

## FAILURE RECOGNITION IN WASTE-WATER TREATMENT PROCESS

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### Abstract

A failure recognition method based on the inverse solution of a linear dynamical model was applied to detect malfunctions of waste-water plant operations. Kalman filter could be successfully employed to eliminate process noise as well as modelling errors.

*Keywords:* failure recognition, linear dynamic model, Kalman filter, waste-water treatment.

### Introduction

Waste-water treatment has become a very important technology in our days when the efficiency of the environment protection must be considerably improved. Waste-waters contain a complex mixture of solids and dissolved components with the latter usually present in very small concentrations. In treatment plants all these contaminants must be reduced to acceptable low concentrations or chemically transformed into inoffensive compounds.

The main component of the activated sludge process is a continuous-flow aerated biological reactor. This aerobic reactor is closely tied to a sedimentation tank in which the liquid is clarified. A portion of the sludge collected in the sedimentation tank is usually recycled to the biological reactor, providing a continuous sludge inoculation. This recycling increases the mean sludge residence time, giving the microorganisms present an opportunity to adapt to the available nutrient. Also, the sludge must reside in the aerobic reactor long enough for adsorbed organics to be oxidized. To improve the performance of the plant this recirculation is usually carried out periodically. The schematic diagram of such an activated sludge process consisting of two reactor stages can be seen in *Fig. 1*.

Although a waste-water treatment plant for a major city is very expensive, the biological reactors contained are usually designed using extremely simplified and idealized models (BAILEY, 1977). Typically, the aeration basin is treated as a perfectly mixed vessel, and sludge is viewed

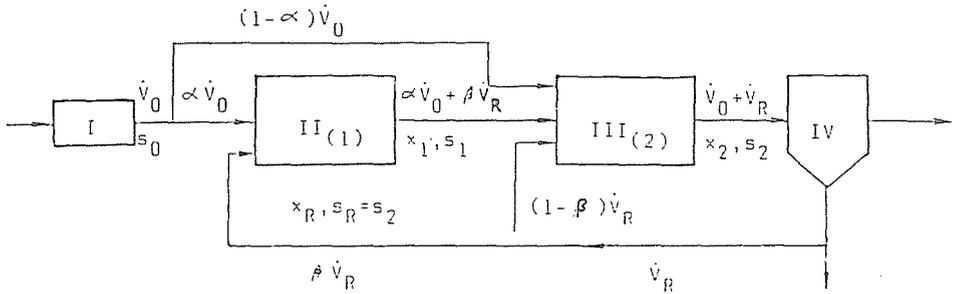


Fig. 1. Schematic diagram of the activated-sludge process

as a single pseudo species whose growth rate follows Monod kinetics, and substrate concentration is usually expressed in terms of BOD (biochemical oxygen demand).

The simulation of this process was performed on the basis of a lumped, nonlinear, dynamical model (KARDOS, 1984)

$$V_1 \frac{ds_1}{dt} = \alpha V_0 s_0 + \beta V_R s_2 - (\alpha V_0 + \beta V_R) s_1 - V_1 \frac{k s_1 x_1}{y}, \quad (1)$$

$$V_1 \frac{dx_1}{dt} = \beta V_R x_R - (\alpha V_0 + \beta V_R) x_1 + V_1 k s_1 x_1, \quad (2)$$

$$V_2 \frac{ds_2}{dt} = (1 - \alpha) V_0 s_0 + (\alpha V_0 + \beta V_R) s_1 + (1 - \beta) V_R s_2 - (V_0 + V_R) s_2 V_2 \frac{k s_2 x_2}{y}. \quad (3)$$

$$V_2 \frac{dx_2}{dt} = (\alpha V_0 + \beta V_R) x_1 + (1 - \beta) V_R x_R - (V_0 + V_R) x_2 + V_2 k s_2 x_2. \quad (4)$$

