PERIODICA POLYTECHNICA SER. CIVIL ENG. VOL. 38, NO. 4, PP. 399–413 (1994)

NONLINEAR ANALYSIS OF PLANE PROBLEMS BY MATHEMATICAL PROGRAMMING

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Received: Febr. 20, 1995

Abstract

A mixed variational principle based on a bilinear material model and on complementary potential energy is applied to the analysis of plane problems. Two discrete models are used to the construction of the fundamental equations. The first model consists of rigid rectangular panels connected along the edges by springs acting in tension, compression and shear and the other one is based on the standard finite element method. The application is illustrated by the solution of a numerical example.

Keywords: bilinear material model, limit analysis, nonlinear programming.

1. Introduction

The nonlinear behaviour of materials is approximated very often by multilinear or bilinear stress–strain relationships. LÓGÓ – TAYLOR proposed a special bilinear material model at which the Total stress is represented as the sum of a linearly elastic and a linearly elastic, pseudo-plastic component. Using this model and the complementary potential energy they developed a mixed extremum principle for the analysis of trusses [1]. Recently, the material model and this principle have been generalized and an other extremum principle based on the potential energy has been derived [2, 3]. Besides, the study has been extended to the optimal design of trusses with bilinear force–deformation characteristics and also to other structural problems [3, 4, 5, 6].

The aim of this paper is to apply the above material model and extremum principles to the investigation of nonlinear plane problems. It will be assumed that the nonlinear behaviour can be approximated by the bilinear material characteristics proposed by LÓGÓ – TAYLOR.

Two different discrete models will be used to the construction of the fundamental equations. The first model consists of rigid rectangular panels connected by springs acting in tension, compression and shear, while the other one is based on the standard finite element approach.

First, we present briefly the bilinear stress-strain relation and then construct the fundamental equations and the extremum principles based on the bilinear material model and on the two different discrete models. Finally, the application will be illustrated by a numerical example.

2. The Bilinear Stress–Strain Characteristics

The bilinear material model proposed by LÓGÓ – TAYLOR [1] has two components. One component is linearly elastic (*Fig. 1a*) with the material law:

$$\hat{\sigma} = E\varepsilon\,, \tag{1}$$

while the other one is linearly elastic, pseudo-plastic (*Fig. 1b*) described by the equations:

$$\overline{\sigma} = \begin{cases} \overline{E}\varepsilon, & \text{if} \quad |\overline{\sigma}| - \overline{\sigma}_0 < 0; \\ \overline{\sigma}_0, & \text{if} \quad |\overline{\sigma}| - \overline{\sigma}_0 = 0. \end{cases} \tag{2}$$

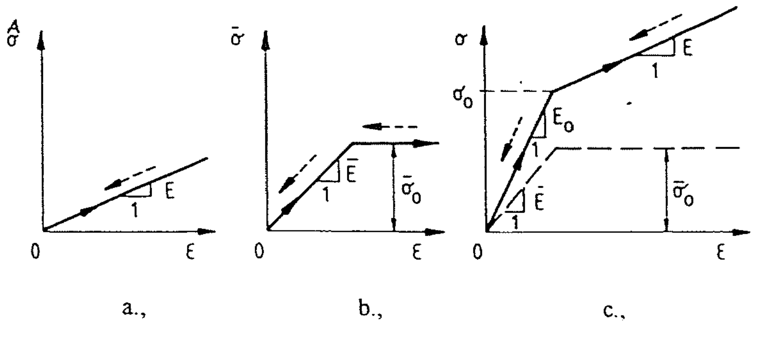

Fig. 1.

By the parallel connection of these two components a material model can be obtained which corresponds to the bilinear stress-strain relation shown in *Fig. 1c*:

$$\sigma = \overline{\sigma} + E\varepsilon\,,$$

where

$$\overline{\sigma} = \begin{cases} \overline{E}\varepsilon, & \text{if} \quad |\overline{\sigma}| - \overline{\sigma}_0 < 0; \\ \overline{\sigma}_0, & \text{if} \quad |\overline{\sigma}| - \overline{\sigma}_0 = 0. \end{cases} \tag{3}$$

The material constants of the 'pseudo-plastic' component can be expressed in terms of the constants E, E_0, and σ_0 of the bilinear model:

$$\overline{E} = E_0 - E\,, \qquad \overline{\sigma}_0 = \left(1 - \frac{E}{E_0}\right)\sigma_0\,. \tag{4}$$

The second component of the model is called 'pseudo-plastic' since it is assumed that loading and unloading take place on the same way without energy dissipation. Hence *Eq.* (3) represents a holonomic bilinear material and the solution methods based on this material model are suitable to the holonomic analysis of the structures.

3. Rigid Panel Model

The rigid panel model consists of rectangular rigid panels connected along the edges by springs acting in tension, compression and shear (*Fig. 2a*). This model has been proposed by KALISZKY et al. and successfully used to the static and dynamic analysis of prefabricated panel buildings [7, 8, 9].

