PERIODICA POLYTECHNICA SER. CIVIL. ENG. VOL. 38, NO. 2, PP. 173–185 (1994)

DETERMINATION OF FRACTAL DISPERSION FRONT WITHIN COMPUTER AND EXPERIMENTAL ENVIRONMENT

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Received: June 30, 1994

Abstract

The displacement of a high-viscosity fluid by a low-viscosity fluid leads to fractal occurrence in a homogeneous porous media. In this paper the possibilities of using a GIS (Geographical Information Software) software in determination of fractal characteristics of immiscible fluids in subsurface environment are presented. In porous media the problem of giving precise answer for transport processes requires new methods in possession of powerful computers. One of the new methods in technical hydraulics is the fractal growth phenomena (VICSEK, 1989).

In this paper we report our first experimental results in the field of analyzing unstable pictures, applying Hele-Shaw cell (HELE-SHAW, 1898) as a laboratory tool and a GIS software to follow the displacement process in the porous media in order to reach a more appropriate hydrodynamical dispersion coefficient.

Keywords: ground-water hydraulics, GIS, fractal theory, viscous fingering, percolation theory, Hele-Shaw cell.

1. Introduction

The geometry of natural objects ranging in size from the atomic scale to the size of universe is central to the models we construct in order to 'understand' nature. The geometry of particle theorem, of hydrodynamic flow lines, of landscapes, in short, the geometry of nature is so central in various fields of natural science that we tend to take the geometrical aspects for granted. Mathematicians have developed a concept that transcends traditional geometry. These complex shapes are called fractals and can be characterized by a noninteger dimensionality. An important field where fractals are observed is that of far-from-equilibrium growth phenomena (VICSEK, 1989).

It is often read in various GIS books that the GIS is such a tool which provides representations of the spatial geographic feature of the world, and these features can be described by some forms of maps. In our experiments we have regarded for the GIS as a picture analysing tool, without any geographical connections. This may be a contradiction, but the GIS is not only just an algorithm package, and the GIS represented services we can easily simulate with writing numerous programmes.

In this paper we deal with the fractal subsurface hydraulics with the help of GIS. After a brief introduction the first chapter, in the second Chapter we show the connection between the GIS and the fractal phenomena. The third chapter deals with theoretical basis of viscous fingering in porous media, then in the fourth the connections between the DLA algorithm and the viscous fingering are presented.

The fifth chapter shows the basis of percolation theory with that we are able to simulate the dispersion easier within a GIS.

The sixth and seventh chapters show our experimental results.

2. GIS and Fractal Phenomena

The word fractal implies properties as in fraction or fragmented: in essence fractal geometry has ideas of fragmentation and self-similarity. Self-similarity is symmetry across different scales: there are patterns within patterns (LAURINI – THOMPSON, 1992).

One of the basic algorithms of generating fractal objects is the Cantor dust (*Fig. 1*). Such a process of pattern formation has two components:

1. initiator (0 level in *Fig. 1*)
2. repetitor (often recursive process is used) (MANDELBROT, 1982).

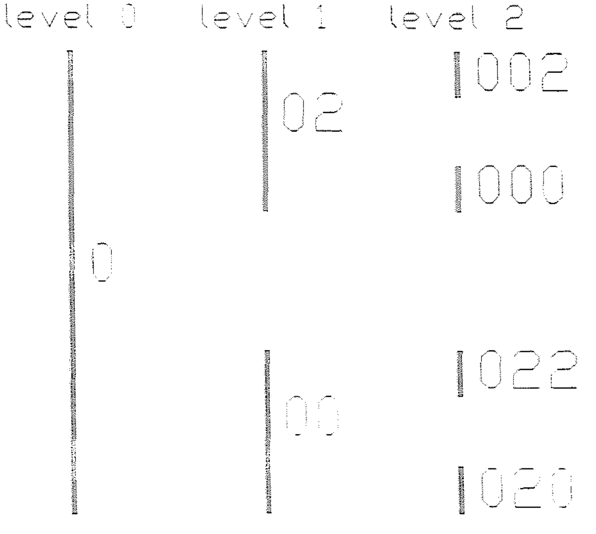


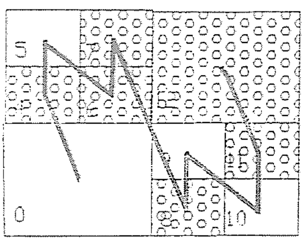
Fig. 1. Cantor dust

An encoding method will serve to distinguish the different points. Identifying the initiator as 0, at the first subdivision we have pieces 0.0 and 0.2. At the second step there are four segments, encoded as shown; at the next step there are eight, and so on. At step N the set consists of 2^N segments, and each has a numerical code with N ternary digits after the 'decimal points'. The segment length is $l \cdot 3^{-N}$, where l is the unit length. For the repetitor, its self-similarity ratio is 1/3.

