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DIMENSIONING OF ESTABLISHMENT SYSTEMS

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Abstract

The paper deals with the dimensioning of project establishments and establishments systems by means of probability theory. One can determine the investment costs, the annual costs of operation and upkeep of the subsystem to dimension, moreover, even the losses in costs due to the missing demands in terms of the logarithm $\ln r$ and $\ln k$ of the reciprocal value of the shouldered risks $1/r$ and $1/k$. The costs evaluated in such a way can be then optimized on the basis of the shouldered risks. Finally the optimal geometrical and technological dimensions of the establishment systems can be determined by successive approximation.

Keywords: establishment system, dimensioning, probability theory, shouldered risk, costs.

Typical Costs of a Project Establishment

For economically optimizing a project establishment, costs of investment, upkeep and operation have to be known, just as losses due to the missing of demands, and to accidental breakdown.

1. Investment Costs

The costs concerned with here refer only to the subsystem to be dimensioned. In designing, for instance, the structural system of a building, only structural costs, other than constructions and installations, are involved.

Unconditionally,

$$\bar{R}(t,\beta) = \bar{S}(t) + \beta\sqrt{[s_R(t)]^2 + [s_S(t)]^2}$$
$$\beta = \beta(a, c, r, k).$$

In the following, the formula above will have to be determined also by means of variation factors (relative deviation scatter).

$$R(t,\beta) = \bar{S}(t)\frac{1+\beta\sqrt{[v_R(t)]^2+[v_S(t)]^2-\beta^2[v_R(t)]^2[v_S(t)]^2}}{1-\beta^2[v_R(t)]^2}$$
$$\beta = \beta(a,c,r,k). \tag{1}$$

According to (1), capacity of the establishment is about linearly varying in β. For afixes distribution, the β value depends only on the risk taken. Variation of B values in an arbitrary function of distribution $F(\beta) = 1-\exp[-g(\beta)]$ for fixed slope a (a) and pointedness (c) is $\beta = \beta_0 + \beta_1 \ln r$ (e.g. for normal distribution, in the range $4 < \ln r < 12$, $\beta_0 = 1.22$, $\beta_1 = 0.26$) with max. 3% of error. Namely, for values $1/r < 0.05$, distributions generally tend asymptotically to zero, and this trend may fairly be approximated by straight sections. For the great family of distributions, $g(\beta)$ is a rational integer function, where in $\ln r$ tending asymptotically to zero may be quite well approximated by a straight line. In a narrow range, investment costs of the part of the establishment underlying the dimensioning increase proportionally with the capacity $C = W_0 + W_1R(t)$. Substituting these terms into each other:

$$C = W_0 + W_1\bar{S}(t) + W_1\beta_0\sqrt{[s_R(t)]^2+[s_S(t)]^2}+$$
$$W_1\beta_1\sqrt{[s_R(t)]^2+[s_S(t)]^2}\ln r, \tag{2}$$

where

$$W_1\beta_0\sqrt{[s_R(t)]^2+[s_S(t)]^2} = C_0c_2,$$
$$W_0 + W_1\left\{\bar{S}(t)+\beta_0\sqrt{[s_R(t)]^2+[s_S(t)]^2}\right\} C_0c_1.$$

Cost C is a magnitude depending also on geometric dimensions, and technology parameters (e.g. in hydraulic dimensioning, cross-section area of the tube, etc.). The C value computed according to the theory of probability may be further optimized within the speciality. For instance, in dynamic dimensioning, probabilistically computed load capacity may be further optimized by the cross-sectional values (for reinforced concrete structures the optimum steel percentage, for steel structures the web height, again for timber structures the optimum beam height, for any material, the cross-section shape affects costs).

So it may be written that:

$$C = C_0(c_0 + c_1 \ln r + c_2 \ln k). \tag{3}$$

Either for dynamic or for hydraulic dimensioning, regressional computation of (3) based on numerical examples is correct at 3% accuracy. C_0 is the investment cost at an optimally assumed risk for optimum geometrical dimensions. (3) may also be interpreted as costs are function of $\ln r$ and $\ln k$. Expanding the functional relationship into power series according to $\ln r$ and $\ln k$, and taking only linear terms into consideration yields (3).

Costs involved in (3) above represent only the part of the total investment cost underlying the dimensioning (structural costs alone, or costs for water outlets, etc.). Other costs in investments other than dimensioned from the given aspect for functional uses but required for the proper use, have to be separately computed. These add up to:

$$C = \sum_{i=1}^{n} C^{(i)}$$

part of them involved in other dimensioning. In thermal design, e.g. the excess costs of thermal insulation of the establishment is $C^{(2)}$. Technology equipment $C^{(3)}$ increasing in proportion to $\ln k$.

2. Costs of Upkeep

fUpkeep costs F of the part of the establishment underlying the dimensioning are function of $\ln k$, $\ln r$ and time t as motivated above. In general,

$$F = F\left(\frac{1}{\ln r}, \ln k, t\right). \tag{4}$$

Expanding (4) into power series according to variables:

$$F(t) = F_0 \left[f_0 + \frac{f_{11}}{\ln r} + \cdots + \frac{f1_n}{(\ln r)^n} + f_{21} \ln k + \cdots + f_{2p}(\ln k)^p + f_{31}t + f_{32}t^2 + \cdots + f_{3s}t^s \right]. \tag{5}$$

