PERIODICA POLYTECHNICA SER. CIVIL ENG. VOL. 37, NO. 2, PP. 69-90 (1993)

GLOBAL GEOPOTENTIAL MODELS IN THE REGION OF HUNGARY

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Received: November 30, 1992

Abstract

The ability of some currently used high order geopotential models - OSU81, GPM2, IFE87E1 and 88E1, OSU86E and 86F, OSU89A and OSU89B - to recover the gravity anomaly field in Hungary has been tested. There were three ways used to compare the geopotential coefficient solutions. One of the tests is based on using the point gravity data published by RENNER and SZILÁRD (1959). A comparison of quasigeoid undulations computed from the potential coefficients with undulations derived from Doppler station positions has been performed. A detailed comparison of several geopotential solutions in terms of differences in gravity anomalies and quasigeoid undulations in the region of Hungary is also presented.

The results have shown that OSU89B model is the most suitable one for use as a reference in the region of Hungary. This model is able to recover gravity anomalies over 54 % of the area of Hungary within 5 mgal and over 81 % of the country with 10 mgal. However, large systematic discrepancies have been observed between different geopotential solutions in the eastern part of Hungary as well as in the eastern and southern Neighbourhood of Hungary due to the lack of real surface gravity data from Romania, and from both the former countries of the Soviet Union and Yugoslavia, etc. in geopotential coefficients computations.

Keywords: geoid, geopotential model, gravity anomaly, geoid undulation, doppler station coordinates.

Introduction

In the past few years a number of high degree ($n, m = 360$) spherical harmonic solutions of the Earth's gravity field have been computed. These geopotential models have a number of different applications in geodetic science and practice. Some of them are: the calculation of reference models for gravimetric predictions; calculation of gravimetric quantities ($N, \Delta g, \xi, \eta, T_{zz}$, etc) on a point by point basis; model for simulation studies involving future gravity field missions; study of the global spectra of the Earth's gravity field; calculation of gravimetric quantities at satellite altitude; geophysical investigations into the Earth's interior and oceanographic studies related to ocean dynamics [TSCHERNING, 1983].

A list of the most recent, high degree, combination solutions, as well as the latest 'satellite-alone fields' that are used in our investigations is given in *Table 1.* GEM-T1 and GEM-T2 are examples of satellite-alone solutions which are derived from satellite observations only. They contain harmonics up to degree 36 and 50, and contain 1406 and 2028 coefficients. Other global geopotential models are obtained by using altimetry and surface gravimetry added to the satellite-only solutions and they usually contain more coefficients. Examples of these are the GEM10C, OSU81, GPM2, OSU86E, OSU86F, OSU89A and OSU89B which contain harmonics up to degree 180, 200, 360 as shown in *Table 1* and contain 32942, 40602 and 130682 coefficients, respectively. The more coefficients there are in a model, the more precise the model usually is since it contains shorter wavelength information on the Earth's gravity field. For more information concerning the various global models, the references listed in *Table 1* are recommended.

The latest high degree global geopotential solutions are the OSU89A and OSU89B models complete to degree and order 360 corresponding to a spatial resolution of approximately 50 km [RAPP and PAVLIS, 1990]. They include accuracy estimates for all coefficients. These models have rigorously used the GEM-T2 coefficients and their variance-covariance matrix. Both models were developed through the combination of GEM-T2 coefficient set, Geos3/Seasat altimeter-derived anomalies and the latest terrestrial gravity data base [KIM and RAPP, 1990] (which firstly contains gravity information from Hungary). OSU89A uses GEM-T2 gravity information in areas where no other terrestrial data exists. In the OSU89B, gravity anomalies in unsurveyed areas were computed from the GEM-T2 model ($n = 2$ to 36) plus topographic and isostatic effects from degree 37 to 360. They are significantly better than the OSU86E/F models. The total geoid error to be expected from the use of OSU89B to degree 360 is ± 63 cm [RAPP and PAVLIS, 1990]. The gravity anomalies and quasigeoid undulations of OSU89B for Hungary and Neighbourhood in GRS80 system is illustrated in *Figs. 1* and *2*, respectively. The quasi-geoid undulations of the OSU89B geopotential model complete to degree and order 360 referring to GRS80 in Hungary range from 39.4 m to 46.4 m (*Fig. 2*) and yield the major part of the quasigeoid, cf. KENYERES (1991).

The purpose of this paper is to compare some current solutions in the region of Hungary. The potential coefficient solutions can be compared in many ways [KEARSLEY and HOLLOWAY, 1989; RAPP, 1986 and 1987; RAPP and CRUZ, 1986; RAPP and PAVLIS, 1990], some of which will be applied here.