Data processing and storage may be more economical if less information can be used to meet the same requirements. Thus, area units may be represented by a zero-dimensional object. A data reduction can also occur if objects could be positioned in only one dimension rather than two or three.

If we define a point as a zero-dimensional object, theoretically speaking it is not possible in the spirit of Euclidean geometry to find a one-dimensional curve passing through the infinity of points. However, if we think of a point as a two-dimensional square the side of which tends toward zero, then, in the spirit of fractal geometry, it is possible to find a curve filling a two-dimensional space. So, now the curve is defined as a sort of ribbon, the width of which tends towards zero (MARK et al., 1986).

The original space filling curve was exhibited in 1890 by the Italian mathematician G. PEANO, and now it is known as the Peano ordering (*Fig. 2*).



OBJECTS	PEANO -KEY	KEY -LENGTH
X	4	1
X	6	1
X	7	1
X	8	1
X	11	1
X	12	1

Fig. 2. Using Peano ordering in computer

In practice, the encoding of space-filling curves uses one coordinate, called key by most practitioners, to stand for two or more coordinates (MEIXLER, 1983).

Generally, the ordered paths have similar shapes at different scale levels (self-similarity). In our case we have used the GIS as follows:

- analyzing fractal pictures;
- generating fictitious porous media (based on RSA algorithm) (HINRICHSEN et al., 1986);

- calculating percolating networks (using the GIS software PC based ARC/INFO NETWORK module) (BAKUCZ, 1993).

3. Viscous Fingering in Porous Media

The problem of viscous fingering in porous media is of central importance in determination of hydrodynamical dispersion. It has recently been shown that viscous fingering in porous media is fractal (CHEN – WILKINSON,

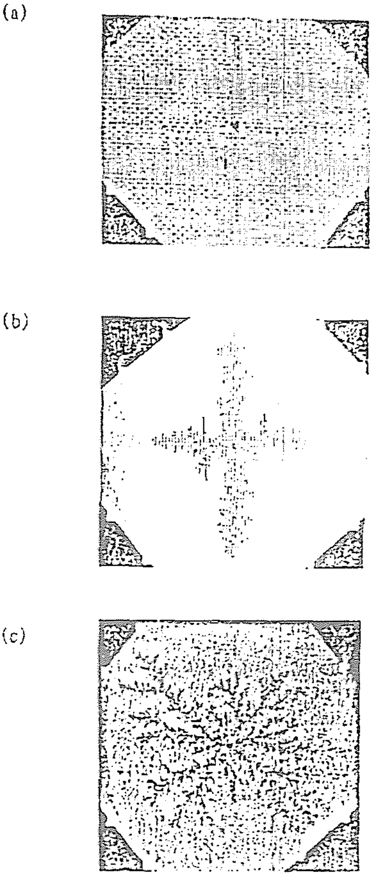


Fig. 3. Experimental results, 'randomness': $a < b < c$ (CHEN – WILKINSON, 1983)

1983). Viscous fingering in porous media is a phenomenon occurring when a less viscous fluid is displacing a more viscous one: a planar boundary between the fluids is unstable against small perturbations, and in the course of time the interface adopts a fingered configuration. Since the interfacial tension provides a stabilizing effect at short distances, the fingering of immiscible fluids in a porous medium is usually a macroscopic phenomenon in which the fingers are large compared to the pore scale (DeGregoria – Schwarz, 1987).

Viscous fingering in porous media is often compared to what occurs in a Hele-Shaw cell. In this cell the flow takes place between two parallel plates. Chen and Wilkinson (1983) demonstrated that when the channel randomness in an experimental Hele-Shaw cell is increasing the received picture shows fractal behaviour. Therefore, the viscous fingering in a porous media is mainly determined by pore geometry with stochastic nature. That is why we can apply the stochastic numerical algorithm to create a porous media, because the porous media is considered as a fractal (Feder et al., 1986; Feder, 1988).