In *Eq.* (5), costs of upkeep in $\ln k$ and t are rational integer functions; in $\ln r$ a rational fractional function, F is upkeep cost of the establishment of rationally assumed risk, geometry, computed at time $t = 0$. As a first approximation, the function is well approximated, taking only linear terms into consideration,

$$F_t^{(1)} = F_0^{(1)} \left(f_0 + \frac{f_1}{\ln r} + f_2 \ln k + f_3 t \right). \tag{6}$$

For the same causes as those for *Eq.* (3), also (6) is justified by regression analysis. Obviously, with increasing risks of breakdown (failure), also upkeep costs are growing. With the decrease of costs shouldered for the missing of demands – with the increase of capacity as concomitant – upkeep costs are again increasing. It is needless to point out that upkeep costs increase with time, since the substance of the establishment degrades with time. Costs of upkeep are divided as described in item 1, and the statements outlined there refer to costs of upkeep to the sense. Overall costs of upkeep of the establishment:

$$F = \sum_{i=1}^{n} F^{(i)}. \tag{7}$$

3. Operation Costs

Essentially, the same are true for operation costs as for costs of upkeep, namely, they being function of $\ln k$ and t. Operation costs are irrelevant to the risk against accidental failure $(1/r)$. Operation costs are increasing with the increase of capacity – at a decrease of missing demands. Operation costs may be spoken of exclusively concerning the dimensioning of technological equipment. Design of structural systems is exempt from operation costs. With the passing of time; of course, also operation costs go increasing. In general:

$$B = B(\ln k, t). \tag{8}$$

Expanding (8) into power series:

$$B(t) = B_0[b_0 + b_{11} \ln k + \cdots + b_{1n}(\ln k)^n + b_{21}t + b_{22}t^2 + \cdots + b_{2q}t^q]. \tag{9}$$

Reckoning only with linear terms in power series (9):

$$B(t) = B_0(b_0 + b_1 \ln k + b_2 t), \tag{10}$$

in *Eq.* (10) B_0 is operation cost at time $t = 0$ of an establishment of optimum risk. Increase with time of operation costs needs no detailed explanation. Operation costs refer to all the establishment.

4. Losses Due to Missing Demands

It has to be reckoned with that during the planned service life, demands may be met only partially, and decreasing or even increasing with time. Capacity of a hotel may be insufficient to meet demands, for some days in a year, but there are days where the hotel has vacancies. For water works the situation may be similar for some days of a very hot and dry summer, limited availability of water has to be reckoned with, while in winter there may be excess water.

Damages due to missing demands being unpredictable, they may be assumed as of uniform distribution. These damages may return every year, and occur similarly to operation costs, excepted that damages are irrelevant to risks $1/r$ shouldered against accidental failure. If the capacity of the establishment is somewhat less than needed, then damages may be assumed to increase with time. Like in the foregoing, the variable is not k, but – advisably – $\ln k$. An increasing capacity shouldered raises increasing unmet demands $1/k$. In general:

$$E = E\left(\frac{1}{\ln k}, t\right). \tag{11}$$

Expanded into power series:

$$E(t) = E_0\left[e_0 + \frac{e_{11}}{\ln k} + \cdots + \frac{e_{1n}}{(\ln k)^n} + e_{21}t + \cdots + e_{2p}t^p\right]. \tag{12}$$

Reckoning only with linear terms:

$$E(t) = E_0\left(e_0 + \frac{e_1}{\ln k} + e_2 t\right). \tag{13}$$

E_0 in *Eqs.* (12) and (13) is the damage of an establishment of optimum risk due to missing demands at time $t = 0$. Regression analysis can always help *Eq.* (12) to be function of the damage due to missed demands in $1/\ln k$ and in time t, a rational integer function. For a damage uniformly distributed in time due to missed demands, E does not vary in time. Term E refers to technology, equipment or to all the establishment.

5. Damages Encountered

During the planned service life, the damage at an accidental breakdown, including missing profits, is function of $\ln r$, with respect to those above.

This relationship involves that with increase of the shouldered risk, an accidental failure has greater losses as concomitant. Accidental damage involves the part of investment costs of the establishment not reckoned with in dimensioning. The damage at an accidental breakdown is independent of the permanence of missing demands but is not time-independent. In general:

$$D = D\left(\frac{1}{\ln r}, t\right). \tag{14}$$

Expanded into power series:

$$D = D_0\left[d_0 + \frac{d_{11}}{\ln r} + \cdots + \frac{d_{1n}}{(\ln r)^n} + d_{21}t + \cdots + d_{2n}t^n\right]. \tag{15}$$

In the simplest case, irrespective of higher-order terms:

$$D = D_0\left(d_0 + \frac{d_1}{\ln r} + d_2 t\right). \tag{16}$$

In *Eqs.* (15) and (16), D_0 is the sum of damages of an establishment of optimum risk at accidental failure at time $t = 0$. Regression analysis may help the expression to be rational integer function in $1/\ln r$.

6. Cost Optimum

Let us calculate all the costs underlying dimensioning of an establishment failing at a time $t < T$, costs that will be capitalized at the time of failure. Sum not written off the investment costs

$$C\frac{q^T(q-1)}{q^T-1}\frac{q^{(T-t)}-1}{q^{(T-t)}(q-1)} = C\frac{q^T-q^T}{q^T-1}. \tag{17}$$

Out of the yearly costs, those not occurring in the least period $(T-t)$ capitalized at the time of failure t:

$$V_0 v_0 \int_0^{T-t} \frac{d\tau}{q^\tau} + V_0 v_1 \int_0^{T-t} \frac{\tau}{q^\tau} d\tau = V_0\left[\frac{q^T-q^t}{q^T(q-1)}\left(v_0 + \frac{v_1}{q-1}\right) - \frac{v_1}{q-1}\frac{(T-t)q^t}{q^T}\right]. \tag{18}$$

Yearly costs in (18) as stated earlier: $V_0(v_0 + v_1 t) = F_0 + B_0 + E_0 + (f_3 + b_2 + e_2)t$. At the time of construction $t = 0 : V = V_0 v_0 = F_0 + B_0 + E_0$.