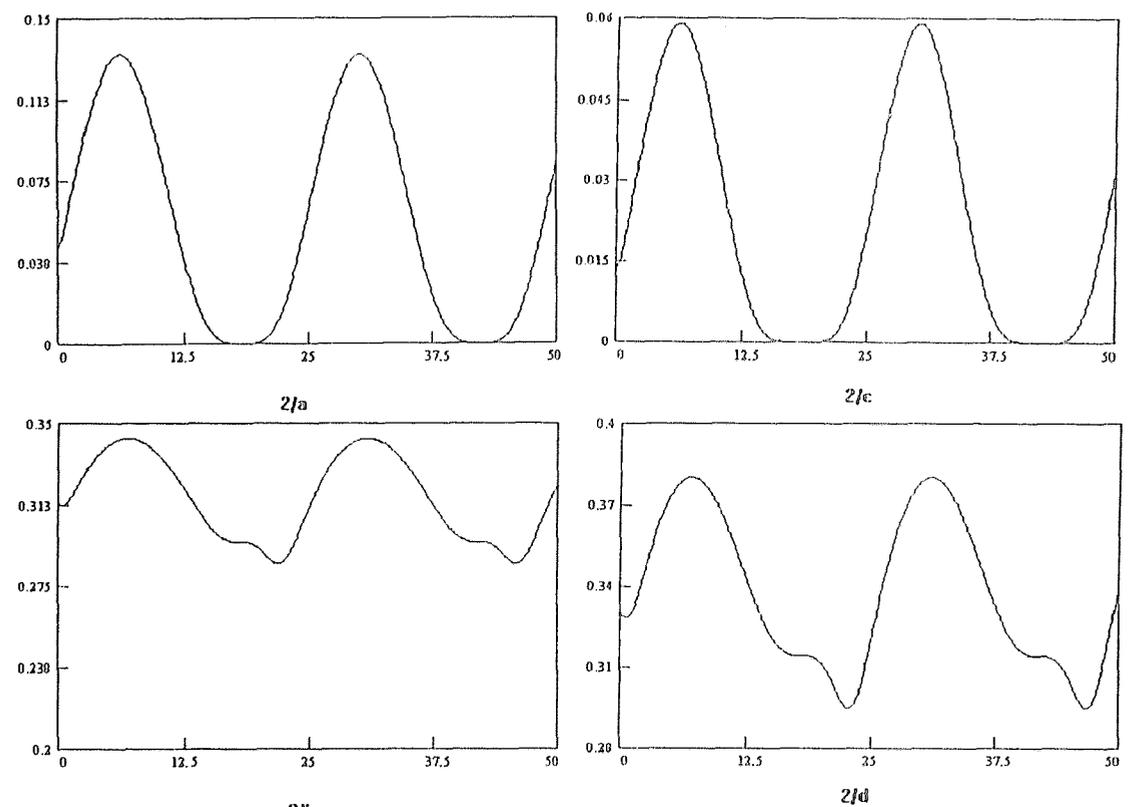
For explanation, see Notation. The details of the modelling can be found in (WINKLER, 1980).

The trajectories of the state variables, contamination and reaction product in the outlet flow of the two reactor stages in case of normal operation are shown in Fig. 2.

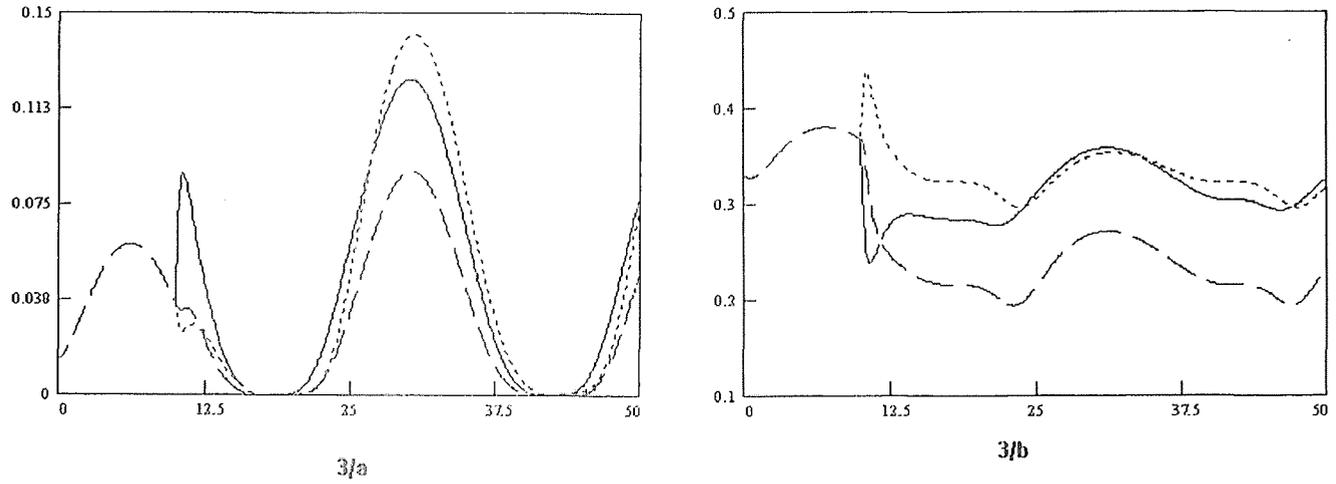
### Failure Recognition

Three different malfunctions were considered:

- failure of the recycling pump,
- failure in valve operation,
- ineffective operation of the sedimentation tank.



