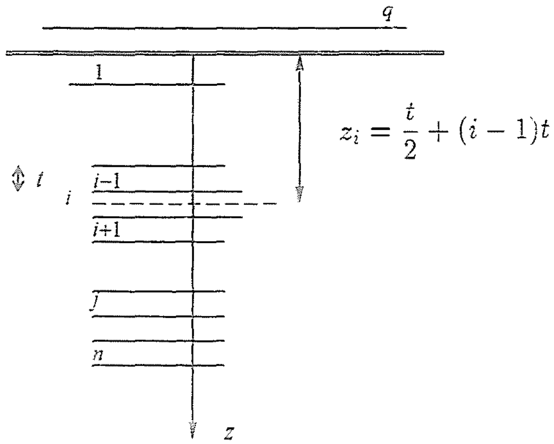

Fig. A1. Given or prescibed data γ_s $e_{\max}$ $e_{\min}$ e° δe λ p_r

Constitutive behaviour of the granular material will be defined by the analytic relationship of

$$e = e(p_z) = e_{\max} + \frac{e_{\max} - e_{\min}}{p_r^2}(p_z^2 - 2p_r p_z) \tag{A1}$$

Structural stability of the grain fabric will be characterized with a threshold void ratio δe, taken deliberately. As a consequence, the structural resistance δp_z can be obtained as

$$\delta p_z = \delta p_z(p_z) = \frac{-p_r^2}{2(e_{\max} - e_{\min})(p_r - p_z)}\delta e \tag{A2}$$

In case of very loose (underconsolidated, unstable) fabrics the structural resistance can be adjusted to the acting geostatic stress by a factor $\lambda \ll 1$ (which can be chosen analogouosly with the 0.2 factor used in common 'limit depth' methods).

Initial State Fields

Consider now n layers of the same thickness t and assume that the initial void ratio $e^0(z)$ has been obtained (by measurement, for instance). Note that this parameter is one of the most natural ones to determine for the initial state. The i-th layer will be characterized by the average value e_i^0 ($i = 1 \dots n$) referred to its centre plane (*Fig. A2*).

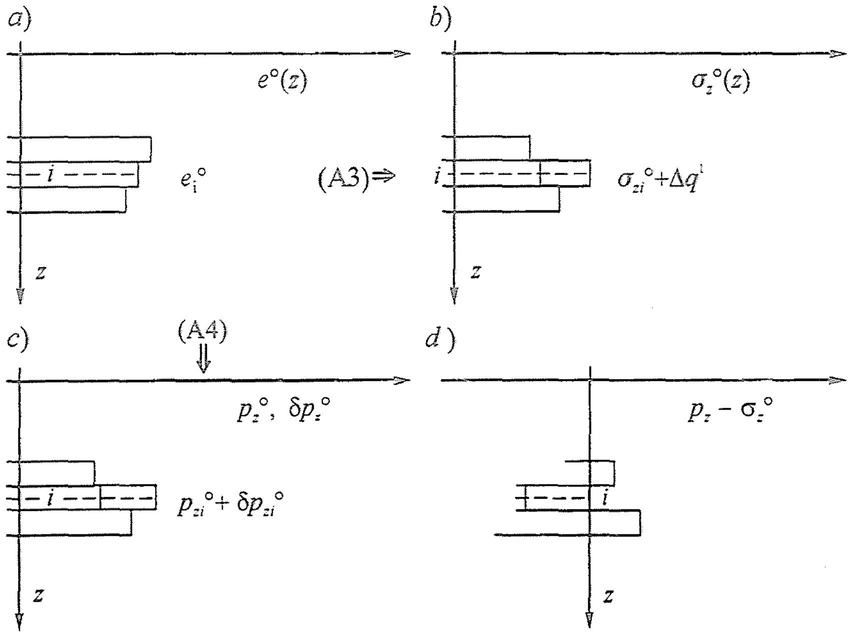

Fig. A2. a) initial void ratio of layers
b) initial geostatic stress and surface load increment
c) initial adjoint stress and structural resistances
d) initial compression sensitivity

Two adjoint stress distributions can be derived now (*Fig. 2.a* $\Rightarrow$ *2b, 2c*)

– geostatic stresses at $z = z_i$ are obtained as:

$$\sigma^0_{zi} = \frac{t}{2(1+e^0_i)}\gamma_s + \sum_{j=1}^{i-1} \frac{\gamma_s}{1+e_i} t \tag{A3}$$