In *Fig. 3* the experimental results can be seen, and in *Fig. 4* the result of numerical simulation are shown.

4. Viscous Fingering and DLA

The fractal structures of viscous fingering in porous media closely resemble those obtained from the diffusion-limited aggregation model of Witten and Sander (1983). Similar structures have also been obtained by fluid–fluid displacement in radial Hele-Shaw cells using non-Newtonian viscous fluids. The relationship between fluid–fluid displacement in porous media and DLA was first discussed by Paterson (1984).

The original lattice model of Witten and Sander for DLA was modified to represent the displacement of a viscous fluid by non-viscous one in a two-dimensional porous medium. In *Fig. 5* we represent an early stage in a small-scale simulation on a square lattice (Lenormand, 1985).

The sites occupied by the zero-viscosity fluid are shaded and the growth sites are represented by open squares. To simulate the viscous fingering process, one of the unoccupied surface sites is selected at random and the random walk is started from that site. After each random walker is launched from an unoccupied surface site the simulation time is incremented by $\frac{1}{N}$, where N is the total number of surface sites. The random walk trajectory is stopped and the site from which the random walk originated is filled if the random walker moves a distance greater than R from original seed. The circle radius represents the edge of the cell. The random walk trajectory t_1 in *Fig. 5* shows a random walk which results in growth. When the random walk trajectory reaches a second unoccupied

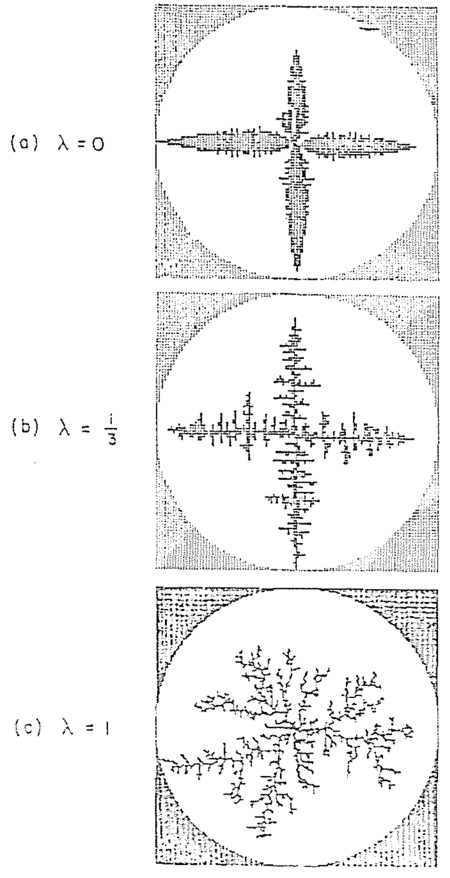


Fig. 4. Numerical results – random variable $a < b < c$ (CHEN – WILKINSON, 1983)

surface site, the random walk is stopped but growth does not occur: t_2 trajectory in *Fig. 5.*

The simulation is continued until the growth reaches the edge of the cell.

5. Percolation Theory and Viscous Fingering

BROADBENT and HAMMERSLEY (1957) discussed the general situation of a fluid spreading randomly through a medium, where the abstract terms fluid and medium could be interpreted according to the context. The randomness can be of two quite different types. In the familiar diffusion process the randomness is the random walks of the fluid particles. The other case

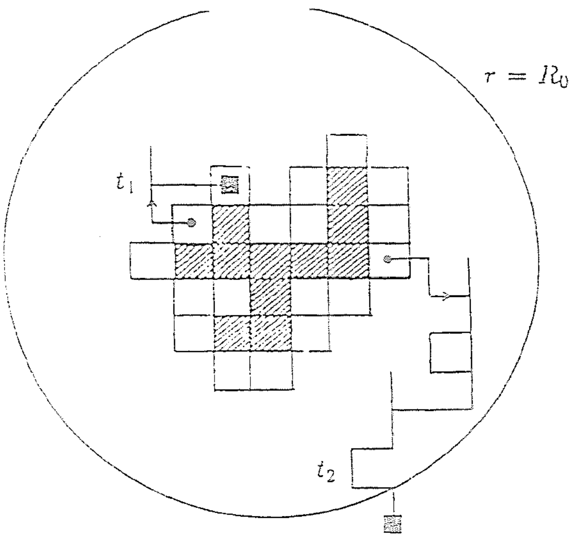


Fig. 5. Schematic representation of the DLA model

in which the randomness is frozen into the medium itself, was denominated a percolation process.