All the costs of the part of establishment underlying the dimensioning at time t:

$$K(t,k,r) = C + V_0\left[\left(v_0 + \frac{v_1}{q-1}\right)\frac{1}{h(T)} - \frac{v_1 T}{q^T(q-1)}\right] + \frac{1}{r}\left\{C\frac{q^T - q^T}{q^T - 1} - V_0\left[\frac{q^T - q^t}{q^T(q-1)}\left(v_0 + \frac{v_1}{q-1}\right) - v_1\frac{(T-t)q^t}{q^T(q-1)}\right] + D_0\left(d_0 + \frac{d_1}{\ln r} + d_2 t\right)\right\}. \tag{19}$$

The first term in (19) is the re-establishment cost of the part of the establishment to be dimensioned, the second term is the capitalized sum of yearly costs, while the third term is the value of costs of an accidental failure multiplied by the shouldered risk. Also this third term is tripartite: first part is the share of parts to be written off of the structure of the establishment failed prematurely, at t, capitalized at the time of failure: the second part, of course, not to be paid, is the value of yearly costs in the period $(T-t)$ capitalized at the time t of accidental failure: at last, the third part comprises all the damages at accidental failure, including the profit missing until re-establishment.

Extreme value of (19) may be where first derivatives equal zero. Deriving first with respect to t, and substituting $\ln q \sim q - 1$:

$$\frac{\partial K}{\partial t} \equiv \frac{1}{r}\left\{-C^{(1)}\frac{q^t}{q^T - 1}(q-1) - V_0\left[\frac{-q^t}{q^T}\left(v_0 + \frac{v_1}{q-1}\right) - v_1\frac{Tq^t(q-1) - tq^t(q-1) - q^t}{q^T(q-1)}\right] + D_0 d_2\right\} = 0. \tag{20}$$

Multiplying every term in (20) by r and q' and dividing by q^t furthermore, substituting $C^{(1)}h(T) = C_1^{(1)}$ and $V_T = V_0(v_0 + v_1 T)$

$$t_0 = \frac{V_T + D_0 d_2 \frac{q^T}{q^t} - C_1^{(1)}}{V_0 v_1}. \tag{21}$$

To decide whether the t_0 value to be obtained from (21) is maximum or minimum, the second derivative has to be formed and its value at $t = t_0$ to be determined:

$$\frac{\partial^2 K^{(1)}}{\partial t^2} \equiv \frac{1}{r}\left\{-C\frac{q^t(q-1)^2}{q^T-1} - V_0\left[\left(v_0+\frac{v_1}{q-1}\right)\left(-\frac{q^t(q-1)}{q^T}\right) - v_1\frac{Tq^t(q-1)^2 - tq^t(q-1)^2 - q^t(q-1) - q^t(q-1)}{q^T(q-1)}\right]\right\}$$

$$\left[\frac{\partial^2 K^{(1)}}{\partial t^2}\frac{rq^T}{q^t(q-1)}\right]_{t_0} = -\frac{V_0 v_1}{q-1} - \frac{D_0 d_2 q^T}{q^{t_0}} < 0, \tag{22}$$

$$t_0 = \frac{V_T + D_0 d_2 \frac{q^T}{q^{t_0}} - C^{(1)}}{V_0 v_1}.$$

The second derivative is zero at $t = t_0$, so its extreme value is maximum. Substituting real numbers in (21), then, if $D_0 d_2$ is not high enough, t_0 may be negative.

Since near the optimum, the function (19) hardly varies, in the following, the time of failure may be put either to $t = 0$ or to $t = T$.

For a failure at time $t = 0$, (19) becomes:

$$K(k,r) = C + V_0\left[\frac{1}{h(T)}\left(v_0+\frac{v_1}{q-1}\right) - \frac{v_1 T}{q^T(q-1)}\right] + \frac{1}{r}\left\{C - V_0\left[\frac{1}{h(T)}\left(v_0+\frac{v_1}{q-1}\right) - \frac{v_1 T}{q^T(q-1)}\right] + D_0\right\}. \tag{23}$$

Deriving (23) with respect to k, at time $t = 0$:

$$\frac{\partial K}{\partial k} \equiv \frac{\partial C}{\partial(\ln k)}\frac{1}{k} + \frac{1}{h(T)}\left[\frac{\partial B_0}{\partial(\ln k)}\frac{1}{k} - \frac{\partial F_0}{\partial(\ln k)}\frac{1}{K} - \frac{\partial E_0}{\partial\left(\frac{1}{nk}\right)}\frac{1}{(\ln k)^2}\frac{1}{k}\right]$$
$$\frac{1}{r}\left\{\frac{\partial C}{\partial(\ln k)}\frac{1}{k} - \frac{1}{h(T)}\left[\frac{\partial B_0}{\partial(\ln k)}\frac{1}{k} + \frac{\partial F_0}{\partial(\ln k)}\frac{1}{k} - \frac{\partial E_0}{\partial\left(\frac{1}{\ln k}\right)}\frac{1}{(\ln k)^2}\frac{1}{k}\right]\right\} = 0. \tag{24}$$