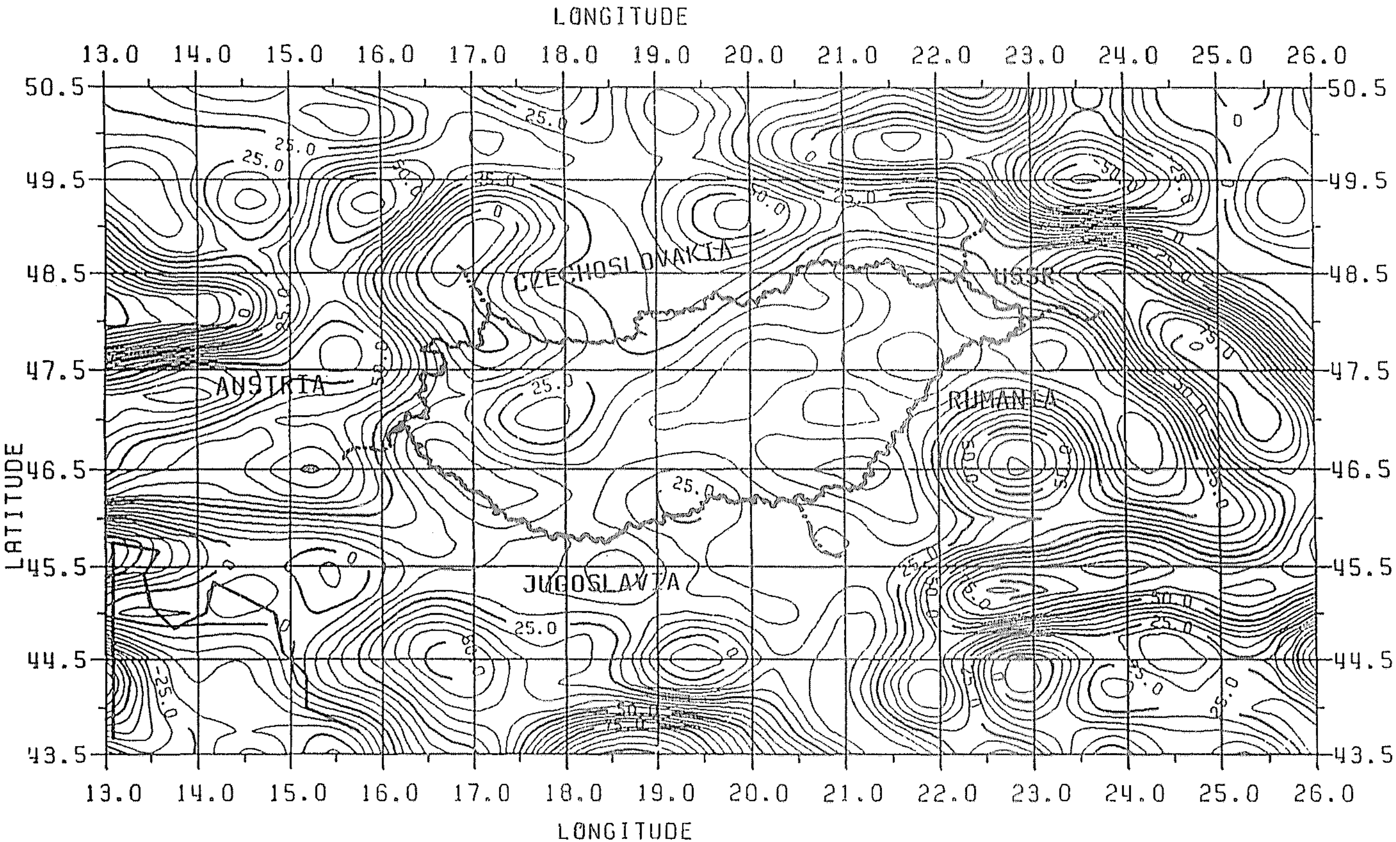

Fig. 1. Gravity anomalies of Hungary and Neighbourhood from the OSU89B geopotential solution complete to degree and order 360. The contour interval is 5 mgal. The data grid was 0.25°.

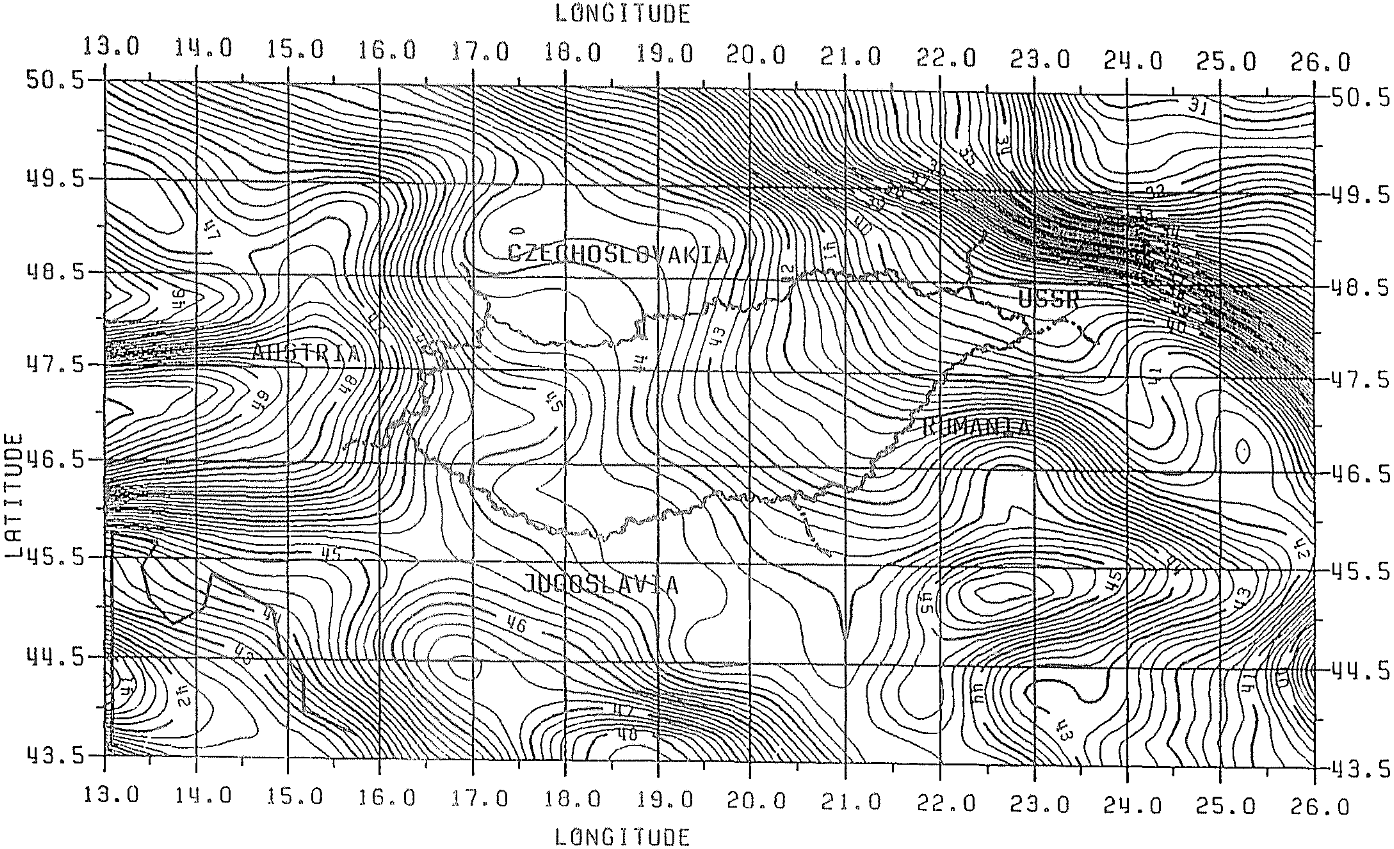

Fig. 2. Quasigeoid undulation of Hungary and Neighbourhood implied by the OSU89B geopotential solution complete to degree and order 360. The contour interval is 0.2 m. The data grid was 0.25°.

Table 1
Some Currently Used Global Geopotential Models

Selected Global Geopotential Models			
Field	Author	Date	Nmax
GEMT1	Marsh, J. G. et al.	1988	36
GEMT2	Marsh, J. G. et al.	1989	50*
GEM10C	Lercs, F. J. et al.	1981	180
OSU81	Rapp, R. H.	1981	180
GPM2	Wenzel, H. -G.	1985	200
IFE87E1	Basic, T.	1989	200
IFE88E1	Basic, T.	1989	360
OSU86E	Rapp, R. H. and J. Y. Cruz	1986	360
OSU86F	Rapp, R. H. and J. Y. Cruz	1986	360
OSU89A	Rapp, R. H. and N. k. Pavlis	1989	360
OSU89B	Rapp, R. H. and N. k. Pavlis	1989	360

Table 2

Geopotential model		Range of the Residual Gravity Anomalies in mGal ($\pm m\delta_g$) 0–5	5–10	10–15	15–20	> 20
GEMT1	– 36	82 (16%)	79 (16%)	99 (20%)	124 (24%)	124 (24%)
GEMT2	– 50	69 (14%)	74 (14%)	103 (20%)	121 (24%)	141 (28%)
GEM10C	– 180	111 (22%)	125 (25%)	114 (22%)	91 (18%)	67 (13%)
OSU81	– 180	109 (21%)	119 (24%)	112 (22%)	87 (17%)	81 (16%)
GPM2	– 200	136 (27%)	148 (29%)	103 (21%)	66 (13%)	55 (10%)
IFE87E1	– 200	144 (28%)	120 (24%)	95 (19%)	77 (15%)	72 (14%)
IFE88E1	– 360	144 (28%)	143 (28%)	102 (20%)	62 (12%)	57 (12%)
OSU86E	– 360	194 (39%)	165 (32%)	80 (16%)	44 (8%)	25 (5%)
OSU86F	– 360	202 (40%)	158 (31%)	75 (15%)	49 (9%)	24 (5%)
	– 180	142 (28%)	151 (30%)	109 (21%)	61 (12%)	45 (9%)
	– 200	195 (38%)	140 (28%)	100 (20%)	43 (8%)	30 (6%)
OSU89A	– 360	268 (53%)	145 (28%)	50 (10%)	29 (6%)	16 (3%)
	– 180	188 (37%)	138 (27%)	96 (19%)	53 (10%)	33 (7%)
	– 200	212 (42%)	155 (30%)	81 (16%)	37 (7%)	23 (5%)
OSU89B	– 360	276 (54%)	138 (27%)	52 (10%)	27 (6%)	15 (3%)
	– 180	186 (37%)	151 (30%)	93 (18%)	44 (9%)	34 (6%)
	– 200	223 (44%)	149 (29%)	78 (16%)	32 (6%)	26 (5%)