The most remarkable feature of percolation process is the existence of a percolation threshold, by which the spreading process is confined to a finite region.

There is no sharp distinction between percolation processes and diffusion in many applications (SAPOVAL et al., 1985).

In our case we discuss the percolation in terms of a fluid wetting pores when injected from a single site. This discussion presupposes the pores are empty so that a fluid may actually enter each pore.

Consider pores that form a connected network and are filled by an incompressible fluid (water). Our problem is to determine the hydrodynamic dispersion on the set of pores. Another fluid (water with potassium-permanganate) that is injected can only displace the water on the backbone of the percolation cluster (where one particle started from one side of the network just reaches the other side at the percolation threshold). The parts of the percolation cluster that are only connected to the backbone by a single site are called dangling ends. The driving fluid cannot enter the dangling ends since the trapped water has no escape rate.

OXAAL et al. have made a physical model (1987) of percolation cluster. The model was molded and has cylindrical pores. The pores are connected by channels. The model was filled with high-viscosity coloured glycerol. The results showed that the displacement process took place on the percolation cluster.

The viscous displacement process on the fractal percolation cluster can be modelled numerically by solving a flow equation (so-called Laplac-equation) with appropriate boundary conditions (SHERWOOD, 1987).

OXAAL et al. showed that the agreement between the experiment and the simulation was very good. In fact 70–80% of the sites invaded by air are common to the experiment and the simulation at any time in the invasion process.

This agreement shows that fluid displacement at the percolation threshold is almost entirely determined by geometrical effects since the numerical simulation does not take into account such factors as interfacial tension and wetting properties that are known to influence ordinary two phase flow in porous media.

We have carried out a numerical simulation to check the results of Oxaal by the help of GIS method. At this position the GIS PC ARC/INFO NETWORK module can be used for the determination of the channel flow velocity. The size of experimental channel network is 170×170. In our procedure we have not taken separately the numerical and the experimental process. It means that we created a GIS coverage from Oxaal's experimental result-picture and in this coverage the ARC/INFO calculated the flow velocities and the Laplace equation for every channel. The agreement supported Oxaal's results (BAKUCZ–L., 1993).

6. Fractal Structure of Hydrodynamic Dispersion

6.1 Hydrodynamic Dispersion

When a tracer is added to a fluid flowing in a porous medium, it disperses because of molecular diffusion and convection. The dispersion front of miscible displacement has a fractal structure as MÅLOY et al. (1988) wrote.

The macroscopic description of dispersion can be written (SAFFMAN, 1959):

$$\frac{\partial C}{\partial t} = \nabla(D \nabla C - UC)\,. \tag{1}$$

Here $C(r,t)$ is the tracer concentration as a function of position (r) and time (t). The dispersion tensor $D(U)$ depends in general, on the imposed hydrodynamic flow velocity (U). For homogeneous porous media this dispersion tensor is only characterized by two independent components: the longitudinal and the transverse dispersivities.

Dispersion of tracers in a stationary fluid ($U = 0$) is due to ordinary molecular diffusion. Måløy et al. showed a new feature when they analyzed the dispersion front itself: contours of constant tracer concentration are self-affine fractal curves confined by the width of the dispersion front.

6.2 Experimental Results

Here we describe the first results of experiments carried out in order to explore the dynamics of hydrodynamic dispersion in a two-dimensional porous medium. The results of these experiments will be compared with simulations carried out using a modified DLA model (FRETTE et al., 1990).