Solving *Eq.* (24) for k:

$$k = \exp\sqrt{\frac{\frac{\partial E_0}{\partial\left(\frac{1}{\ln k}\right)}}{h(T)\frac{\partial C}{\partial(\ln k)}\frac{r+1}{r-1} + \frac{\partial B_0}{\partial(\ln k)} + \frac{\partial F_0}{\partial(\ln k)}}}. \tag{25}$$

Eq. (25) will be simpler if (3), (6), (10) and (13) hold, and $r = 100$, then:

$$k = \exp\sqrt{\frac{E_0 e_1}{h(T)C_0 c_2 + B_0 b_1 + F_0 f_2}}. \tag{26}$$

From (26) it appears that the k value is independent of the risk r of failure. (26) is a minimum, since, in general:

$$\frac{\partial^2 K}{\partial k^2} = \frac{1}{k^2}\left\{\left[\frac{\partial^2 C_0^{(1)}}{\partial(\ln k)^2} - \frac{\partial C}{\partial(\ln k)}\right]\left(1+\frac{1}{r}\right) + \frac{1}{h(T)}\left[\frac{\partial^2 B_0}{\partial(\ln k)^2} - \frac{\partial B_0}{\partial(\ln k)} + \frac{\partial^2 F_0}{\partial(\ln k)^2} - \frac{\partial F_0}{\partial(\ln k)}\right]\left(1-\frac{1}{r}\right) + \frac{1}{h(T)}\left[\frac{\partial^2 E}{\partial\left(\frac{1}{(\ln k)^2}\right)}\frac{1}{(\ln k)^2} + \frac{\partial E}{\partial\left(\frac{1}{\ln k}\right)}\frac{2}{(\ln k)}\right]\left(1-\frac{1}{r}\right)\frac{1}{(\ln k)^2}\right\}. \tag{27}$$

If (3), (6), (10) and (13) are true, then

$$\left[\frac{\partial^2 K}{\partial k^2}\right]_{\mathrm{k\ opt}} = \exp\left[-2\sqrt{\frac{E_0 e_1}{h(T)C_0 c_2 + B_0 b_1 + F_0 f_2}}\right]\left\{-C_0 c_2 \frac{2}{r} + \frac{2}{h(T)}\left(1-\frac{1}{r}\right)(h(T)C_0 c_2 + B_0 b_1 + F_0 f_2)\sqrt{\frac{h(T)C_0 c_2 + B_0 b_1 + F_0 f_2}{E_0 e_1}}\right\} > 0, \tag{28}$$

since every letter in (28) is positive. The first factor is an exponential term, hence always positive, also the second factor is essentially positive, although its first term is negative, but $r = 100$, so its value is nearly zero. If (3), (6), (10) and (13) are wrong, then further examinations are needed in the given case to determine the extreme values.

For an accidental failure at time $t = T$, the cost function becomes:

$$K(k,r) = C + [F(T) + B(T) + E(T)]\frac{1}{h(T)} + \frac{1}{r}(C + D_T), \tag{29}$$

which, derived with respect to k:

$$\frac{\partial K}{\partial k} \equiv \frac{\partial C}{\partial(\ln k)}\frac{1}{k} + \frac{1}{h(T)}\left[\frac{\partial B(T)}{\partial(\ln k)}\frac{1}{k} + \frac{\partial F(T)}{\partial(\ln k)}\frac{1}{k} - \frac{\partial E(T)}{\partial\left(\frac{1}{\ln k}\right)}\frac{1}{(\ln k)^2}\frac{1}{k}\right] = 0. \tag{30}$$

Solving (30) for k:

$$k = \exp \sqrt{\frac{\frac{\partial E(T)}{\partial\left(\frac{1}{\ln k}\right)}}{h(T)\frac{\partial C}{(\ln k)} + \frac{\partial B(T)}{\partial(\ln k)} + \frac{\partial F(T)}{\partial(\ln k)}}}. \tag{31}$$

(31) is about the same as (25), so, if (3), (6), (10) and (13) are true, then at time $t = T$, k is expressed by (26). Of course, E_0, B_0 and F_0 have to be replaced by $E(T)$, $B(T)$ and $F(T)$, respectively. Thus, the formula for k is independent of the time of failure, partial derivatives being equal:

$$\frac{\partial E(T)}{\partial\left(\frac{1}{\ln k}\right)} = \frac{\partial E_0}{\partial\left(\frac{1}{\ln k}\right)}, \frac{\partial B(T)}{\partial(\ln k)} = \frac{\partial B_0}{\partial(\ln k)} \text{ and } \frac{\partial F(T)}{\partial(\ln k)} = \frac{\partial F_0}{\partial(\ln k)}.$$

Deriving (23) with respect to r, at time $t = 0$:

$$\frac{\partial K}{\partial r} \equiv \frac{\partial C}{\partial(\ln r)}\frac{1}{r} - \frac{1}{h(T)}\frac{\partial F_0}{\partial\left(\frac{1}{\ln r}\right)}\frac{1}{(\ln r)^2}\frac{1}{r} - \frac{1}{r^2}\times$$
$$\left[C_0^{(1)} - (F_0 + B_0 + E_0)\frac{1}{h(T)} + D_0\right] + \frac{1}{r}\left\{\frac{\partial C}{\partial(\ln r)}\frac{1}{r} - \right.$$
$$\left.\left[\frac{\partial D}{\partial\left(\frac{1}{\ln r}\right)} - \frac{\partial F_0}{\partial\left(\frac{1}{\ln r}\right)}\right]\frac{1}{(\ln r)^2}\frac{1}{r}\right\} = 0. \tag{32}$$