In the first way, we used the statistics of residual gravity anomaly $\delta g(i)$,

$$\delta g(i) = \Delta g(i) - \Delta g^*(i), \tag{1}$$

where $\Delta g(i)$ is the free-air gravity anomaly from gravimetric survey and $\Delta g^*(i)$ is the gravity anomaly generated from the geopotential model.

For these tests, $\Delta g^*(i)$ was generated on 0.25° grid across the Hungarian region. A value of $\Delta g^*(i)$ was estimated by interpolation at each gravity point in the used data set given by RENNER and SZILÁRD (1959), and $\delta g(i)$ were obtained by *Eq. (1)*. The $\delta g(i)$ were then analysed to obtain the mean and root mean square for the population.

In the second way, the geopotential fields have been tested through comparison of Doppler station quasigeoid undulations (ζ_D) with undulations from various geopotential models (ζ_G) by

$$\delta\zeta(i) = \zeta_D(i) - \zeta_G(i). \tag{2}$$

This can only be done after the Doppler station co-ordinates have been converted to a geocentric, true scale system. The accuracy of a geopotential coefficient model may also be judged in this way.

In the third way, differences in gravity anomalies and undulations between different geopotential solutions were statistically analyzed in the region of Hungary.

For the generation of the reference gravity anomalies and geoid undulations from the geopotential model coefficients, the efficient algorithm of Rizos was used (see e.g. TSCHERNING et al., 1983).

Discussion of Results for Testing the Geopotential Models

A number of tests have been carried out on the ability of various geopotential models to represent the gravity field in the area of Hungary and Neighbourhood, in order to establish their value as a reference field for geoid solutions in the region. We have tested the models for both the point gravity data and the geoid (or quasigeoid) heights derived from Doppler positions in Hungary. (Note that according to an investigation by BIRÓ (1961), the difference between geoid and quasigeoid heights in Hungary reaches values of maximum a few cm). We have also compared some high degree solutions to each other, for interest.

Comparison of Geopotential Model and Surface Point Values of Δg

The test of the fit of high order geopotential models to the gravity field in Hungary is based on using the point data published by RENNER and SZILÁRD (1959). These gravity data were obtained on 16 first order base stations and on 492 second order base stations while establishing a network of gravity bases across Hungary in the years 1950-1955. *Fig. 3* shows the location of the gravity stations. The average spacing between gravity stations is 15 to 20 km. The free-air gravity anomalies are shown in *Fig. 4*. This data set was transformed into the GRS80 and IGSN system.

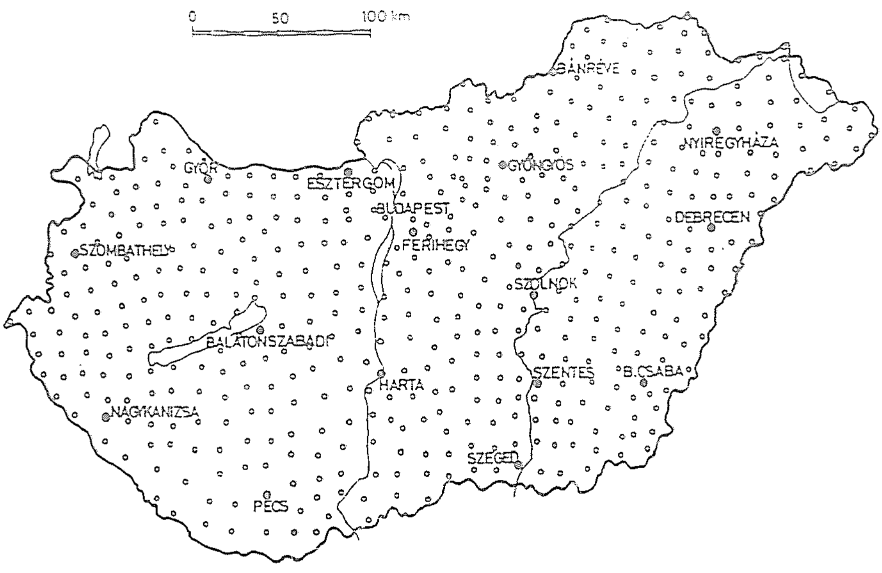

Fig. 3. Distribution of point gravity data in Hungary used for test computation (after Renner and Szilárd, 1959)

In the comparison, the global geopotential solutions presented in *Table 1* are used. The 'fitness' of these models by computing reference gravity anomalies and comparing them to terrestrial gravity anomalies over the Hungarian territory has been investigated. Comparisons were made at all gravity points by calculating the residual gravity anomalies. They were then placed into five categories (bins) in increasing magnitude. The results are summarized in *Table 2*.

The results of this table show that the ability of high order geopotential models to model the geoid and terrestrial gravity varies according to

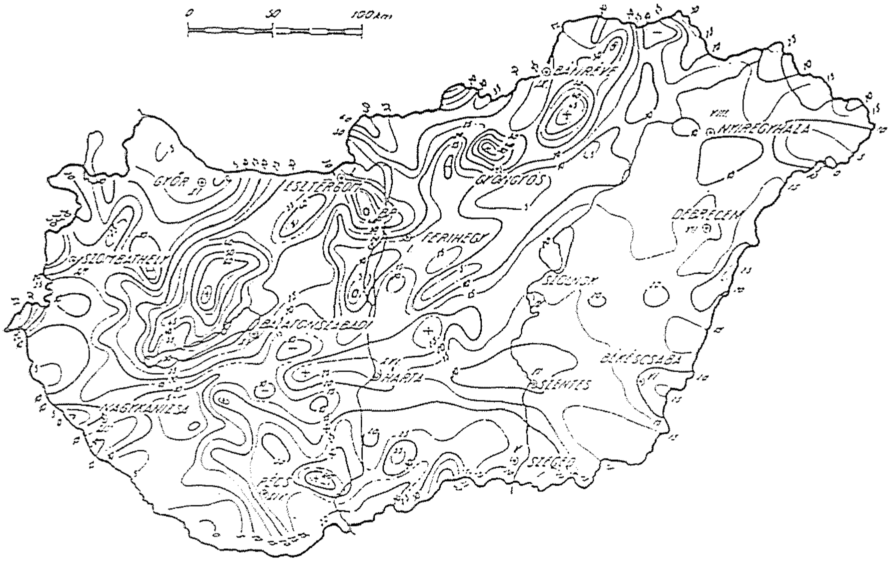

Fig. 4. Representation of the free-air gravity anomalies computed by the use of Int. Gravity Formula (1930) from point gravity values referred to the Potsdam gravity system. The contour interval is 5 mgal (after Renner and Szilárd, 1959)

the model used, the degree and order of the model and the behaviour of the quasigeoid in the region in which the computation is made. The results have also been summarized in the histograms in *Fig. 5*, allowing a direct comparison between the models.