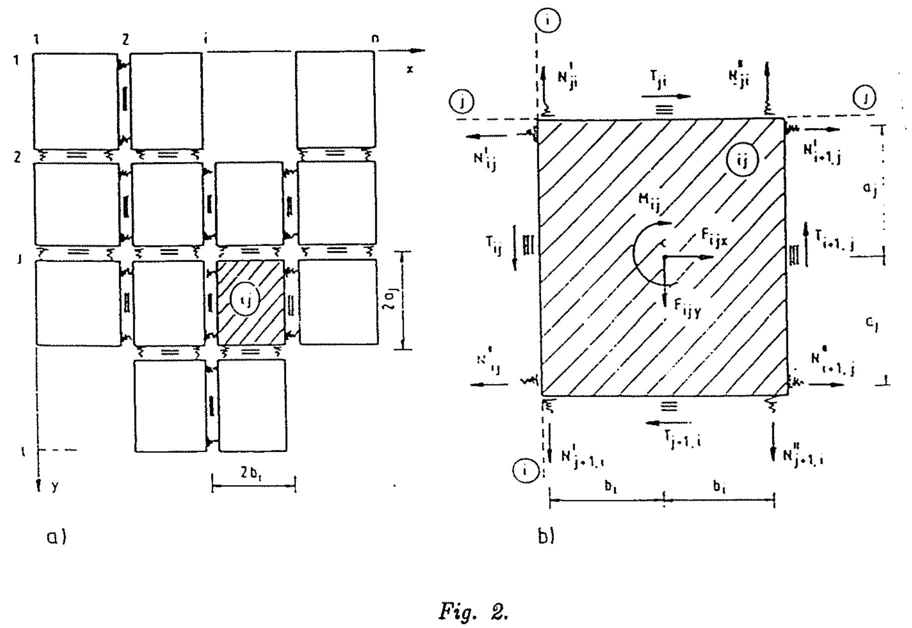

Fig. 2.

The disc (wall) under consideration and shown in *Fig. 2a* is divided by $i = 1, 2, \ldots, n$ vertical and by $j = 1, 2, \ldots, l$ horizontal lines (edges) into rectangular panels. It is assumed that the panels are perfectly rigid and all the deformations are concentrated in the springs interconnecting the panels and acting in tension, compression and shear (*Fig. 2b*). The disc is subjected to proportional loading with the load parameter m. The load consists of concentrated forces and couples acting at the centroids of the panels. Additional loads along the free edges of the disc can also be applied by replacing the springs by external forces. The external forces and the spring forces acting on the panel ij and the corresponding deformations of the springs are listed in the vectors

$$P_{ij} = [\, F_{ijx} \quad F_{ijy} \quad M_{ij} \,]^T \,, \tag{5}$$

$$Q_{ij} = \left[N'_{ij}\, N''_{ij}\, T_{ij} \mid N'_{ji}\, N''_{ji}\, T_{ji} \mid N'_{j+1,i}\, N''_{j+1,i}\, T_{j+1,i} \mid N'_{i+1,j}\, N''_{i+1,j} T_{\, i+1,j} \right]^T \,,$$

$$q_{ij} = \left[n'_{ij}\, n''_{ij}\, t_{ij} \mid n'_{ji}\, n''_{ji}\, t_{ji} \mid n'_{j+1,i}\, n''_{j+1,i}\, t_{j+1,i} \mid n'_{i+1,j}\, n''_{i+1,j}\, t_{i+1,j} \right]^T \,. \tag{6}$$

Note that here the subscripts of N, T, n and t cannot be interchanged.

3.1 Constitutive Equations

We assume that the force–displacement relations of the springs can be characterised by the bilinear material model described in Chapter 2. The spring coefficients r, r_0, N_0, f, f_0 and T_0 shown in *Fig. 3a – b* can be determined on the basis of theoretical consideration [7, 8, 9]. In case of panel buildings the springs represent the in situ connections of the prefabricated panels therefore their force–deformation relations can be obtained by experiments [10]. In the knowledge of the above coefficients the constants of the 'pseudo-plastic' components are determined by the relations

$$\left. \begin{aligned} \overline{r} &= r_0 - r \,, & \overline{f} &= f_0 - f, \\ \overline{N}_0 &= \left(1 - \frac{r}{r_0}\right) N_0 \,, & \overline{T}_0 &= \left(1 - \frac{f}{f_0}\right) T_0 \,. \end{aligned} \right\} \tag{7}$$

Then the constitutive equations of the springs are expressed in the following forms:

$$N_i = \overline{N}_i + r_i n_i \,, \qquad \text{where} \qquad \overline{N} = \begin{cases} \overline{r}_i n_i \,, & \text{if} \quad |\overline{N}_i| - \overline{N}_{0i} < 0 \,; \\ \overline{N}_{0i} \,, & \text{if} \quad |\overline{N}_i| - \overline{N}_{0i} = 0 \,. \end{cases} \tag{8}$$

$$T_i = \overline{T}_i + f_i t_i \,, \qquad \text{where} \qquad \overline{T} = \begin{cases} \overline{f}_i t_i \,, & \text{if} \quad |\overline{T}_i| - \overline{T}_{0i} < 0 \,; \\ \overline{T}_{0i} \,, & \text{if} \quad |\overline{T}_i| - \overline{T}_{0i} = 0 \,. \end{cases} \tag{9}$$