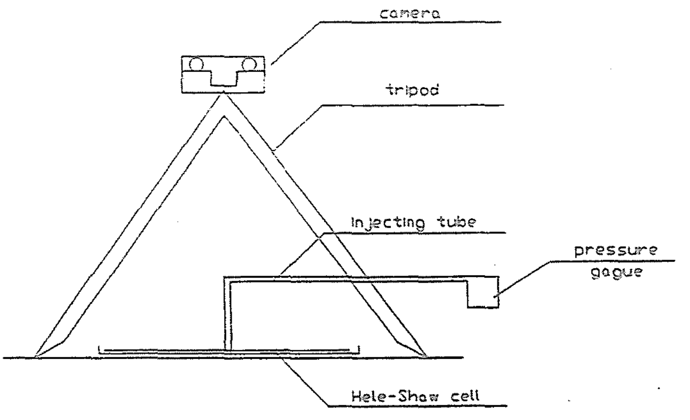


Fig. 6. Experimental setup sketch for central Hele-Shaw cell

We have used linear and central Hele-Shaw cells. The central Hele-Shaw cell setup is shown sketchily in *Fig. 6*, and the linear Hele-Shaw setup can be seen in *Fig. 7*. The porous model consists of the following sequence of disks : $d \leq 0.20$ mm, 0.20–0.30 mm, 0.30–0.50 mm, 0.50–1.00 mm and 1.50–2.00 mm.

The model was made by coating a plexiglass disk (5 mm thick and 40 cm in diameter) with a 0.1–(0.2) mm layer of transparent two phase epoxy and on it the previously enumerated fractions. After the epoxy layer had hardened, the excess of glass spheres was removed, leaving a monolayer. In our experiment we have injected firstly clear water at the center of the model, filling the pore space of the model. Then coloured water is displaced with a stable pressure using a pressure regulator. The resulting finger structure was photographed firstly with 37 mm camera and next with videocamera controlled by IBM PC AT. A typical time between each picture in the first case was 10 s, in the second case 0.2 s. In the analyzing

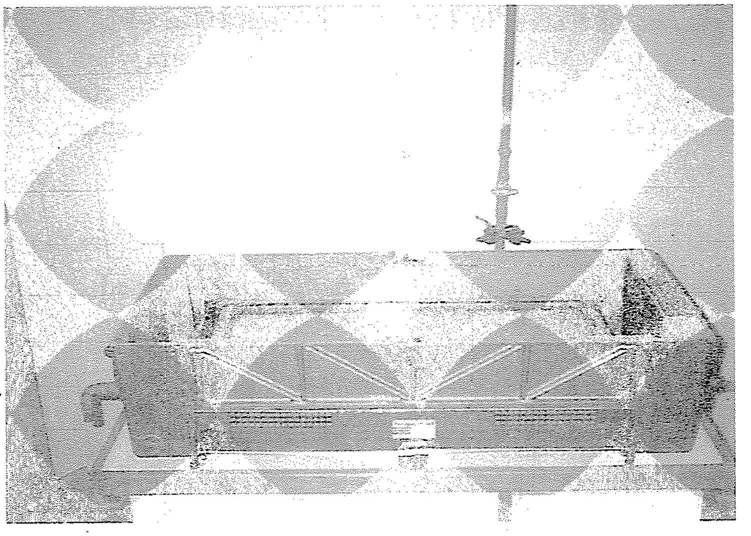


Fig. 7. Experimental setup for linear Hele-Shaw cell

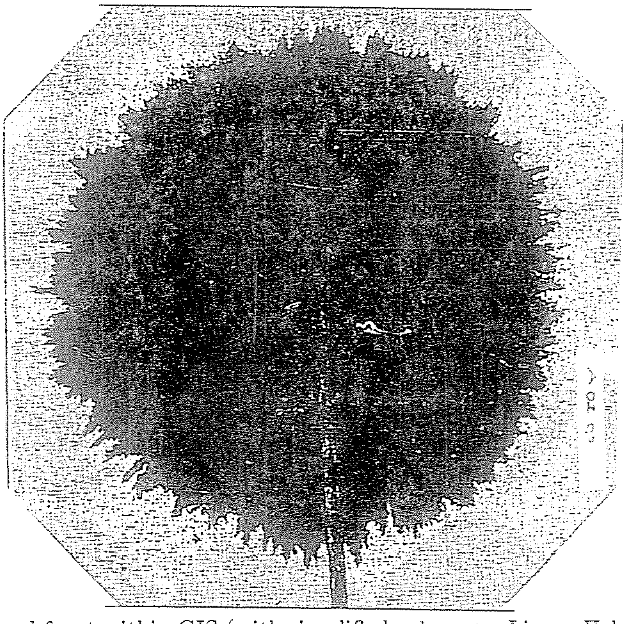


Fig. 8. Digitized front within GIS (with simplified colours). (Linear Hele-Shaw cell with 1.5–2.0 mm porous layer)

process we used ARC/INFO PC 3.4. D GIS software as a digitizing tool connection to the cameras. Within GIS a macro (AML) can be started which directs the process to determine the fractal dimension of the fractal