Solving (32) for r:

$$r = \frac{C_0 - \frac{1}{h(T)}(F_0 + B_0 + E_0) + D - \frac{\partial C^{(1)}}{\partial(\ln r)} + \frac{\partial D}{\partial\left(\frac{1}{\ln r}\right)}\frac{1}{(\ln r)^2}}{\frac{\partial C_0^{(1)}}{\partial(\ln r)} - \frac{1}{h(T)}\frac{\partial F_0}{\partial\left(\frac{1}{\ln r}\right)}\frac{1}{(\ln r)^2}} - \frac{\frac{1}{h(T)}\frac{\partial F_0}{\partial\left(\frac{1}{\ln r}\right)}\frac{1}{(\ln r)^2}}{\frac{\partial C_0^{(1)}}{\partial(\ln r)} - \frac{1}{h(T)}\frac{\partial F_0}{\partial\left(\frac{1}{\ln r}\right)}\frac{1}{(\ln r)^2}}. \tag{33}$$

If (3), (6), (10) and (13) are true, then

$$r = \frac{C_0^{(1)}(1 - c_1) + D_0\left[1 + \frac{d_1}{(\ln r)^2}\right] - \left[F_0 + B_0 + D_0 + \frac{F_0 f_1}{(\ln r)^2}\right]\frac{1}{h(T)}}{C_0^{(1)}c_1 - \frac{1}{h(T)}\frac{F_0 f_1}{(\ln r)^2}}. \tag{34}$$

(34) is a minimum, namely:

$$\left[\frac{\partial^2 K}{\partial r^2}\right]_{r_{\rm opt}} = \frac{1}{r^3_{\rm opt}}\left[\left(C_0^{(1)}c_1(r_{\rm opt}-1)+\frac{q^T-1}{q^T(q-1)}\frac{F_0 f_1}{(\ln r)^3}\times\right.\right.$$
$$\left.(r_{\rm opt}-1)(\ln r_{\rm opt}+2)+\frac{D_0 d_1}{(\ln r_{\rm opt})^3}(\ln r_{\rm opt}+2)\right]>0. \qquad (35)$$

All terms in (35) are positive, thus, also the second derivative is so. If (3), (6), (10), (13) and (16) are wrong, then demonstrating the second derivative to be positive needs special examinations.

If the time of accidental failure $t=T$ is the end of the planned service life, then the cost function is (29), deriving with respect to r:

$$\frac{\partial K}{\partial r}\equiv\frac{\partial C}{\partial(\ln r)}\frac{1}{r}-\frac{1}{h(T)}\frac{\partial F(T)}{\partial\left(\frac{1}{\ln r}\right)}\frac{1}{(\ln r)^2}\frac{1}{r}-\frac{1}{r^2}(C+D_T)+$$
$$\frac{1}{r}\left[\frac{\partial C}{\partial(\ln r)}\frac{1}{r}-\frac{\partial D_T}{\partial\left(\frac{1}{\ln r}\right)}\frac{1}{(\ln r)^2}\frac{1}{r}\right]=0. \qquad (36)$$

Solving (36) for r:

$$r=\frac{C+D_T-\frac{\partial C}{\partial(\ln r)}+\frac{\partial D_T}{\partial(\frac{1}{\ln r})}\frac{1}{(\ln r)^2}}{\frac{\partial C^{(1)}}{\partial(\ln r)}-\frac{1}{h(T)}\frac{\partial F_T}{\partial(\frac{1}{\ln r})}\frac{1}{(\ln r)^2}}. \qquad (37)$$

If (3), (6), (10), (13) and (16) are true, then:

$$r=\frac{C_0(1-c_1)+D_T\left(1+\frac{d_1}{(\ln r)^2}\right)}{C_0 c_1-\frac{1}{h(T)}\frac{F_0 f_1}{(\ln r)^2}}. \qquad (38)$$

(38) is a minimum, since the second derivative is positive, just as in (35).

Certainly, $r_{\rm opt}$ according to (34) is less – whatever slightly – than that in (38), but in the following, $r_{\rm opt}$ at $(t=T)$, end of the service life, has to be reckoned with.

The sum of magnitudes in numerators of (34) and (38) other than damage D, will be omitted, just as the second term to be deduced from the denominator, so:

$$r\sim\frac{1}{c_1}\left(\frac{D_T}{C_0}+1\right). \qquad (39)$$

According to (39), reciprocal value of the risk taken in a dynamic analysis is $\frac{1}{c_1=50}$ to 80 times the share of damage. Numerically: for $\left[\frac{D_r}{C_0^{(1)}}\right] \sim 200$, $r = 10000 \div 16000$, that is to say, the optimum risk taken ranges from 10^{-4}–$6 \cdot 10^{-5}$. In hydraulic computations, for $1/c_1 \sim 6 - 10$ as loss percentage, for $\frac{D_0}{C_0} \sim 20$, $r \sim 120 \div 200$ that is, the risk taken ranges from $8 \cdot 10^{-3}$ to $5 \cdot 10^{-3}$.

7. Dimensioning of Multifunctional Establishments

Establishments for several purposes, (e.g. barrage) or those with a single technological function but comprising several subsystems, are called establishment systems. For instance, a hotel consists of a technological, a structural, a water supply, a canalization, a heating, a gas supply, etc. subsystems. An establishment generally consists of m subsystems.