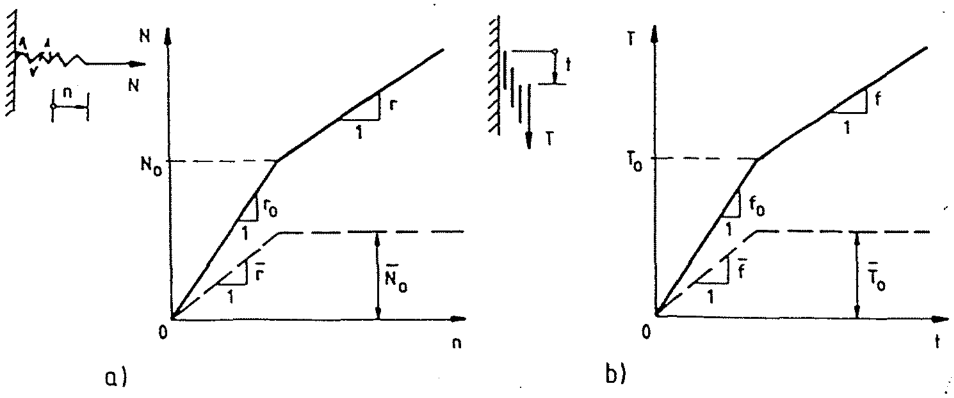

Fig. 3.

In *Eq.* (8) we tacitly assumed that the response of the springs is the same in tension and compression. Introducing additional constraints, however, the sign-dependent behaviour of the springs can also be taken into account.

Making use of *Eqs.* (8) and (9) and introducing the vectors of the spring coefficients

$$k_{ij} = \left[r'_{ij}\, r''_{ij}\, f_{ij} \,|\, r'_{ji}\, r''_{ji}\, f_{ji} \,|\, r'_{j+1,i}\, r''_{j+1,i}\, f_{j+1,i} \,|\, r'_{i+1,j}\, r''_{i+1,j}\, f_{i+1,j}\right]^T , \quad (10)$$

$$\overline{k}_{ij} = \left[\overline{r}'_{ij}\, \overline{r}''_{ij}\, \overline{f}_{ij} \,|\, \overline{r}'_{ji}\, \overline{r}''_{ji}\, \overline{f}_{ji} \,|\, \overline{r}'_{j+1,i}\, \overline{r}''_{j+1,i}\, \overline{f}_{j+1,i} \,|\, \overline{r}'_{i+1,j}\, \overline{r}''_{i+1,j}\, \overline{f}_{i+1,j}\right]^T , \quad (11)$$

$$\overline{Q}_{0ij} =$$

$$\left[\overline{N}'_{0ij}\overline{N}''_{0ij}\overline{T}_{0ij}|\overline{N}'_{0ji}\overline{N}''_{0ji}\overline{T}_{0ji}|\overline{N}'_{0j+1,i}\overline{N}''_{0j+1,i}\overline{T}_{0j+1,i}|\overline{N}'_{0i+1,j}\overline{N}''_{0i+1,j}\overline{T}_{0i+1,j}\right]^T , \quad (12)$$

the constitutive equation of the springs attached to panel ij becomes

$$Q_{ij} = \overline{Q}_{ij} + k^T_{ij} q_{ij} , \qquad \text{where} \qquad \overline{Q}_{ij} = \begin{cases} \overline{k}^T_{ij} q_{ij}, & \text{if } |\overline{Q}_{ij}| - \overline{Q}_{0ij} < 0; \\ \overline{Q}_{0ij}, & \text{if } |\overline{Q}_{ij}| - \overline{Q}_{0ij} = 0. \end{cases} \quad (13)$$

Here the vector

$$\overline{Q}_{ij} = \left[\overline{N}'_{ij}\, \overline{N}''_{ij}\, \overline{T}_{ij} \,|\, \overline{N}'_{ji}\, \overline{N}''_{ji}\, \overline{T}_{ji} \,|\, \overline{N}'_{j+1,i}\, \overline{N}''_{j+1,i}\, \overline{T}_{j+1,i} \,|\, \overline{N}'_{i+1,j}\, \overline{N}''_{i+1,j}\, \overline{T}_{i+1,j}\right]^T \quad (14)$$

collects the 'pseudo-plastic' components of the spring forces. Note that in *Eqs.* (10 – 12) and (14) the subscripts of r, f, $\overline{Q}_0$ and $\overline{Q}$ cannot be interchanged.