2/b *Fig. 2.* State variables in case of normal operation  
 2/a Outlet substrate concentration at the first stage  
 2/b Outlet sludge concentration at the first stage  
 2/c Outlet substrate concentration at the second stage  
 2/d Outlet sludge concentration at the second stage



*Fig. 3. a* Outlet substrate concentration at the second stage in case of different malfunctions  
*b* Outlet sludge concentration at the second stage in case of different malfunctions

Three malfunctions were simulated, and the results can be seen in Fig. 3/a, 3/b.

The figures show the outlet concentrations of the second stage. It can be seen that the trajectories are fairly similar to each other in the different cases. Consequently, it is not easy to identify the malfunction which has taken place.

### The Recognition Method

As the first step, we approximate the nonlinear dynamics of the process by a linear model:

$$\dot{\mathbf{x}}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t), \quad (5)$$

where  $\mathbf{x}$  —  $N$  dimensional vector of the measurable state variables

$\mathbf{A}$  —  $N * M$  dimensional system matrix

$\mathbf{B}$  —  $N * M$  dimensional indicator matrix

$\mathbf{u}$  —  $M$  dimensional indicator vector

$t$  — time

The elements of  $\mathbf{A}$  and  $\mathbf{B}$  matrices can be determined by a 'teaching' process (PALÁNCZ, 1990), which means that we simulate the different malfunctions employing the nonlinear model and minimizing the following functional:

$$I(a_{i,j}, b_{i,j}) = \sum_{m=1}^M \int_0^T [(\mathbf{x}^m - \mathbf{A}\mathbf{x}^m - \mathbf{B}\mathbf{u}^m)(\mathbf{x}^m - \mathbf{A}\mathbf{x}^m - \mathbf{B}\mathbf{u}^m)] dt, \quad (6)$$

where  $M$  — the number of the malfunctions

$\mathbf{x}^m$  — the vector of the state variables in case of the  $m$ -th malfunction

$T$  — the length of the monitoring window, its time-span

and

$$\mathbf{u}_k^m = \begin{cases} 1 & m = k, \\ 0 & \text{otherwise.} \end{cases}$$

Once the matrices  $\mathbf{A}$  and  $\mathbf{B}$  are known, the linear model, Eq. (5) can approximate the trajectories of the malfunction transients which have been involved in the teaching process.

In the case of the  $k$ -th malfunction  $u_k \equiv 1$ , and the other elements of the indicator vector are zero.

During diagnostic process the components of  $\mathbf{x}(t)$  are measured. Now, we will express  $\mathbf{u}(t)$  from Eq. (5) explicitly and compute it on the basis

of the measured trajectories. Using discrete time solution and employing parameter estimation technique, one may get (see Appendix):

$$\mathbf{u}^{n+1} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \left( \frac{2}{\Delta T} x^{n+1} - \frac{2}{\Delta T} \bar{\Phi} x^n - \bar{\Phi} \mathbf{B} \mathbf{u}^n \right), \quad (7)$$

where

$\Delta T$  - time step

$\bar{\Phi}$  - resolvent matrix,  $\bar{\Phi} = \exp(\mathbf{A} \Delta T)$

$\mathbf{u}$  -  $\mathbf{u}(n \Delta T)$

$x^n$  -  $x(n \Delta T)$

If  $\mathbf{u}_j^n \equiv 1$  for every  $n$ , this means that the  $j$ -th malfunction has taken place. If  $\mathbf{u}_j^n \equiv 0$ , then this malfunction has not occurred. In practice, integral form of Eq. (8) ensures better stability properties of the algorithm, namely

$$\mathbf{u}(t) = \frac{1}{T} \int_0^t \mathbf{u}(\lambda) d\lambda \quad (8)$$

#### 4. Application of the Method for Deterministic System

In order to employ Eq. (5) to the waste-water plant problem, we considered the difference of the trajectories of the normal operation and that of the malfunction transients. According to Fig. 3 the malfunction occurs at  $t = 10$ , and because the time-span of the monitoring window is 10, too, the considered time interval is  $10 \leq t \leq 20$ . Fig. 4 shows the time history of the state variables in the case of imperfect valve operation.