Investments costs of subsystems i:

$$C_i = C_i(\ln k_1, \ln k_2 \dots \ln k_m, \ln r_1, \ln r_2 \dots \ln r_m).$$

Costs of upkeep:

$$F_i = F_i\left(\ln k_1, \ln k_2 \dots \ln k_m, \frac{1}{\ln r_1}, \frac{1}{\ln r_2} \dots \frac{1}{\ln r_m}, t\right),$$

operation costs:

$$B_i = B_i(\ln k_1, \ln k_2 \dots \ln k_n, t), \tag{40}$$

losses due to missing demands:

$$E_{ij} = E_{ij}\left(\frac{1}{\ln k_1}, \frac{1}{\ln k_2}, \dots, \frac{1}{\ln k_n}, t\right),$$

losses due to accidental breakdown:

$$D_{ij} = D_{ij}\left(\frac{1}{\ln r_1}, \frac{1}{\ln r_2}, \dots, \frac{1}{\ln r_m}, t\right).$$

E_{ij} is damage in the establishment part (subsystem) i due to the missing of technical demand in establishment part j that may occur year-wise. D_{ij} is damage in part establishment (subsystem) i at the failure or breakdown of part establishment j.

Damages E_{ij} or D_{ij} may be assumed to be independent of each other and of damages due to accidental breakdown or to missing of annual cyclic technical demands.

Overall costs of the system of establishments:

$$K(t,k_1,k_2,\ldots,k_m,r_1,r_2,\ldots,r_m) = \sum_{i=1}^{m} C_i + \sum_{i=1}^{m} \frac{1}{h(T_i)}(v_{0i} + \frac{v_{1i}}{q-1} - \frac{v_{1i}T_i}{q^{T_i}(q-1)} + \sum_{i=1}^{m} \frac{1}{r_i}\left[C_i \frac{q^{T_i}-q^t}{q^{T_i}-1} - \frac{q^{T_i}-q^t}{q^{T_i}(q-1)}\left(v_{0i} + \frac{v_{1i}}{q-1}\right) - v_{1i}\frac{(T_i-t)q^t}{q^{T_i}(q-1)} + D_i\right]. \quad (41)$$

In (41), yearly costs are linear in time.

$$V_i = V_{0i}(v_{0i} + v_{1i}t) = F_{0i} + B_{0i} + E_{0i}^{*} + (f_{3i} + b_{2i} + e_{2i})t$$

as stated above. C_i is the cost of remaking the establishment part (subsystem) i, to be written as follows, by analogy to (3):

$$C_i = C_{0i}(c_{i0} + c_{i1}^{(k)} \ln k_1 + \cdots + c_{im}^{(k)} \ln k_m + c_{i1}^{(r)} \ln r_1 + \cdots + c_{im}^{(r)} \ln r_m). \quad (42)$$

(42) has been proven for a single independent establishment part. Obviously, if the subsystem is not independent of the other subsystems, then its relationship is *Eq.* (42). Magnitudes E_{0i}^{*} and D_{0i}^{*}

$$E_{0i}^{*} = E_{0ij} + \frac{E_{0ij+1}}{\ln k_{j+1}} + \frac{E_{0ij+2}}{\ln k_{j+1} \ln k_{j+2}} + \ldots \frac{E_{0im}}{\ln k_1 \ldots \ln k_{j-1} \ln k_{j+1} \ldots \ln k_m}$$

and

$$D_i^{*} = D_{ij} + \frac{D_{ij+1}}{\ln r_{j+1}} + \frac{D_{ij+2}}{\ln r_{j+1} \ln r_{j+2}} + \ldots \frac{D_{im}}{\ln r_1 \ldots \ln r_{j-1} \ln r_{j+1} \ldots \ln r_m}. \quad (43)$$

F_{0ij} and D_{ij} mean maximum damage in part establishment (subsystem) i $(j = 1, 2, \ldots, m)$.

$\frac{E_{0ij+1}}{\ln k_{j+1}}$ and $\frac{D}{}$ are maxima among terms $\frac{E_{0ij+2}}{\ln k_{j+1} \ln k_{j+2}}$ and $\frac{D_{ij}}{}$ while $\frac{E}{\ln k \ln k'}$ and $\frac{D}{}$ are maxima among terms $E \ldots$ and D, etc. Values of k and of r varying in the range of 10 to 10^4, it is meaningless to calculate with other than the first two terms in (43). $\ln k$ and $\ln r$ in (43) are overestimations of damages on the safety side.

Determination of extreme values of (43) may be done by means of (20) and (21)

$$\frac{\partial K}{\partial t} \equiv \sum_{i=1}^{m} \frac{1}{r_i} \left\{ -C_i \frac{q^t(q-1)}{q^{T_i}-1} - V_{0i} \left[\frac{-q^t}{q^{T_i}} \left(v_{0i} + \frac{v_{1j}}{q-1} \right) - v_{1i} \frac{q^1}{q^{T_i}} \left(T_i - t - \frac{1}{q-1} \right) \right] + D_{0i}^* d_{2i}^* = 0. \quad (44)$$

From (44):

$$t_0 = \frac{\sum\limits_{i=1}^{m} \frac{1}{r_i q^{T_i}} \left[V_{0i} \left(v_{0i} + v_{1i} T_i \right) - C_{1i} + D_{0i}^* d_{2i}^* \right]}{\sum\limits_{i=1}^{m} \frac{V_{0i} v_{1i}}{r_i q^{T_i}}}. \quad (45)$$