Finally, the constitutive equation of the entire structure can be obtained by compilation of the relations of *Eq.* (13):

$$\mathbf{Q} = \overline{\mathbf{Q}} + \mathbf{k}^T\mathbf{q}, \qquad \text{where} \qquad \overline{\mathbf{Q}} = \begin{cases} \overline{\mathbf{k}}^T\mathbf{q}, & \text{if } |\overline{\mathbf{Q}}| - \overline{\mathbf{Q}}_0 < 0; \\ \overline{\mathbf{Q}}_0, & \text{if } |\overline{\mathbf{Q}}| - \overline{\mathbf{Q}}_0 = 0. \end{cases} \tag{15}$$

3.2 Equilibrium Equation

Omitting the details the equilibrium of the panel ij shown in *Fig. 2b* is expressed by the equation

$$G_{ij}Q_{ij} + mP_{ij} = 0, \tag{16}$$

where G_{ij} is the equilibrium matrix of panel ij:

$$G_{ij} = \left[\begin{array}{ccc|ccc|ccc|ccc} -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & 1 & 1 & 0 & 0 & 0 & -1 \\ -a_j & a_j & -b_i & b_i & -b_i & a_j & -b_i & b_i & a_j & a_j & -a_j & -b_i \end{array}\right]. \tag{17}$$

By compilation of *Eq.* (16) of all panels the equilibrium equation of the entire structure becomes:

$$\mathbf{GQ} + m\mathbf{P} = 0. \tag{18}$$

Note that in *Eq.* (16) external forces along the free edges are not taken into consideration, however, they can be incorporated in *Eq.* (18). Substituting the constitutive *Eq.* (15) in *Eq.* (18) we can express the equilibrium of the structure in the following form

$$\mathbf{G}\left(\overline{\mathbf{Q}} + \mathbf{k}^T\mathbf{q}\right) + m\mathbf{P} = 0, \tag{19.a}$$

$$|\overline{\mathbf{Q}}| - \overline{\mathbf{Q}}_0 \le 0. \tag{19.b}$$

3.3 The Extremum Principle

The mixed extremum principle elaborated by LÓGÓ – TAYLOR for trusses [1, 2] can be formulated in the following modified form for the present problem.

Among all states of the discretized structure under consideration which satisfy the equilibrium and constitutive conditions and correspond to an assumed level $\overline{\Pi}_0$ of the complementary potential energy that is the actual one at which the load multiplier m assumes its maximum value.

Hence, using *Eq.* (19) the unknown state variables $\overline{\mathbf{Q}}$, $\mathbf{q}$ and the corresponding load multiplier m can be determined by solving the following extremum problem:

$$\underset{(m,\overline{\mathbf{Q}},\mathbf{q})}{m} = \max! \tag{20}$$

subject to

$$\mathbf{G}\left(\overline{\mathbf{Q}} + \mathbf{k}^T\mathbf{q}\right) + m\mathbf{P} = 0\,, \tag{21}$$

$$|\overline{\mathbf{Q}}| - \overline{\mathbf{Q}}_0 \leq 0\,, \tag{22}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{l}\left[\frac{\left(\overline{N}'_{ij}\right)^2}{2\overline{r}'_{ij}} + \frac{\left(\overline{N}''_{ij}\right)^2}{2\overline{r}''_{ij}} + \frac{\left(\overline{T}_{ij}\right)^2}{2\overline{f}_{ij}} + \frac{r'_{ij}\left(n'_{ij}\right)^2}{2} + \frac{r''_{ij}\left(n''_{ij}\right)^2}{2} + \frac{f_{ij}\left(t_{ij}\right)^2}{2}\right] +$$
$$+\sum_{j=1}^{l}\sum_{i=1}^{n}\left[\frac{\left(\overline{N}'_{ji}\right)^2}{2\overline{r}'_{ji}} + \frac{\left(\overline{N}''_{ji}\right)^2}{2\overline{r}''_{ji}} + \frac{\left(\overline{T}_{ji}\right)^2}{2\overline{f}_{ji}} + \frac{r'_{ji}\left(n'_{ji}\right)^2}{2} + \frac{r''_{ji}\left(n''_{ji}\right)^2}{2} + \frac{f_{ji}\left(t_{ji}\right)^2}{2}\right] -$$
$$-\overline{\Pi}_0 \leq 0\,. \tag{23}$$

Here in *Eq.* (23) the first and the second summation terms express the complementary strain energy of the springs attached to the vertical and horizontal edges, respectively. The proof of the principle can be found elsewhere [1, 2].

4. Finite Element Model

The bilinear stress-strain relation of Chapter 2 can easily be incorporated in the standard finite element approach. To construct the fundamental equations let us consider a plane strain problem with $e = 1, 2, \ldots, s$ triangular elements and $i = 1, 2, \ldots, n$ nodes (*Fig. 4*). The nodes are subjected to proportional loading defined by the forces

$$\mathbf{F} = \left[F_{1x}\, F_{1y},\, F_{2x}\, F_{2y},\, \ldots,\, F_{ix}\, F_{iy},\, \ldots,\, F_{nx}\, F_{ny}\right]^T \tag{24}$$

and by the load parameter m. We assume homogeneous state of stress and strain in the elements defined by the vectors

$$\sigma^e = \left[\,\sigma_x^e \quad \sigma_y^e \quad \sigma_{xy}^e\,\right]^T\,, \tag{25.a}$$

$$\varepsilon^e = [\,\varepsilon_x^e \quad \varepsilon_y^e \quad \varepsilon_{xy}^e\,]^T \,. \tag{25.b}$$

Hence, inside the elements the equilibrium and the compatibility conditions are satisfied.