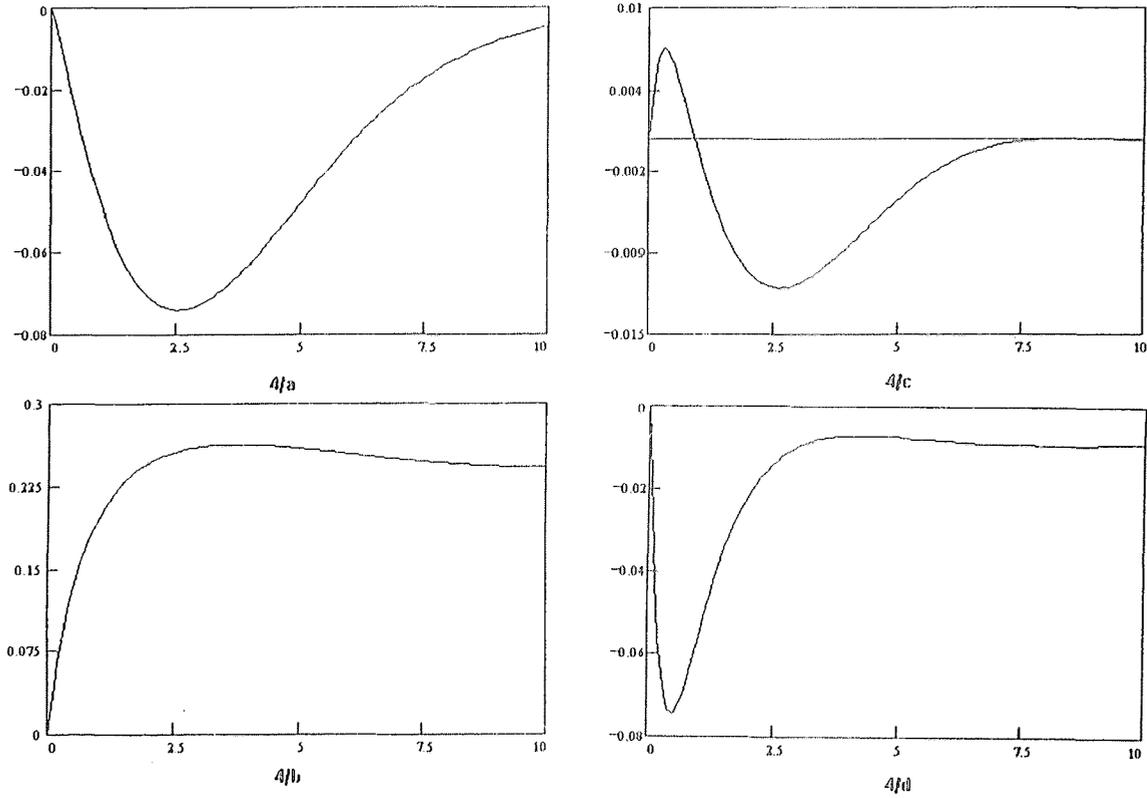
The elements of  $\mathbf{A}$  and  $\mathbf{B}$  were computed employing Eq. (6) :

$$\mathbf{A} = \begin{bmatrix} - & 0.105 & 0.251 & 1.154 & 0.164 \\ & 0.055 & -0.584 & -0.425 & -0.968 \\ - & 0.056 & -0.196 & -0.481 & 0.315 \\ - & 0.259 & 0.450 & 0.108 & -0.934 \end{bmatrix},$$