To determine the optimum time of failure, in (45) the risks taken $1/r_i$ $(i = 1, 2, \ldots, m)$ have to be preassumed. t_0 may have a negative value since in the term in square brackets the yearly write-off share $C_{\ln i}$ may exceed the sum of yearly costs by the end of the planned service life, and the sum of time-dependent damages. Whether the optimum is a maximum or a minimum may be answered by the second derivative. Differentiating (44) and substituting $t = t_0$:

$$\frac{d^2 K}{dt^2} = q^t (q-1)^2 \sum_{i=1}^{m} \frac{1}{r_i q^{T_i}} \left\{ V_{0i} [v_{0i} + v_{1i}(T_i - t)] - C_{1i} - \frac{v_{1i}}{q-1} \right\} < 0. \quad (46)$$

Since near the optimum, the function in (41) but slightly varies, subsequently, the time of failure will be assumed at $t = T$. (See *Eqs.* (34) and

(38)). Function (41) for $t = T$ becomes:

$$K(k_1, k_2 \dots k_m, r_1 \dots r_m) =$$
$$\sum_{i=1}^{m} \left\{ C_i + \frac{1}{h(T_i)}(F_{Ti} + B_{Ti} + E^*_{Ti}) + \frac{1}{r_i} \left[C_i - \frac{1}{h(T_i)}(F_{Ti} + B_{Ti} + E^*_{Ti}) + D^*_{Ti} \right] \right\}. \tag{47}$$

(31) will be replaced by:

$$k_j = \exp \sqrt{\sum_{i=1}^{m} \frac{\frac{\partial E^*_{0i}}{\partial(\ln kj)}}{h(T_i)\frac{\partial C_i}{\partial(\ln k_j)}\frac{r_{i+1}}{r_{i-1}} + \frac{\partial B_{0i}}{\partial(\ln k_j)} + \frac{\partial F_{0i}}{\partial(\ln k_j)}}}$$
$$(j = 1, 2, \dots, m). \tag{48}$$

If also the right-hand side of the relationship comprises terms k then the equation system of m equations may only be solved by gradual approximation.

(33) is replaced by:

$$r_j = \frac{C_j - \frac{1}{h(T_j)}(F_{0j} + B_{0j} + E^*_{0j}) + D^*_j + \left[\frac{\partial D^*_j}{\partial\left(\frac{1}{\ln r_j}\right)} - \right.}{\sum_{i=1}^{m} \left[\frac{\partial C_i}{\partial(\ln r_j)} - \frac{1}{h(T_i)} \frac{\partial F_{0i}}{\partial\left(\frac{1}{\ln r_j}\right)} \right] + \sum_{\substack{i=1 \\ j+1}}^{\substack{j-1 \\ m}} \frac{1}{r_i} \left\{ \frac{\partial C_i}{\partial(\ln r_j)} - \right.}$$
$$\frac{\left. - \frac{1}{h(T_j)} \frac{\partial F_{0i}}{\partial(\ln r_j)} \left(\frac{1}{\ln r_j} \right) \right]^2 - \frac{\partial C_i}{\partial(\ln r_j)}}{\left. - \left[\frac{1}{h(T_i)} \frac{\partial F_{0i}}{\partial\left(\frac{1}{\ln r_j}\right)} + \frac{\partial D^*_i}{\partial\left(\frac{1}{\ln r_j}\right)} \right] \left(\frac{1}{\ln r_j} \right)^2 \right.}. \tag{49}$$

$$(j = 1, 2, \dots, m).$$

Values of r_j in (49) can only be determined by gradual approximation. Values of $1/r_i$ in the denominator have to be preassessed estimated. In addition also the D^*_i values have to be preassessed. For a failure at time $t = T$, then of course, k_j values equal those in (48), r_j values being, rather than those in (49).

$$r_j = \frac{C_j - \frac{\partial C_j}{\partial(\ln r_j)} + D_j^* + \frac{D_j^*}{\partial\left(\frac{1}{\ln r_j}\right)} \frac{1}{(\ln r_j)^2}}{\sum_{i=1}^{m} \left[\frac{\partial C_j}{\partial(\ln r_j)} - \frac{1}{h(T_i)} \frac{F_{0i}}{\partial\left(\frac{1}{\ln r_j}\right)} \right] + \sum_{\substack{i=1\\ j+1}}^{\substack{m\\ j-1}} \frac{1}{r_i} \left\{ \frac{\partial C_i}{\partial(\ln r_j)} - \left[\frac{1}{h(T_i)} \frac{\partial F_{0i}}{\left(\frac{1}{\ln r_j}\right)} + \frac{\partial D_i^*}{\partial\left(\frac{1}{\ln r_j}\right)} \right] \frac{1}{(\ln r_j)^2} \right.},$$

$$(j = 1, 2, \ldots, m). \quad (50)$$

r_j from (50) exceeds that from (49), so r_j has to be determined according to (50).