4.1 Constitutive Equations

Let us assume that the property of the material is characterised by the bilinear material model described in Chapter 2 and the material constants E_0, G_0, σ_0 and E, G corresponding to the two linear stages of the material are given. Then the constants of the 'pseudo-plastic' component of the material can be obtained:

$$\overline{E} = E_0 - E\,, \qquad \overline{G} = G_0 - G\,, \qquad \overline{\sigma}_0 = \left(1 - \frac{E}{E_0}\right)\sigma_0\,. \tag{26}$$

Let us suppose that the ratios of the Young's Modulus and Shear modulus are equal i. e. $\frac{E_0}{G_0} = \frac{E}{G}$ in both stages of the material. Then it can easily be shown that the Poisson ratios ν_0, ν, and $\overline{\nu}$ corresponding to $(E_0,\, G_0)$, $(E,\, G)$ and $\left(\overline{E}, \overline{G}\right)$, respectively, are equal and become:

$$\nu_0 = \nu = \overline{\nu} = \frac{E}{2G} - 1\,. \tag{27}$$

Following the concept of the bilinear material model we split the stresses of *Eq.* (25) in two parts

$$\sigma^e = \overline{\sigma}^e + \hat{\sigma}^e\,, \tag{28}$$

where

$$\hat{\boldsymbol{\sigma}}^e = \mathbf{D}^e \boldsymbol{\varepsilon}^e\,, \tag{29}$$

$$\overline{\boldsymbol{\sigma}}^e = \begin{cases} \overline{\mathbf{D}}^e \boldsymbol{\varepsilon}^e\,, & \text{if} \quad f\left(\overline{\sigma}^e, \overline{\sigma}_0\right) < 0\,, \\ \overline{\boldsymbol{\sigma}}_0^e\,, & \text{if} \quad f\left(\overline{\sigma}^e, \overline{\sigma}_0\right) = 0\,, \end{cases} \tag{30}$$

where $f\left(\overline{\sigma}^e, \overline{\sigma}_0\right)$ is the yield function of the pseudo-plastic component and in case of plane strain the flexibility matrices of the two material components are

$$\mathbf{D}^e = E\frac{(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \dfrac{\nu}{1-\nu} & 0 \\ \dfrac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \dfrac{1-2\nu}{1-\nu} \end{bmatrix}, \qquad \overline{\mathbf{D}}^e = \frac{\overline{E}}{E}\mathbf{D}^e\,. \tag{31}$$

In addition, we assume that the constraint by which the pseudo-plastic component $\overline{\sigma}^e$ is controlled is expressed by the Tresca yield condition [11]. Hence, the constitutive equation of element e can be written in the following form

$$\boldsymbol{\sigma}^e = \overline{\boldsymbol{\sigma}}^e + \mathbf{D}^e \boldsymbol{\varepsilon}^e \,, \tag{32.a}$$

$$\left(\overline{\sigma}^e_{xx} - \overline{\sigma}^e_{yy}\right)^2 + 4\left(\overline{\sigma}^e_{xy}\right)^2 - \overline{\sigma}_0^2 \leq 0 \tag{32.b}$$

and for the entire structure it becomes

$$\boldsymbol{\sigma} = \overline{\boldsymbol{\sigma}} + \mathbf{D}\boldsymbol{\varepsilon} \,, \tag{33.a}$$

$$\left(\overline{\sigma}^e_{xx} - \overline{\sigma}^e_{yy}\right)^2 + 4\left(\overline{\sigma}^e_{xy}\right)^2 - \overline{\sigma}_0^2 \leq 0 \,, \qquad (e = 1,\ 2,\ s,\ s). \tag{33.b}$$

Here the vectors $\boldsymbol{\sigma}$, $\overline{\boldsymbol{\sigma}}$ and $\boldsymbol{\varepsilon}$ contain the stresses σ^e, $\overline{\sigma}^e$ and the strains ε^e, respectively, of the elements $e = 1, 2, \ldots, s$.

4.2 Equilibrium Equation

Omitting the details well known from the literature [12] and making use of *Eq.* (32) the equilibrium equation of the joint i loaded by the external force $m\mathbf{F}_i$ becomes:

$$\sum_{e=1}^{h} \mathbf{B}_i^e \left(\overline{\boldsymbol{\sigma}}^e + \mathbf{D}^e \boldsymbol{\varepsilon}^e\right) + m\mathbf{F}_i = 0 \,. \tag{34}$$

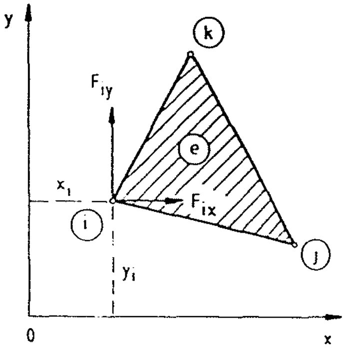

Fig. 4.