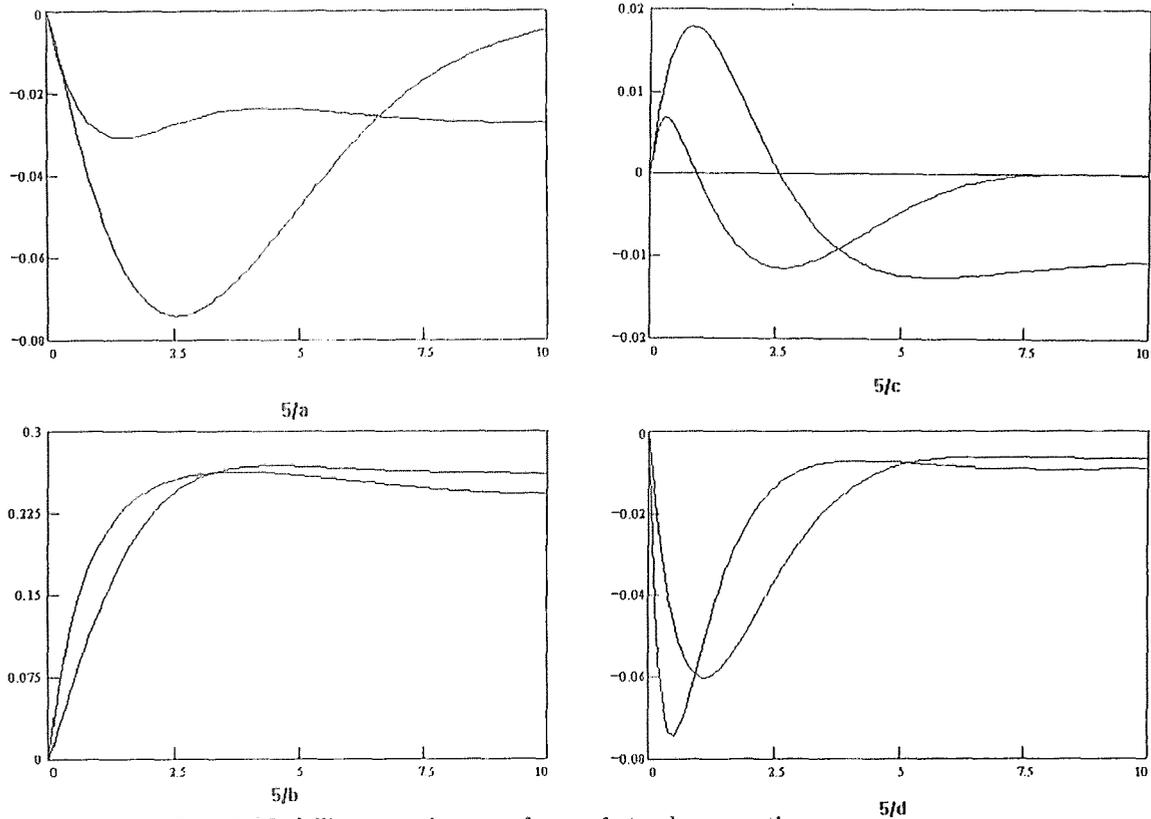
$$\mathbf{B} = \begin{bmatrix} 0.114 & 0.055 & -0.035 \\ -0.247 & 0.143 & 0.150 \\ -0.103 & 0.047 & -0.011 \\ 0.241 & -0.130 & 0.048 \end{bmatrix}.$$

In Fig. 5 the deviations between the trajectories of the linear and nonlinear model indicate the modelling failure.

Using formulas (7) and (8), the integral form of the indicator vector  $\mathbf{u}(T)$  has been computed for all of the three malfunctions, see Table 1.



*Fig. 4.* State variables in case of imperfect valve operation  
 4a. Outlet substrate concentration at the first stage  
 4b. Outlet sludge concentration at the first stage  
 4c. Outlet substrate concentration at the second stage  
 4d. Outlet sludge concentration at the second stage



*Fig. 5.* Modelling error in case of imperfect valve operation  
 5a. Outlet substrate concentration at the first stage  
 5b. Outlet sludge concentration at the first stage  
 5c. Outlet substrate concentration at the second stage  
 5d. Outlet sludge concentration at the second stage

Table 1

	$M_1$	$M_2$	$M_3$
$\hat{u}_1(T)$	1.0377	0.1012	0.094
$\hat{u}_2(T)$	-0.0270	0.9882	0.076
$\hat{u}_3(T)$	-0.1660	-0.263	0.970

### 5. Application of the Method in Noisy Environment

In practical situations, there is always considerable process and measurement noise, which deteriorates the quality of the event recognition. To eliminate the effect of these disturbances a Kalman filter with constant gain matrix  $K$  has been used. However, Kalman filter can fail when there is an instrument error in the measuring system. In that case the to detect outliers the most sophisticated technique must be used (POTTER, 1977). The filter equations are the following:

$$\bar{x}^{n+1} = \Phi x^n + \frac{\Delta T}{2} (\Phi B \hat{u}^n + B \hat{u}^{n+1}), \quad (9)$$

$$\hat{x}^{n+1} = \hat{x}^n + K(\bar{x}^{n+1} - x^{n+1}), \quad (10)$$

where  $\hat{x}^{n+1}$  - estimated state vector from the  $x^{n+1}$  - measured state vector

The only problem with the application of Eq. (9) and (10) is that  $u(t)$  is not known before the filtering.