One can see that the OSU89B model is able to recover gravity anomalies over 54 % of the Hungarian territory to within 5 mgal and over 81 % of the country to within 10 mgal. Tests show that the best agreement occurs when the geopotential model is taken to its maximum degree and order.

The mountainous regions, the north-east part of Hungary and the central region of western part of Hungary are poorly represented by the high order geopotential models. This is most probably due to short wavelength quasigeoidal features in these areas not detected by the geopotential models. *Fig. 6* shows the residual gravity anomalies based on OSU86F model to degree and order 360 [ÁDÁM and DENKER, 1990].

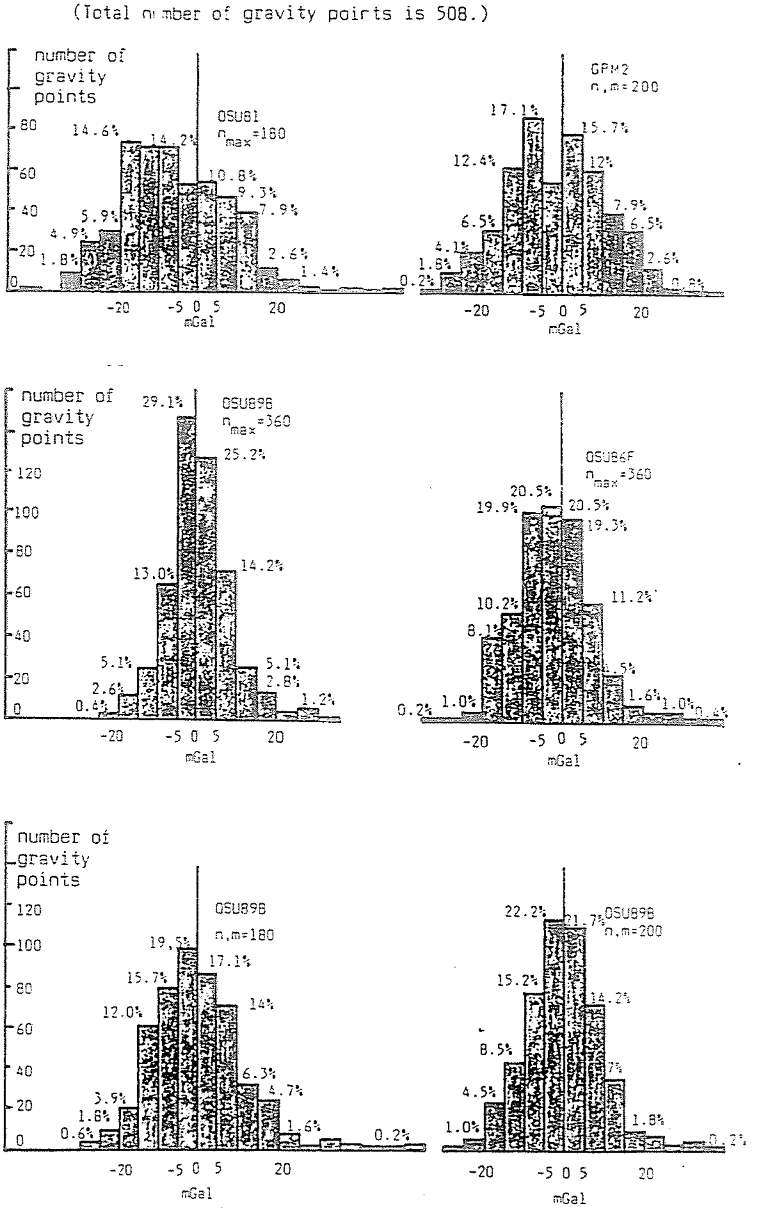

Fig. 5. Distribution of residuals of Geopotential Model minus observed point values of Δg

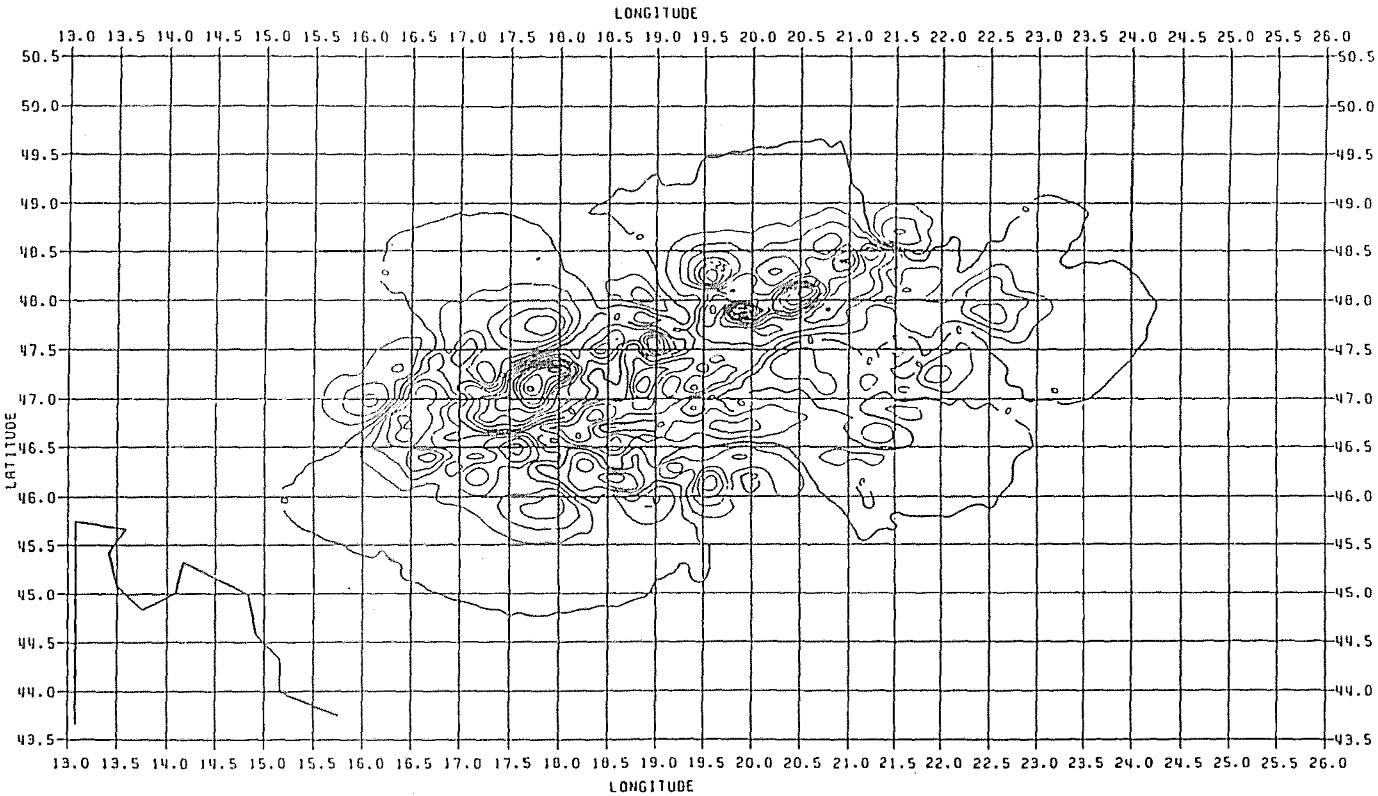

Fig. 6. A representation of the residual gravity anomaly field: OSU86F complete to degree and order 360 minus point gravity anomalies published by Renner and Szilárd (1959). Values are given on 0.1° × 0.1° grid. Contour interval is 5 mgal. Gravity anomalies refer to the Geodetic Reference System 1980 (GRS80)

Table 3 shows the statistics of the residual gravity anomalies. The mean values and RMS errors are small in the case of OSU89 models.