Here for a typical element e shown in *Fig. 4*

$$\mathbf{B}_i^e = \frac{1}{2}\begin{bmatrix} b_i^e & 0 & c_i^e \\ 0 & c_i^e & b_i^e \end{bmatrix}, \tag{35}$$

$$b_i^e = y_j^e - y_k^e, \qquad c_i^e = x_k^e - x_j^e, \tag{36}$$

and $e = 1, 2, \ldots, h$ denote the elements which contain the node i. By the compilation of *Eq.* (34) for all nodes $i = 1, 2, \ldots, n$ the equilibrium equation of the entire structure becomes:

$$\mathbf{B}(\overline{\boldsymbol{\sigma}} + \mathbf{D}\boldsymbol{\varepsilon}) + m\mathbf{F} = 0. \tag{37}$$

4.3 The Extremum Principle

The mixed extremum principle proposed by LÓGÓ – TAYLOR for trusses [1, 2, 3] for the present problem can be formulated in the following form.

Among all states of the plane strain problem under consideration which satisfy the equilibrium and constitutive equations and correspond to an assumed level $\overline{\Pi}_0$ of the complementary potential energy that is the actual one at which the load multiplier m assumes its maximum value.

Hence, using *Eqs.* (33) and (37) the unknown state variables $\overline{\boldsymbol{\sigma}}$, $\boldsymbol{\varepsilon}$ and the corresponding load multiplier m can be determined by solving the following extremum problem:

$$\underset{(m,\overline{\sigma},\varepsilon)}{m} = \max! \tag{38}$$

subject to

$$\mathbf{B}(\overline{\boldsymbol{\sigma}} + \mathbf{D}\boldsymbol{\varepsilon}) + m\mathbf{F} = 0, \tag{39}$$

$$\left(\overline{\sigma}_{xx}^e - \overline{\sigma}_{yy}^e\right)^2 + 4\left(\overline{\sigma}_{xy}^e\right)^2 - \overline{\sigma}_0^2 \le 0, \qquad (e = 1,\ 2,\ s,\ s), \tag{40}$$

$$\sum_{e=1}^{s} \Delta^e \left\{ \frac{1}{e\overline{E}} \left[\left(\overline{\sigma}_{xx}^e\right)^2 + \left(\overline{\sigma}_{yy}^e\right)^2 - 2\nu\left(\overline{\sigma}_{xx}^e\right)\left(\overline{\sigma}_{yy}^e\right) + 2(1+\nu)\left(\overline{\sigma}_{xy}^e\right)^2 \right] + \right.$$

$$\left. + \frac{E}{2(1+\nu)} \left[\left(\varepsilon_{xx}^e\right)^2 + \left(\varepsilon_{yy}^e\right)^2 + \frac{\nu}{1-2\nu}\left(\varepsilon_{xx}^e + \varepsilon_{yy}^e\right)^2 + 2\left(\varepsilon_{xy}^e\right)^2 \right] \right\} - \overline{\Pi}_0 \le 0. \tag{41}$$

Here *Eq.* (42) is the complementary potential energy and Δ^e denotes the area of the element e.

5. Solution Techniques and Applications

The above principles are stated in form of constrained, nonsmooth, nonlinear mathematical programming. There are several methods in the literature to solve them [13] and among others the bundle method [14] is one of the most suitable. The basic algorithm solves unconstrained, nonlinear programming problems with either a smooth or nonsmooth objective function and the constraints can be taken into account by formulating an L1-penalty function or using some barrier function techniques. Since our problem consists of smooth intervals, in these intervals smooth algorithms can be used. Then a search direction for the variables is obtained and a line search is performed to get a new iteration [15]. On the basis of the above solution techniques a computer program was elaborated on FORTRAN 77 language for IBM 3090 and HP 9730 computers.

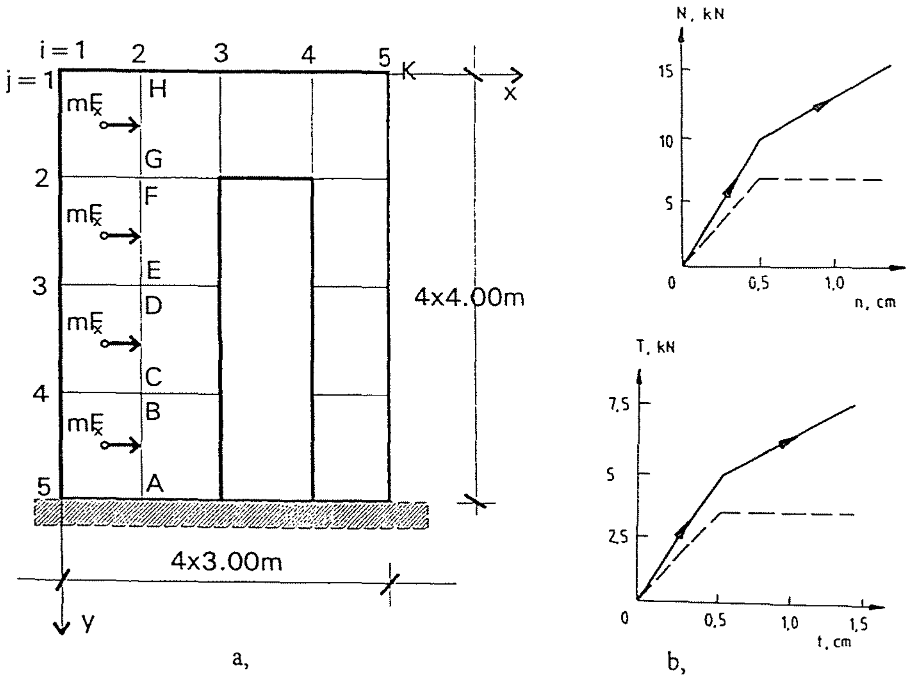

Fig. 5.