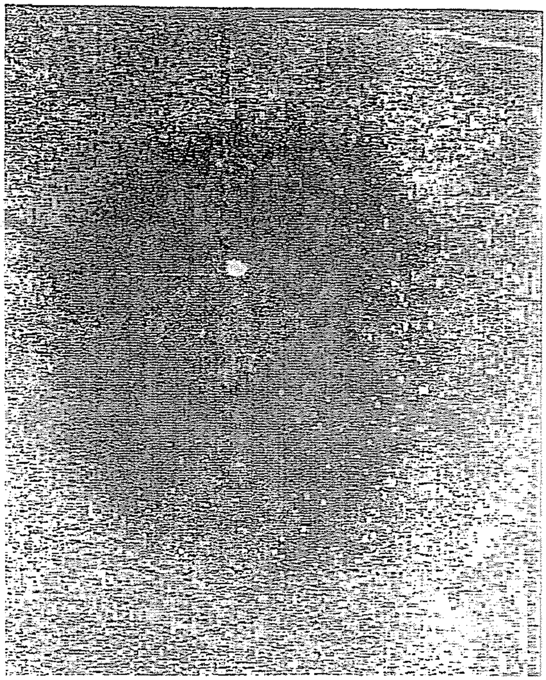


Fig. 9. Digitized front within a GIS (with simplified colours). (Central Hele-Shaw cell with 0.2–0.3 mm porous layer)

structure. In order to identify the active growth zone at a given time, and to filter out noise from the coverage in ARC/INFO, we subtracted the picture taken at the previous time. The earlier coverage was subtracted by superimposing the negative of the earlier picture below the positive of the last coverage. In this way we can determine the growth zone, that we use in order to point out the fractal behaviour of the dispersion. An example of digitized picture is shown in *Fig. 8* and in *Fig. 9.*

The simplest geometry for solving the transport equation is the case where tracers are added as a step function in a linear geometry at $x = 0$. The concentration profile then is given by

$$C(x,t) = 0.50 \cdot \left[1 - \operatorname{erf}\left[\frac{(x-R_0)}{w}\right]\right] . \tag{2}$$

Here $\operatorname{erf}(x)$ is the error function, $R_0 = Ut$ is the position of the front, U is the velocity and the width of the dispersion front is $w = 2(D_{||}t)^{1/2}$. ($D_{||}$ is the longitudinal dispersion coefficient (BEAR, 1972)).

In our case we have to use a more complicated situation for solving a transport equation because of radial geometry. However, for the present purposes sufficient accuracy is obtained by our estimating the average concentration:

$$C(r,t) = \sum c(r_i,t) 2\pi r_i \,, \tag{3}$$

where the sum extends over the observed concentrations $c(r_i, t)$ in the pixel labelled by i at the t-th moment. In possession of set $C(r,t)$ the hydrodynamic dispersion can be calculated by the help of a functional relation. (BEAR, 1972). This function can be built into ARC/INFO GIS as an AML macro, and the digitizing coverage can be analyzed in an easier way than using other software package.

7. Conclusion

The subsurface fractal hydraulics have been studied in a GIS environment. We showed the basics of viscous fingering, percolation theory and DLA algorithm. We have pointed out the connections between GIS and fractal phenomena.

The GIS is a useful tool when we want to analyze our picture. A growing unstable zone can be seen as a picture, on which we are able to try out a lot of statistical METHODS. A kind of statistical method is the determination of fractal dimension which is very important in the field of calculating the hydrodynamics dispersion.

Using the percolation theory a result has been shown that analyzed the fractal picture from Oxaal (OXAAL et al., 1987). We have found a good agreement (BAKUCZ, 1993).

We developed an experimental setup (after MÅLØY et al., 1987) in which we solved a GIS based analyzing system to determine in the Hele-Shaw growing viscous finger.

We have to continue our research in the field of fractal hydraulics in GIS environment as follows:

- comparing more experimental results,
- developing an exact mathematical basis for fractals in a GIS with particular regard to the hydraulical problems,
- creating the connection between the unstable thermodynamical formalism and the hydrodynamical dispersion by the help of chaotic systems in GIS.

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