The results of this test comparison show that the OSU89B model describes the quasigeoid most closely in Hungary. This model to its degree and order 180 or 200 fits the gravity anomaly field of Hungary in the same measure or better than the other previous OSU geopotential solutions.

Note that a number of detailed tests have been carried out on the ability of various geopotential models to represent to gravity field in the Australian region by KEARSLEY and HOLLOWAY (1989), KEARSLEY and FORSBERG (1991) and KEARSLEY (1991). The results of their tests show a better fit of geopotential models to the gravity of Australia than to that of Hungary. The reason for the less favourable recovery of gravity field in Hungary by these models is most probably due to the lack of real surface gravity data from the eastern and southern Neighbourhood of Hungary (the former Soviet Union, Romania, and former Yugoslavia, etc.) in geopotential coefficients computations.

Table 3

Geopotential modell		Range of the Residual Gravity Anomalies [mGal]				
		MEAN	STD.DEV.	RMS	MIN.	MAX.
GEMT1	– 36	−11.72	11.23	16.23	−33.23	50.19
GEMT2	– 50	−12.79	11.28	17.05	−34.35	50.24
GEM10C	– 180	−8.41	11.34	14.11	−37.89	52.49
OSU81	– 180	−5.59	13.75	14.84	−41.85	49.09
GPM2	– 200	−1.28	12.78	12.84	−40.95	43.77
IFE87E1	– 200	−3.52	13.12	13.58	−44.81	45.77
IFE88E1	– 360	−0.64	12.84	12.86	−42.57	61.27
OSU86E	– 360	−1.39	10.64	10.73	−30.74	51.53
OSU86F	– 360	−1.94	10.61	10.78	−31.21	51.18
	– 180	−2.53	12.45	12.70	−32.52	52.89
	– 200	−2.30	10.92	11.16	−31.36	50.26
OSU89A	– 360	0.59	8.78	8.80	−24.10	60.21
	– 180	−0.43	11.84	11.84	−32.98	54.61
	– 200	−0.38	10.47	10.48	−26.76	56.27
OSU89B	– 360	0.82	8.53	8.57	−23.49	60.33
	– 180	0.07	11.62	11.62	−29.30	55.25
	– 200	0.20	10.39	10.39	−26.65	57.76

Comparison of Doppler-derived Undulations with Values from Different Geopotential Models

We next turn to a comparison of the quasigeoid undulations derived from the various geopotential models with undulations derived from Doppler-derived positions in Hungary. *Fig. 7* shows the distribution of 24 Doppler stations in Hungary. In all our comparisons we used the parameters to convert the Doppler station co-ordinates from the Broadcast Ephemeris Doppler system to a geocentric system given in [ÁDÁM, 1987a and 1987b].

Fig. 7. Distribution of Doppler stations in Hungary. (Hungarian Doppler Observation Campaign in 1980, 1982 and 1985)

The comparative computations – which are helpful in revealing the relative strengths between solutions – have been carried out with the OSU81, OSU86F, OSU89A and OSU89B models. The mean difference (Doppler minus model) and the RMS of the difference is given in *Table 4* for different Doppler program and adjustment solutions of each Doppler observation campaign.

From this table one can conclude that in Hungary, from among the degree 180 solutions, the OSU89 models are better than the OSU81 model. The degree 360 fields show a slight improvement over their 180 counterparts. Among the 180 and 360 solutions, both OSU89A and OSU89B solutions give essentially the same results. Concerning the 360 fields, both

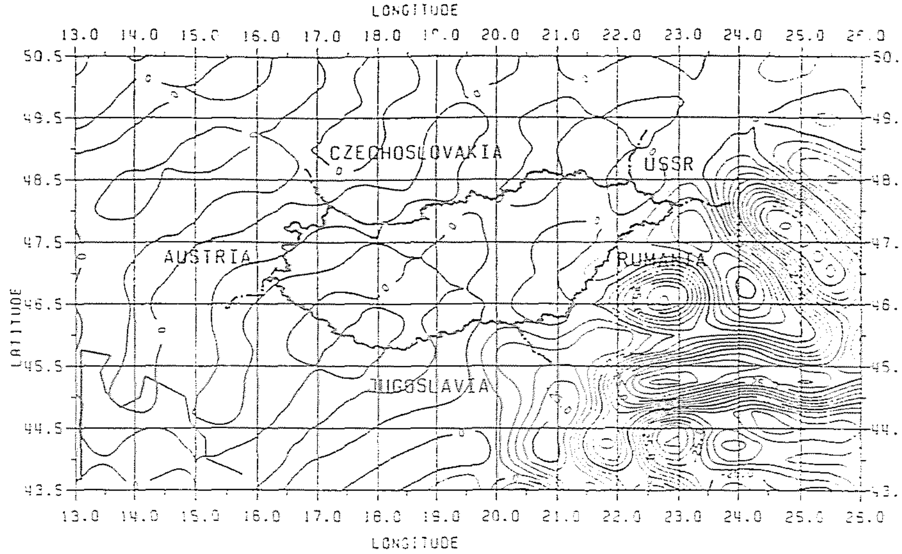

Fig. 8. Gravity Anomaly Differences: OSU89B minus OSU89A complete to degree and order 360. The contour interval is 5 mgal. The data grid was 0.25°.

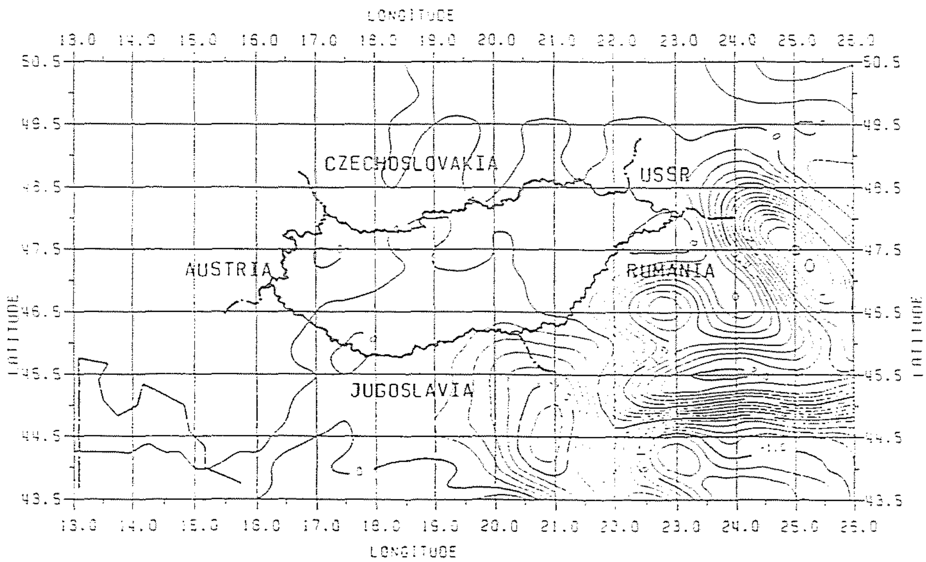

Fig. 9. Undulation Differences: OSU89B minus OSU89A complete to degree and order 360. The contour interval is 0.2 m. The data grid was 0.25°.