In order to study a specific state of the structure one has to assume an appropriate value $\overline{\Pi}_0$ of the complementary potential energy. Then, using the principle and the solution techniques described above one can determine

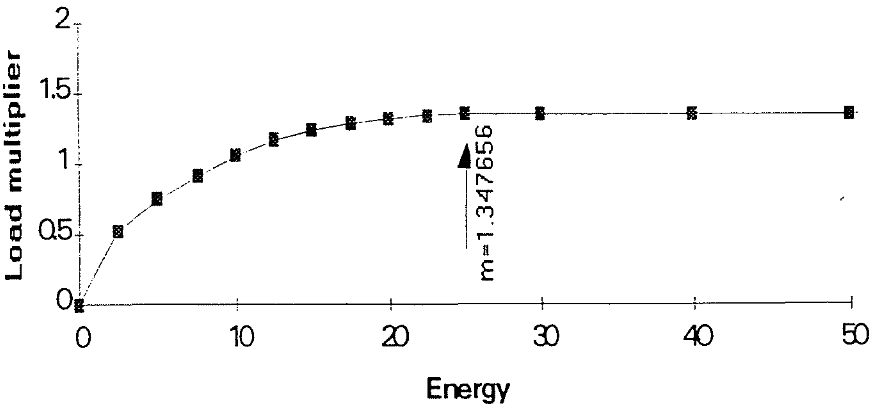

Fig. 6. Load multiplier-energy

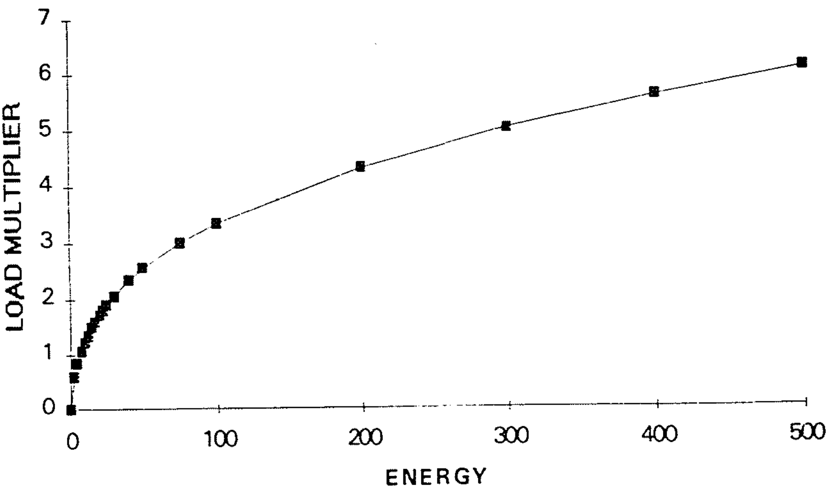

Fig. 7. Load multiplier-energy

the load intensity m and all the state variables which correspond to the assumed energy level of the structure. Repeating this procedure one can conduct a load history analysis or can easily find the state of the structure which corresponds to a requested load intensity.

To illustrate the application let us consider the structure of *Fig. 5a* already subdivided into rigid panels and subjected at the centroids of the panels 1,1; 1,2; 1,3 and 1,4 to equal ($F_x = 2kN$) horizontal forces with a common load multiplier m. For the sake of simplicity we assume that all