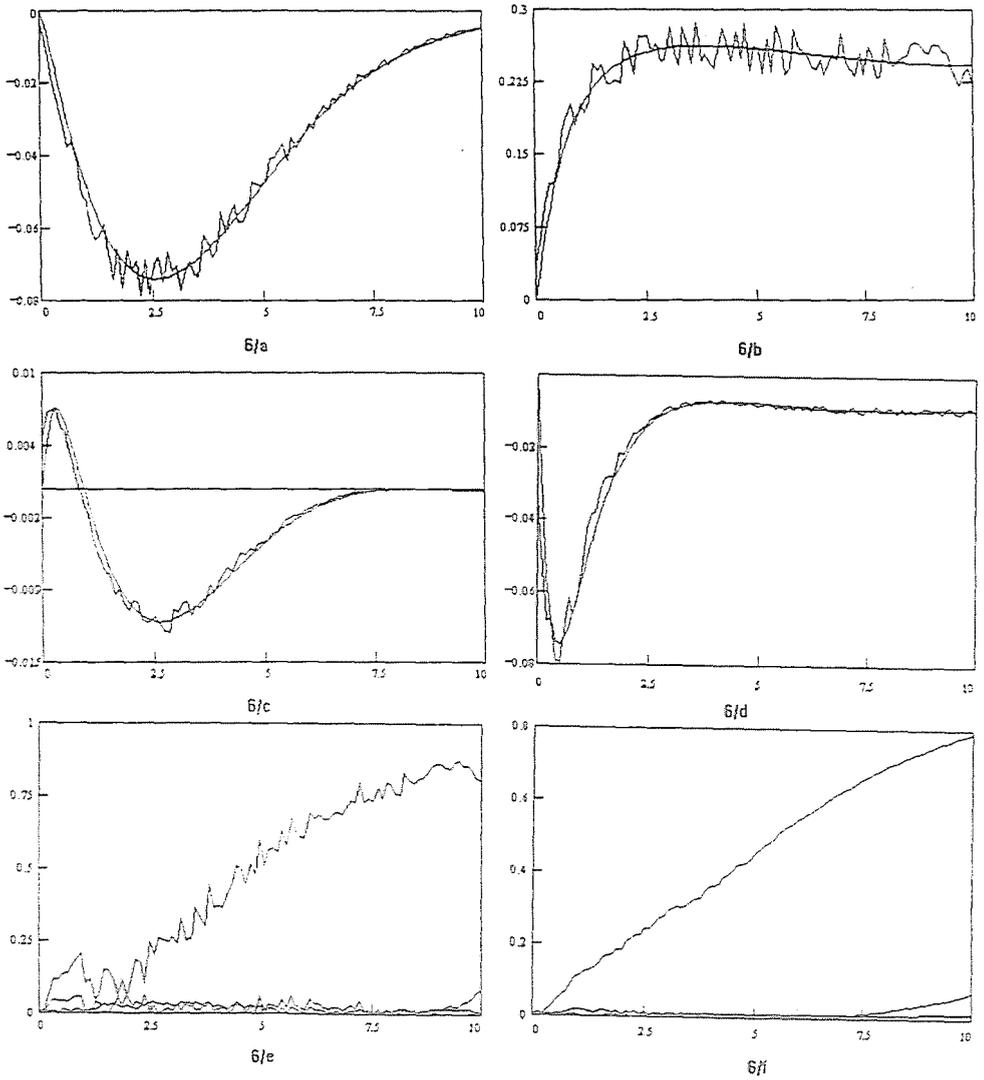
To overcome this difficulty, the following iteration technique can be used:

1. Compute  $\hat{u}(t)$  on the basis of unfiltered measurement,  $x^n$
2. Employ  $u(t) \equiv \hat{u}(T)$  to compute the estimated state vector  $\hat{x}^n$  from Eq. (9) and (10)
3. Compute  $\hat{u}(t)$  on the basis of  $\hat{x}^n$
4. Check  $\|\hat{u}(T)_{k+1} - \hat{u}_k(T)\| \leq \epsilon$ , where  
 $\hat{u}_k(T)$  - the indicator vector after  $k$ -th iteration  
 $\epsilon$  - error limit
5. If the condition in 4. is not satisfied, then make a new estimation with  $\hat{u}_{k+1}(T)$

The convergence of this iteration is very fast.