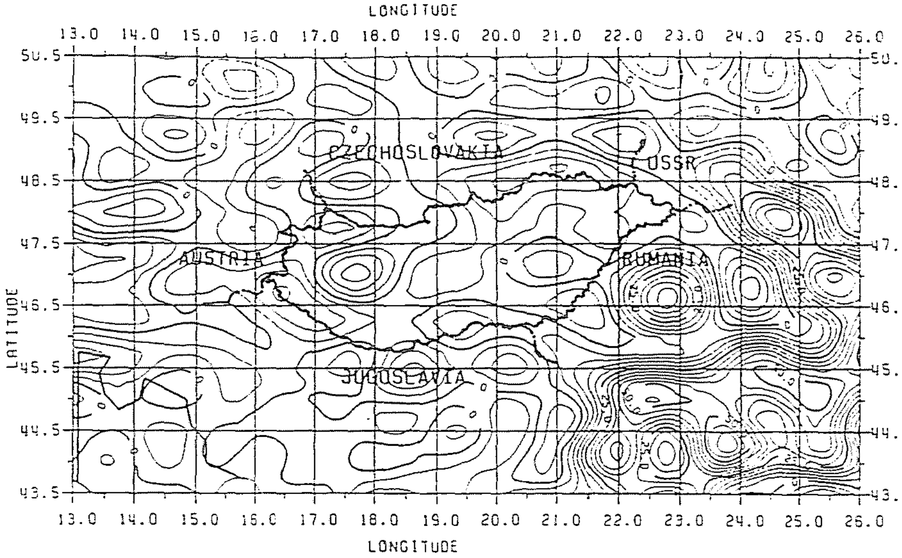

Fig. 10. Gravity Anomaly Differences: OSU89B minus OSU86F complete to degree and order 360. The contour interval is 5 mgal. The data grid was 0.25°.

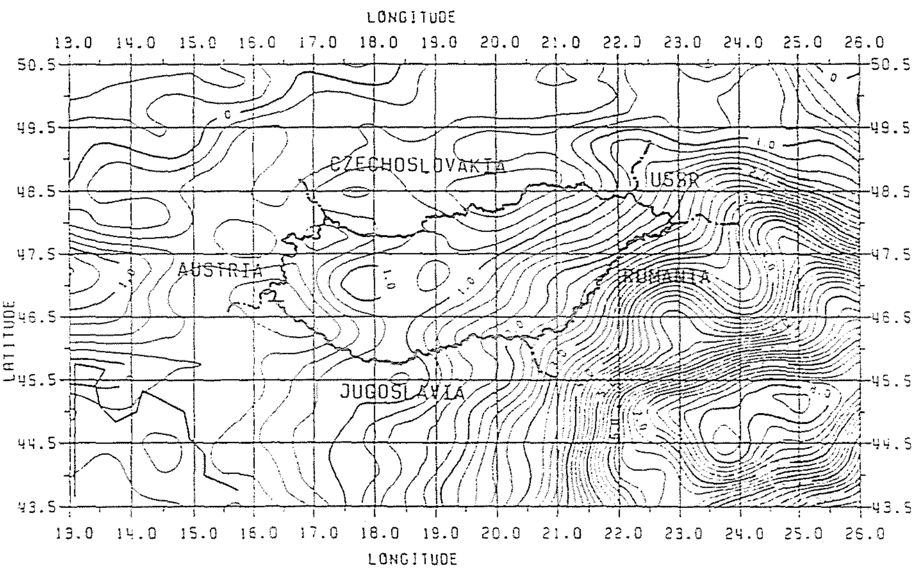

Fig. 11. Undulation Differences: OSU89B minus OSU86F complete to degree and order 360. The contour interval is 0.2 m. The data grid was 0.25°.

OSU89 geopotential models are more accurate than the OSU86F model. Considerable improvement occurs at the Doppler positions in hilly area and in the eastern part of Hungary. The OSU89 models in degree 180 field are even better than the OSU86F model in degree 360 field.

A comparison with some local quasigeoid solutions is also included in *Table 4*, for interest. In this test, OSU86FR solution is derived by FFT using the OSU86F geopotential model up to degree 360 as a reference and the terrestrial point gravity values published by RENNER and SZILÁRD (1959) [ÁDÁM and DENKER, 1990]. The Hungarian Quasigeoid Solutions HGQ90A and HAQ90A were determined by KENYERES (1991). HGQ90A is a gravimetric, HAQ90A is an astrogravimetric quasigeoid solution. Both solutions are based on the OSU89B spherical harmonic model up to degree 180 as a reference and terrestrial gravity data with very dense distribution in Hungary. Furthermore, a set of astrogeodetic data is used for HAQ90A quasigeoid solution due to the applied Molodensky's astrogravimetric method. All three local quasigeoid solutions show (in some cases considerable) improvement over their geopotential model counterparts.

Comparison of Gravity Anomalies and Undulations of Different Geopotential Models

The next set of comparison is to indicate the regional, root mean square, geoid undulation and gravity anomaly differences between several solutions. The geoid undulations (height anomalies) and gravity anomalies were computed with a direct evaluation on a $0.25°$ grid in the area $[40° < \phi < 55°; 10° < \lambda < 30°]$ involving Hungary and Neighbourhood. A total of 4941 $0.25°$ grid values each for geoid undulations and gravity anomalies were computed. Differences in gravity anomalies and undulations between geopotential solutions computed by *Eqs.* (1) and (2) were statistically analysed. The results of these comparisons are summarized in *Table 5*.

Although the difference between various solutions is small, there are large discrepancies that reach some tens of mgals in gravity anomalies and some metres in undulations. For example, between OSU89B and OSU86F solutions complete to degree and order 360, the maximum and minimum difference in gravity anomalies and undulations are 64.41 mgal and -65.86 mgal, respectively, 8.34 m and -2.31 m in this regional area. Between OSU89B ($n, m = 180$) and OSU81 complete to degree and order 180, the corresponding maximum and minimum differences, in gravity anomalies and undulations are 59.91 mgal and -108.47 mgal, respectively, 7.66 m and -4.03 m (*Table 5*).