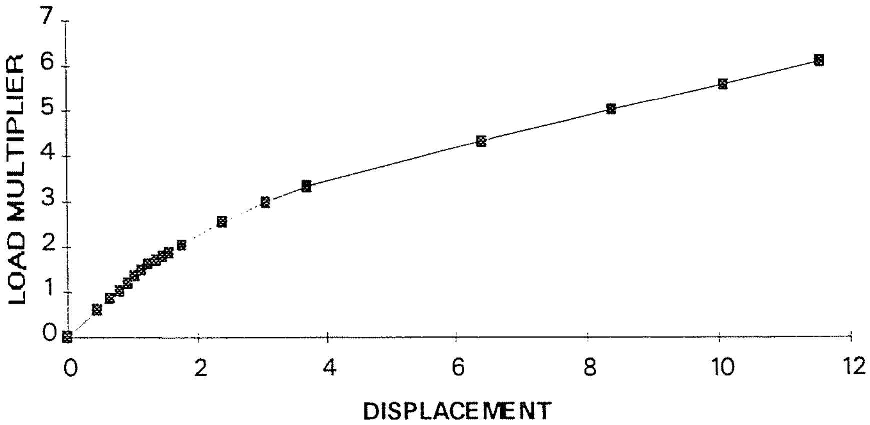

Fig. 8. Load multiplier-horizontal displacement at K

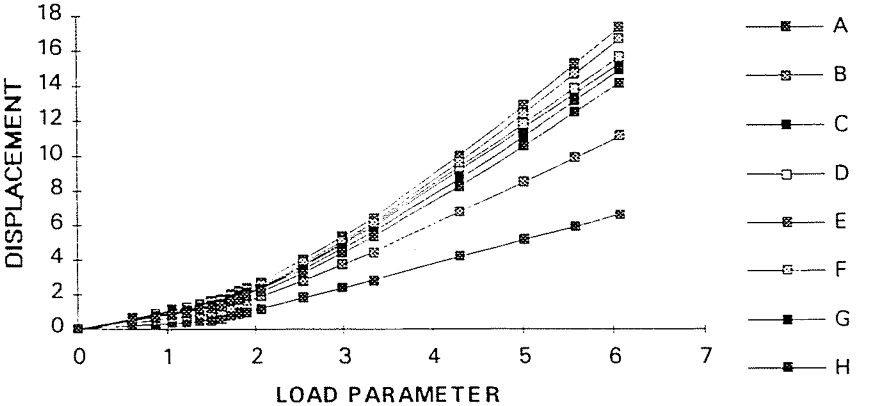

Fig. 9. Horizontal displacement – load multiplier in vertical section at $i = 2$

the springs have the same coefficients (*Fig. 5b*):

$$r_0 = 20\,\text{kN/cm}, \qquad f_0 = 10\,\text{kN/cm},$$
$$r = 5\,\text{kN/cm}, \qquad f = 2.5\,\text{kN/cm},$$
$$N_0 = 10\,\text{kN}, \qquad T_0 = 5\,\text{kN}.$$

Then using *Eq.* (7) the constants of the 'pseudo-plastic' components are as follows:

$$\overline{r} = 15\,\text{kN/cm}, \qquad \overline{t} = 7.5\,\text{kN/cm},$$
$$\overline{N}_0 = 7.5\,\text{kN}, \qquad \overline{T}_0 = 3.75\,\text{kN}.$$

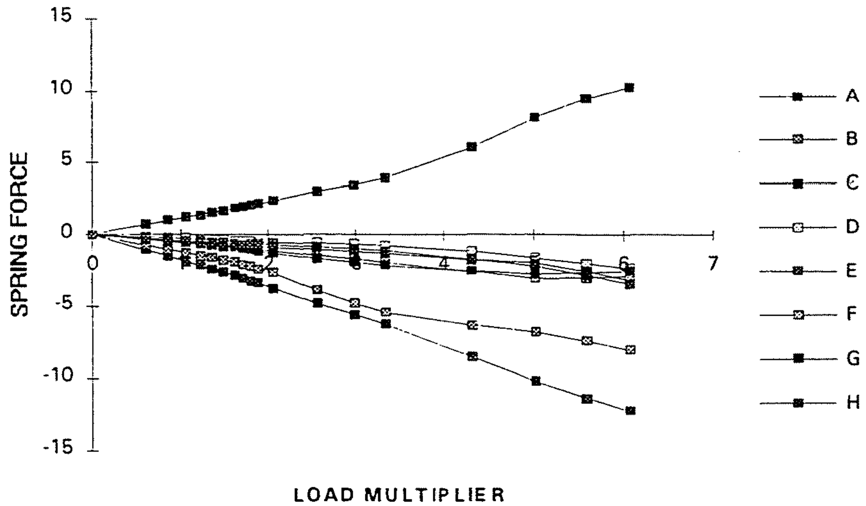

Fig. 10. Normal spring force – load multiplier in vertical section at $i = 2$

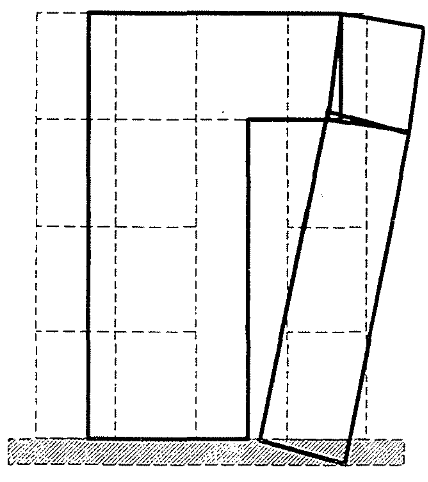

Fig. 11. The collapse mechanism of the structure

Some results of the load history analysis are shown in *Figs. 6 – 11*. *Figs. 7 – 10* contain the results obtained by the general bilinear material characteristics given above, while the diagram of *Fig. 6* shows the results of the elasto-plastic structure when $r = 0$ and $f = 0$. Finally *Fig. 11* illustrates the collapse mechanism of the structure at the collapse load multiplier $m_{coll} = 1.3476$.

Acknowledgements

The present study was supported by the Hungarian National Scientific and Research Foundation (OTKA #683).

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