If terms F_i, B_i and E_i^* may be assumed to be similar to those in (6), (10), (13) and (16), then

$$\left.\begin{aligned}
F_i &= F_{0i} \left(f_{i0} + \frac{f_{i0}^{(r)}}{\ln r_1} + \cdots + \frac{f_{im}^{(r)}}{\ln r_m} + f_{i2}^{(k)} \ln k_1 + \cdots + f_{im}^{(k)} \ln k_m + F_i t \right) \\
B_i &= B_{0i} \left(b_{i0} + b_{i1}^{(k)} \ln k_1 + \cdots + b_{im}^{(k)} \ln k_m + b_i t \right) \\
E_i^* &= E_{0i}^* \left(e_{i0}^* + \frac{e_{i1}^*}{\ln k_1} + \cdots + \frac{e_{i1n}^*}{\ln k_m} + e_i t \right) \\
D_i^* &= D_{0i}^* \left(d_{i0}^* + \frac{d_{i1}^*}{\ln r_1} + \cdots + \frac{d_{im}^*}{\ln r_m} + d_i t \right) \quad (i = 1, 2, \ldots, m).
\end{aligned}\right\} . \quad (51)$$

Terms marked in (51) are:

$$E_{0i}^* = E_{0ij} + \frac{E_{0ij+1}}{\ln k_{j+1}} + \cdots + \frac{E_{0im} \ln k_j}{\prod_{j=1}^{m} \ln k_j},$$

$$e_{i1}^* = e_{ijl} + \frac{e_{ij+1k}}{\ln k_{j+1}} + \cdots + \frac{e_{im} \ln k_j}{\prod_{j=1}^{m} \ln k_j},$$

$$D_{0i}^* = D_{0ij} + \frac{D_{0ij+1}}{\ln r_{j+1}} + \cdots + \frac{D_{0im} \ln r_j}{\prod_{j=1}^{m} \ln r_j},$$

$$d_i^* = d_{ij} + \frac{d_{ij+1}}{\ln r_{j+1}} + \cdots + \frac{d_{im} \ln r_j}{\prod_{j=1}^{m} \ln r_j},$$

$$(i = j = 1, 2, \ldots, m, \qquad = 0, 1, 2, \ldots, m). \tag{52}$$

To determine terms marked *, the k and r values have to be preassumed. If there are significant differences compared to k and r values in (52), then the computation has to be repeated. If (51) and (52) are true, and $r_i = 100$, then (48) and (49)

$$k_j = \exp \sqrt{\sum_{i=1}^{m} \frac{E_{0i}^* e_{ij}^*}{\frac{1}{h(T_i)} C_{0i} c_{ij}^{(k)} + B_{0i} b_{ij}^{(k)} + F_{0i} f_{ij}^{(k)}}}$$

$$(j = 1, 2, \ldots, m) \tag{53}$$

$$r_j = \frac{C_j - \frac{1}{h(T_j)}(F_{0j} + B_{0j} + E_{0j}^*) + D_{0j}^* - C_{0j} c_{ij}^{(r)} +}{\sum_{i=1}^{m} C_{0i} c_{ij}^{(r)} - \frac{1}{h(T_i)} F_{0i} f_{ij}^{(r)} +} \qquad \frac{\frac{D_{0j}^* d_{jj}^* - \frac{1}{h(T_j)} F_{0j} f_{jj}^{(r)}}{(\ln r_j)^2}}{+ \sum_{\substack{i=1 \\ j=1}}^{j-1} \frac{1}{r_i} \left[C_{0i} c_{ij}^{(r)} - \frac{D_{0i}^* d_{ij}^* - \frac{1}{h(T_i)} F_{0i} f_{ij}^{(r)}}{(\ln r_i)^2} \right]} \tag{54}$$

$$(j = 1, 2, \ldots, m).$$

If the time of failure $t = T$, then the k values are those in

$$r_j = \frac{C_{0j}(1 - c_{jj}^{(r)}) + D_{rj}^* \left[1 + \frac{d_{jj}^*}{(\ln r_j)^2} \right]}{\sum_{i=1}^{m} \left[C_{0j} c_{ij}^{(r)} - \frac{1}{h(T_i)} \frac{F_{0i} f_{ij}^{(r)}}{(\ln r_j)^2} \right] + \sum_{\substack{i=1 \\ j=1}}^{m} \frac{1}{r_i} \left[C_{0i} c_{ij}^{(r)} - \frac{1}{h(T_i)} \frac{F_{0i} f_{ij}^{(r)}}{(\ln r_j)^2} + \frac{D_{0i}^* d_{ij}^*}{(\ln r_j)^2} \right]} \tag{55}$$

$$(j = 1, 2, \ldots, m).$$

r_j from (55) exceeds that from (54), hence the higher one, obtained from (55), is valid.

Forming second derivatives of (47) and substituting (53) and (54) in them, all of them will be positive: so (53) and (54) yield minima.

It appears from (48) and (49) that the k_j and r_j values are essentially mutually independent. Practical course of dimensioning an entire establishment:

a) First step: determination of random characteristics of technical demand and capacity $[m_{\mathrm{si}}(t), s_{\mathrm{si}}(t), \ldots]$.
b) Second step: determination of investment costs (c_i) of establishment parts (subsystems), taking various technology parameters and risks into consideration. Setting up of (42).
c) Third step consists in determining costs of upkeep F_i and of operation B_i, either as functions of simple proportionality according to (51), or rational integer or fractional functions comprising also higher-order terms.
d) As the fourth step, damage functions (E_i^* and D_i^*) have to be determined. These functions are either inverted proportions according to (51), or rational fractional functions comprising ever higher-order terms.
e) The fifth step will be to determine cost function K according to (47).
f) The sixth step will be to determine optimum geometrical dimensions, and the risks optimally taken (k_j, r_j), either according to (31) and (33), or to (48) and (50).
g) The seventh step will be to compute capacity of part establishment or subsystem $j(R_j(t))$ according to the risk optimally taken, from (1).
h) As the eighth step, in knowledge of capacity ($R_j(t)$) geometrical dimensions and technology parameters have to be determined to be optima.

The outlined procedure is justified only in the design of costly, grandious (monumental) establishment systems, where this lengthy procedure may result in important savings (> 3%). By the way, the subsystems may be assumed to be independent of each other, much simplifying their dimensioning.