Table 4
Comparison of Doppler Undulations in Hungary with Values from Geopotential Models
(m)

Gravity Field			HDOC80				HDOC80				HDOC85			
			SADOSA	SPPENC	GEODOP(TL)	GEODOP(SP)	SADOSA	SPPENC	GEODOP(ML)	GEODOP(SP)	SADOSA	SPPENC	GEODOP(ML)	GEODOP(SP)
OSU81	(180)	MEAN	−1.12	−3.40	−3.83	−1.30	3.46	0.92	1.06	1.69	2.30	2.70	−1.72	−0.92
		RMS	±0.70	±0.79	±1.05	±1.03	±0.82	±1.43	±0.94	±1.14	±1.10	±1.86	±0.89	±1.44
OSU89A	(180)	MEAN	−1.03	−3.30	−3.74	−1.20	3.44	0.90	1.04	1.67	2.21	2.81	−1.80	−1.09
		RMS	±0.30	±0.85	±0.73	±1.14	±0.57	±1.21	±0.67	±0.89	±0.92	±1.48	±0.72	±1.26
OSU89B	(180)	MEAN	−1.01	−3.29	−3.42	−1.18	3.41	0.86	1.00	1.64	2.20	2.47	−1.82	−1.08
		RMS	±0.36	±0.86	±0.64	±1.17	±0.59	±1.22	±0.69	±0.93	±0.94	±1.67	±0.75	±1.28
GPM2	(200)	MEAN	−1.24	−3.52	−3.96	−1.42	3.26	0.62	0.85	1.49	2.03	2.42	−1.99	−1.25
		RMS	±0.56	±1.04	±0.75	±1.32	±0.75	±1.42	±0.85	±1.06	±1.04	±1.89	±0.82	±1.20
OSU86F	(360)	MEAN	−2.29	−4.57	−5.00	−2.50	2.17	0.26	−0.22	0.41	1.32	1.80	−2.70	−2.16
		RMS	±0.71	±0.71	±1.27	±1.31	±0.65	±1.43	±1.11	±1.28	±0.64	±1.50	±0.57	±1.15
OSU89FR	(360)	MEAN	−2.56	−4.84	−5.28	−2.77	1.91	0.51	−0.49	0.14	1.13	1.61	−2.89	−2.35
		RMS	±0.62	±0.65	±1.34	±1.36	±0.48	±1.29	±1.06	±1.22	±0.47	±1.18	±0.28	±1.04
OSU89A	(360)	MEAN	−1.16	−3.44	−3.87	−1.34	3.35	0.81	0.95	1.58	2.19	2.58	−1.83	−1.11
		RMS	±0.39	±0.89	±0.85	±1.23	±0.43	±1.16	±0.61	±0.83	±0.85	±1.59	±0.61	±1.15
OSU89B	(360)	MEAN	−1.14	−3.42	−3.86	−1.32	3.35	0.81	0.94	1.54	2.19	2.58	−1.78	−1.11
		RMS	±0.38	±0.88	±0.85	±1.22	±0.45	±1.17	±0.63	±0.78	±0.84	±1.60	±0.67	±1.16
HGQ90A		MEAN	−0.44	−2.72	−3.15	−0.39	4.01	1.47	1.61	2.24	2.86	3.26	−1.16	−0.45
		RMS	±0.35	±0.86	±0.85	±1.34	±0.52	±1.21	±0.67	±0.91	±0.86	±1.62	±0.62	±1.16
HAQ90A		MEAN	−0.40	−2.69	−3.12	−1.59	4.12	1.58	1.71	2.35	3.10	3.50	−0.92	−0.19
		RMS	±0.46	±0.75	±1.07	±1.28	±0.39	±1.21	±0.80	±1.00	±0.57	±1.42	±0.36	±1.21

Table 5
Geopotential Coefficient Differences in Terms of Gravity Anomalies (mgal) and Geoid Undulations (m) in the Region ($10^\circ \leq \lambda \leq 30^\circ$; $40^\circ \leq \varphi \leq 55^\circ$).

Geopotential solution	Δg [mGal] MEAN	RMS	MIN.	MAX.	N [m] MEAN	RMS	MIN.	MAX.
1 OSU86F (360) – OSU86E (360)	−0.46	6.88	−47.94	39.58	−0.19	0.62	−3.45	2.75
2 – IFE88E1 (360)	−0.89	11.92	−78.89	73.17	−0.39	0.80	−4.02	2.29
3 OSU86F (180) – OSU81 (360)	−0.94	11.48	−71.65	49.29	−0.35	1.30	−4.27	4.01
4 OSU86F (200) – GPM2 (200)	−1.95	10.04	−52.58	30.17	−0.95	1.66	−5.46	1.43
5 OSU86F (360) – OSU86F (180)	−0.08	9.63	−76.10	49.04	0.00	0.28	−1.96	1.26
6 – GEMT1 (360)	−0.21	24.83	−134.94	94.59	−0.10	2.86	−8.79	8.57
7 – OSU81 (180)	−1.03	14.88	−90.99	72.29	−0.35	1.32	−4.15	4.53
8 – GPM2 (200)	−2.00	12.82	−74.63	47.48	−0.95	1.67	−5.57	1.45
9 – IFE87E1 (200)	−2.20	12.11	−71.71	66.45	−0.84	1.37	−4.90	2.29
10 OSU89B (360) – OSU89A (360)	−0.17	7.71	−41.49	61.96	−0.05	0.45	−1.62	3.06
11 – OSU86F (360)	2.12	12.79	−65.86	64.41	0.97	2.24	−2.31	8.34
12 – OSU86E (360)	1.67	12.60	−67.84	75.83	0.78	2.00	−2.65	7.89
13 – IFE88E1 (360)	1.24	14.85	−87.33	77.36	0.58	1.93	−3.67	7.89
14 OSU89A (360) – OSU86E (360)	1.83	11.04	−68.19	32.78	0.83	2.00	−2.70	6.25
15 – OSU86F (360)	2.29	12.69	−66.21	72.23	1.02	2.26	−2.36	9.10
16 – OSU89A (180)	−0.05	11.95	−91.84	64.02	0.00	0.32	−2.33	1.74
17 – OSU89A (200)	−0.04	10.86	−81.08	50.73	0.00	0.27	−1.96	1.31
18 OSU89A (200) – OSU89A (180)	0.00	4.96	−17.19	19.33	0.00	0.17	−0.59	0.67
19 OSU89B (360) – OSU89B (180)	−0.08	12.85	−89.92	63.33	0.00	0.35	−2.29	1.64
20 – OSU89B (200)	−0.04	11.58	−79.38	52.18	0.00	0.29	−1.92	1.29
21 OSU89B (200) – OSU89B (180)	−0.04	5.53	−20.09	18.75	0.00	0.19	−0.69	0.65
22 – GPM2 (200)	0.16	11.29	−55.49	46.83	0.02	1.17	−3.91	4.80
23 – OSU89A (200)	−0.17	5.98	−31.49	41.04	−0.05	0.44	−1.75	2.58
24 OSU89B (180) – OSU89A (180)	−0.14	5.52	−29.78	34.72	−0.05	0.43	−1.71	2.36
25 – OSU81 (180)	1.17	16.50	−108.47	59.91	0.62	2.33	−4.03	7.66

In order to obtain more information, we have plotted regional undulation and anomaly difference maps between geopotential solutions. These maps were made by contouring data gridded at a $0.25^\circ \times 0.25^\circ$ interval.

In *Figs. 8* and *9*, the gravity anomaly and undulation differences between OSU89B and OSU89A are illustrated. Large systematic discrepancies occur in the Carpathians (Transylvanian Alps) within the investigated area having a maximum and minimum difference of 61.96 mgal and −41.49 mgal in gravity anomalies and of 3.06 m and −1.62 m in undulations, respectively. The differences between these two solutions depend only on the way data in gap areas were treated.