In the case of the second malfunction Fig. 6 shows the effectivity of the filtration after the first iteration. The elements of  $K$  are the following:

$$K = \langle 0.05 \quad 0.01 \quad 0.20 \quad 0.01 \rangle$$



*Fig. 6.* Effect of Kalman filter after first iteration  
 6.a-d Outlet concentrations in case of second malfunction  
 6.e. Indicator vectors in case of non-filtered data  
 6.f. Indicator vectors using Kalman filter

The different gains in  $\mathbf{K}$  ensure the correction of the modelling error in Eq. (9).

Random error, 10% of the nominal state values was considered as process noise.

## 6. Conclusion

The failure recognition method proposed above seemed to be very efficient even in the case of periodical operation like water treatment processes. According to our numerical experiments Kalman filter can successfully eliminate environmental and process noise and decrease modelling error. However, further investigations are necessary with more complex models where other factors, e.g. time delay can be considered.

## Appendix

In the case  $M > N$  the inverse solution presented in (PALÁNCZ, 1990) can be employed to express  $u(t)$  explicitly.

Now, we have more equations than variables, then therefore the parameter estimation technique should be applied to the problem.

Let us introduce the following variables

$$\mathbf{Y} \cong \frac{2}{\Delta T} \mathbf{x}^{n+1} - \frac{2}{\Delta T} \bar{\Phi} \mathbf{x}^n - \bar{\Phi} \mathbf{B} \mathbf{u}^n, \quad (\text{A1})$$

$$\mathbf{X} \cong \mathbf{B} \quad (\text{A2})$$

$$p \cong \mathbf{u}^n \quad (\text{A3})$$

Then

$$\mathbf{x}^{n+1} = \bar{\Phi} \mathbf{x}^n + \frac{\Delta T}{2} (\bar{\Phi} \mathbf{B} \mathbf{u}^n + \mathbf{B} \mathbf{u}^{n+1}) \quad (\text{A4})$$

can be considered as parameter estimation problem

$$\mathbf{Y} = \mathbf{X} \mathbf{p}. \quad (\text{A5})$$

Its solution is:

$$\mathbf{p} = (\mathbf{x}^T \mathbf{x})^{-1} \mathbf{x}^T \mathbf{y}, \quad (\text{A6})$$

consequently,

$$\mathbf{u}^{n+1} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \left( \frac{2}{\Delta T} \mathbf{x}^{n+1} - \frac{2}{\Delta T} \bar{\Phi} \mathbf{x}^n - \bar{\Phi} \mathbf{B} \mathbf{u}^n \right) \quad (\text{A7})$$

## Notation

$k$  – coefficient of the reaction kinetics

$s$  – substrate concentration

$t$  – time

$x$  – sludge concentration

$y$  – sludge substrate ratio

$V$  – reactor volume

$\dot{V}$  – flow rate

### *Greek letters*

$\alpha\beta$ – recirculation parameters

### *Indices*

1,2– first and second stage of the reactor

$o$  – inlet

$R$  – recirculation

## References

- BAILEY, J. E. – OLLIS, D. F. (1977): *Biochemical Engineering Fundamentals*, McGraw-Hill, 1977.
- KARDOS, J. – ZIRLIN, A. M. – BÖHME, B. – LORENZ, K. – SAJEW, A. (1984): *Modellierung und optimale Steuerung*, Akademie V., Berlin, 1984.
- PALÁNCZ, B. (1990) : Application of Inverse Solution to Recognition of Malfunction in Unit Operation Equipment. *IV. Chemical Engineering Conference*, Budapest, 1990. V.30-VI.1., Vol. II. p. 636.
- POTTER, J. E. – SUMAN, M. C. (1977): *Thresholdless Redundancy Management with Arrays of Skewed Instruments* NATO AGARDOGRAPH-224, pp. 15–25, 1977.
- WINKLER, M. (1980) : *Ein Beitrag zur Analyse und Steuerung biologischer Abwasserreinigungsanlagen*, Dissertation A, IH Köthen, 1980.