– imaginary (theoretic) stresses corresponding to the 'normally consolidated' state can be obtained from the constitutive equations as

$$p^0_{zi} = p_r \left(1 - \sqrt{\frac{e^0_i - e_{\min}}{e_{\max} - e_{\min}}}\right) \tag{A4}$$

The difference $p^0_{zi} - \sigma^0_{zi}$ (as shown on *Fig. A2–d* is not, in general, equal to zero. Its sign and value, as an index of initial compression sensitiveness is of great importance regarding the anticipated behaviour. Particularly, considering the effect of the surface load increment Δq, two characteristic initial states of layers are to be distinguished:

$p^0_{zi} > \sigma^0_{zi}$ (relatively overconsolidated layer):

No further compression occurs until the actual stress $\sigma^0_{zi} + \Delta q$ exceeds threshold value $p^0_{zi} + \delta p^0_{zi}$ (where the second term can be obtained from *Eq.* (A2). In other words, mobilizing threshold surface load increments

$$\Delta q_i = p^0_{zi} + \delta p^0_{zi} - \sigma^0_{zi}$$

can be adjoined to these layers;

$p^0_{zj} < \sigma^0_{zj}$ (layer of compressional instability):

Due to some finite stress increment Δq_j the fabric collapses and the void ratio decreases to the value corresponding with the stress $\sigma^0_{zj} + \Delta q_j$. In this case, either δp^0_{zj} can be taken as upper limit for Δq_j or Δq_j can be defined by the geostatic stress as $\Delta q_j = \lambda \sigma_{zj}$.

Threshold values obtained in accordance with these considerations are shown on *Fig. A3*.

Computing of Settlements

For taking into account the sequential character of loading and soil response

$$\Delta q^1 = \min - q_i - \quad (i = 1 \dots n) \tag{A5}$$

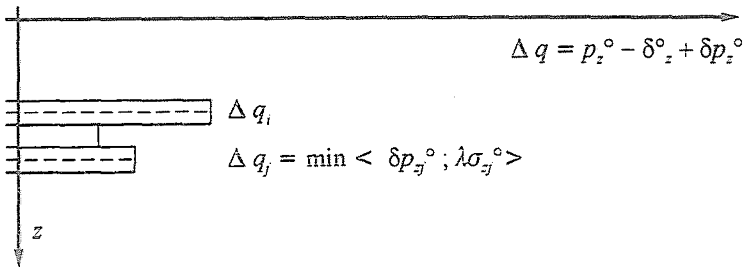

Fig. A3. Initial threshold values for surface load increments

is to be chosen as first load increment. Due to this surface load, the void ratio will decrease at least in one layer – let it be denoted with k. In this case we obtain

$$e_k^1 = e_{\min} + \frac{e_{\max} - e_{\min}}{p_r^2}(p_r - \sigma_{zk}^0 - \Delta q^1)^2. \tag{A6}$$

The elementary settlement due to this change can be expressed as

$$ds_k^1 = \frac{e_k^0 - e_k^1}{1 + e_k^0} \cdot t \tag{A7}$$

In case of several layers with the same threshold surface load increment Δq^1, the elementary settlements are to be summed up. No other layers with higher threshold values will be compressed yet. They remain quasi-rigid under the increment Δq^1.

Due to the effect of the first load increment the state characteristics are to be modified as follows:

- void ratios of those layers affected are to be changed in accordance with *Eq.* (A6);
- in the layer (or layers) affected the difference $p_{zk}^1 - \sigma_{zk}^1$ disappears (we assume that the layers become normally consolidated) – therefore new threshold values of δp_{zj}^1 are to be computed and substituted;
- values of geostatic stresses in each layer are to be increased by Δq^1.

After these modifications we arrive at the end of the first cycle and the next increment

$$\Delta q^2 = \min\{q_i\}$$

is to be chosen over the set of modified data. Iterative character of the computation is obvious.

Remarks

1. Extension of the active domain considered is not to be defined in advance, particularly in cases where $e_i^0 \gg e_{\min}$ On the other hand, taking of too big n can be avoided comparing e_i with $e_{\min}$ and q with δp_i^0

2. Of course, rate of convergence depends both on the e-distribution and on the magnitude of q , moreover can be influenced by the choice of δe, as well.