Fig. 10 shows a map of gravity anomaly difference between the OSU89B and OSU86F solutions with large discrepancies noted. A similar map is shown in *Fig. 11* for the undulation differences between these two solutions. The eastern part of the region attracts large discrepancies. Increasing systematic discrepancies between these two solutions are seen by going from the eastern part of Hungary to the Carpathian Mountains in Romania. Inconsistencies between these solutions in undulations reach values of two metres in the eastern part of Hungary. Most of the large differences between OSU89B and OSU86F solutions are due to the use of different data and the difference in the treatment of data gaps.

In *Figs. 12* and *13*, the corresponding comparisons are given between the OSU89B solution (up to $n, m = 180$) and the OSU81 solution. Although these plots are very similar to *Figs. 10* and *11*, respectively, there is an important difference to be noted from the comparison of 12 with 10 (or 11 to 13). The differences between OSU89B and OSU86F are smaller in the western part of the region as compared to the differences between OSU89B (up to $n, m = 180$) and OSU81. Such behaviour should be expected since OSU81 fails to recover higher frequency features of the gravity field, while the opposite is true for OSU89B and OSU86F.

From the examination of *Figs. 8, 9, 10, 11, 12* and *13*, it becomes apparent that the large systematic differences in both gravity anomalies and undulations between different geopotential solutions occur in the eastern Neighbourhood of Hungary due to the difference in the treatment of data gaps in this region. The discrepancies in this area (especially in Carpathians) are quite large (e.g. 7 m undulation differences are observed in *Figs. 11* and *13*).

Conclusions

Hungary is in need of a precise quasigeoid, primarily to enable the use of the GPS for surveying engineering (e.g. levelling). In the local geoid

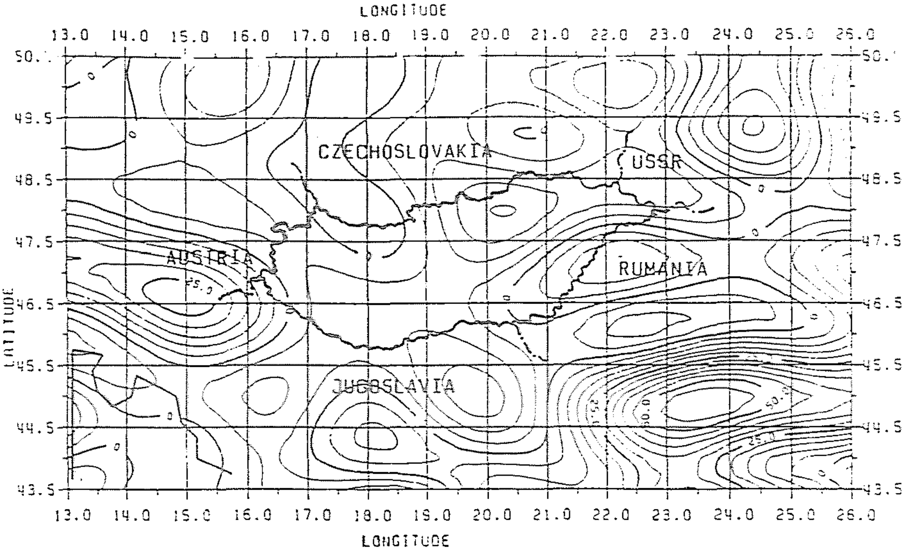

Fig. 12. Gravity Anomaly Differences: OSU89B ($n, m = 180$) minus OSU81 ($n, m = 180$). The contour interval is 5 mgal. The data grid was 0.25°.

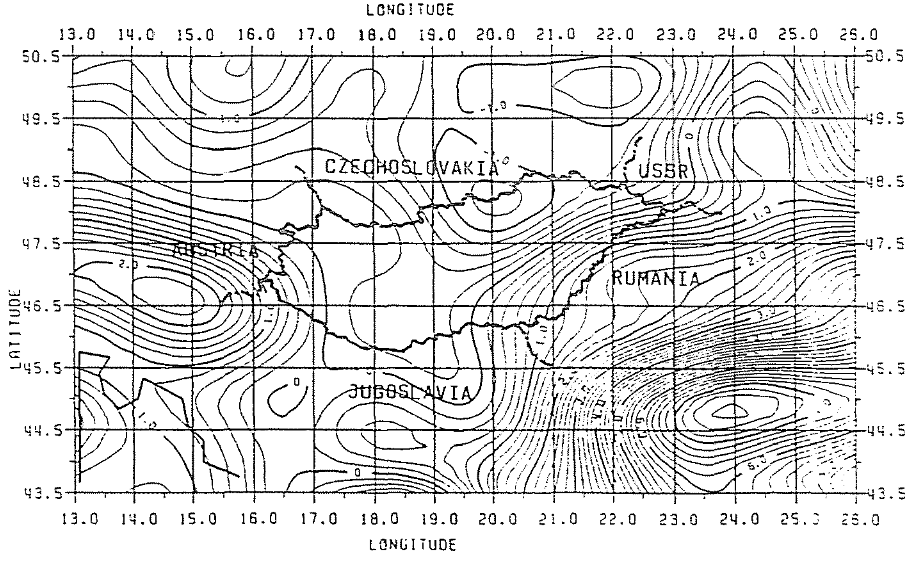

Fig. 13. Undulation Differences: OSU89B ($n, m = 180$) minus OSU81 ($n, m = 180$). The contour interval is 0.2 m. The data grid was 0.25°.

(quasigeoid) determination, we need a geopotential solution as the reference model. Therefore it is necessary to decide which model describes the geoid most closely in Hungary.

An examination is presented to show how well the gravity anomalies and quasigeoid undulations are able to be recovered in Hungary by using different high order global geopotential solutions. The results showed that OSU89B model is most suitable for use as a reference in the region of Hungary. However, large systematically discrepancies have been observed between different geopotential solutions in the eastern part of Hungary as well as in the eastern Neighbourhood of Hungary. Therefore, in order to have an appropriate absolute location of the Hungarian quasigeoid solutions, a number of five EUREF GPS stations will be occupied in Hungary. For different geodetic purposes including geoid determination demands, we have designed a national GPS control network to realize in October–November, 1991 [BORZA, 1991; CZOBOR, 1991]. This GPS network with EUREF positions should control and improve the Hungarian quasigeoid solutions.

Acknowledgement

Most computations of this work were carried out while the author was staying at the Department of Geodetic Science and Surveying, The Ohio State University, Columbus, invited by Professor Ivan I. Mueller. Special thanks go to him to offer the possibility for me to perform gravimetric computations beside my Space VLBI basic research work. I would like to also thank Professor Richard H. Rapp for sharing his expertise in gravimetric geodesy with me through his lectures and for providing the OSU89 geopotential model coefficients. *Figs. 1, 2, 6* and *8* to *13* were prepared with the GSPP plot package made by H. SÜNKEL (1980). Computer support at The Ohio State University was provided by the Instruction and Research Computer Center through the Department of Geodetic Science and Surveying.

Financial support for this study has partly been given by the Ministry of Agriculture and by the National Science Foundation of the Hungarian Academy of Sciences in the frame of OTKA 5-204 'Global and local geoid examinations' supervised by Professor Péter Bíró.

This paper was presented at the XXth IUGG/IAG General Assembly (Vienna, Austria, August 11-24, 1991) as a poster presentation and it is included in the Final Report of a research project titled 'Global and Local Geoid Investigations' for the period 1988-1991 supported by National Research Foundation (OTKA) contract No. 5-204.

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