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TREND MODELS IN THE LEAST-SQUARES PREDICTION OF FREE-AIR GRAVITY ANOMALIES[1]

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Abstract

The different wavelength components of the anomalous gravity field were treated as trend, signal and noise parts of this field. Since signal models have already been investigated extensively by others, therefore the role of deterministic information used in the prediction process was emphasized. In order to improve the reliability of prediction, several trend models were tested on regional and local data sets in the Pannonian Basin. The results show that the prediction errors can be significantly reduced by applying simple and generalized physical trend models, although it is a laboursome task to produce high quality prediction below ±1 mgal R.M.S., even if the data point density is high (e.g. 1 point/km^2) Since the method of Least-Squares Prediction is not an automatic process (that is its use requires an a priori analysis of the physical-statistical content of input data), beyond the practical results useful information can be gained from the solution to a prediction problem about the features of the gravity field.

Keywords: free-air gravity anomaly, trend determination, topographical and crustal information, high quality prediction.

Introduction

The determination of the geoid in the error range of a few centimeters requires accurate gravity data with a deviation less than ±1 mgal. This accuracy requirement can be easily fulfilled if the usual precision (±0.01 mgal) of relative gravity measurements is considered. Due to the geophysical assumptions used in the computations of gravity anomalies, however, the final reliability of the gravity material processed in geodetic computations is generally less than ±0.1–0.5 mgal. Obviously, this deviation range refers to points where measurements were carried out, so the accuracy decreases if a new set of data (e.g. a set of gridded values) is derived from the given set of gravity anomalies. The gridding of scattered data is un-

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avoidable if new and very efficient methods, like FFT and Fast Collocation are used in the determination of the geoid. Therefore, the investigation of interpolation methods used in gravity field prediction is very important because of the high accuracy requirements set up for the derived values.

There are many methods to create a regular grid from scattered data and the Least-Squares Prediction (generally called Least-Squares Collocation) is only one but efficient method among them. Its power lies in its analytical approach. A measured or given quantity (e.g. Δg_i) is considered as a sum of (1) a deterministic trend $\mathbb{A}_i\mathbb{X}$, (2) a stochastic signal s_i, and (3) a random noise n_i:

$$\Delta g_i = \mathbb{A}_i\mathbb{X} + s_i + n_i, \tag{1}$$

Such a presentation *Eq.* (1) provides a possibility to connect (1) the trend component to the global or regional (but in any case the regular, 'exactly computable'), usually long wavelength features of the Earth's gravity field, (2) the signal component to its usually medium and high frequency part which cannot be modelled deterministically and (3) the noise to the errors and uncertainties which are present in the given gravity anomalies. In this method it is also possible to vary the deterministic and stochastic models and their model parameters, that is, to approximate the reality as far as the information content and density of data make it possible.

Since earlier investigations [e.g. KRAIGER, 1988, MORITZ, 1976] extensively discussed the role of different stochastic models in the process of prediction and collocation, therefore the effect of trend removal on the accuracy and on the behaviour of the prediction will be only examined and demonstrated by test computations.

General Outline of the Prediction Method

Since the method is widely known [MORITZ, 1980; HEISKANEN and MORITZ, 1967] the fundamental equations of the prediction are only repeated here. In the case of gravity anomalies the predicting equation has the following form:

$$\Delta g_p = \mathbb{C}_{ps}\mathbb{C}_{sn}^{-1}(\Delta g - \mathbb{A}\mathbb{X}) + \mathbb{A}_p\mathbb{X}, \tag{2}$$

where Δg is a vector of known gravity anomalies around or near the point to be predicted, Δg_p is the predicted value, $\mathbb{C}_{ps}$ is the vector of covariances between the location of $\Delta g - s$ and Δg_p, $\mathbb{C}_{sn}^{-1}$ is the inverse variance-covariance matrix of $\Delta g - s$ (subscript sn indicates that the signal and the noise components are supposed to be uncorrelated, so their autocovariances can be simply included in one matrix), $\mathbb{A}$ is the shape matrix of the trend model belonging to the locations of $\Delta g - s$, $\mathbb{X}$ is the vector of model parameters, $\mathbb{A}_p$ is the shape vector of the trend model at the computational point.

The mean square error of the prediction m_p^2 is given by *Eq.* (3) according to the error propagation law

$$m_p^2 = C_o - \mathbb{C}_{ps}^T \mathbb{C}_{sn}^{-1} \mathbb{C}_{ps}, \tag{3}$$

where C_o is the variance of the signal and the other symbols have been already explained.

Theoretically, the autocovariance of gravity anomalies separated by distance r are given by *Eq.* (4), if homogeneity and isotropy are assumed

$$C(r) = M\{\Delta g_i \cdot \Delta g_j\}, \quad \text{where} \quad r = \overline{P_i P_j}. \tag{4}$$

This expected value can be computed on the sphere by the triple integral of (5)

$$M\{\Delta g_i \Delta g_j\} = \frac{1}{8\pi^2} \int\limits_{\lambda=o}^{2\pi} \int\limits_{\vartheta=o}^{\pi} \int\limits_{\alpha=o}^{2\pi} \Delta g(\vartheta_i\, \lambda_i) \cdot \Delta g(\vartheta_j\, \lambda_j) \sin\vartheta \; d\vartheta d\lambda d\alpha. \tag{5}$$

[Moritz, 1972]

A spherical harmonic expansion of *Eq.* (5) derived by Tscherning and Rapp (1974) is usually used for the global case. Although there is a possibility to use the global ACF in local computations [Lachapelle, 1975] for local purposes some plane approximations of *Eq.* (5) described in textbooks and many papers [Moritz, 1980; Jordan, 1972; Kasper, 1971] are usually more suitable due to the convenience and the higher computational speed.

The Role of Trend Removal in the Least-Squares Prediction

There are two reasons why deterministic information should be removed from the given set of data. (1) From the modification *Eq.* (6) of *Eq.* (3), the scaling factor of the estimated error variance of a predicted gravity anomaly is variance (C_o) of the input data; and (2) according to the theory of the method, stochastic processes having 0 mean value can be predicted, therefore any trend being present in the data violates this theoretical condition

$$m_p^2 = C_o[1 - f(r_{ps}) f(r_{ss}) f(r_{ps})]. \tag{6}$$

Fortunately, a proper trend removal usually resulting in 0 mean residuals (so *Eq.* (7) holds) decreases the variance of residuals according to *Eq.* (8) and improves their statistical conditions as well

$$M\{(\Delta g_i - \mathbb{A}_i\mathbb{X})\} \cong 0, \tag{7}$$

$$C_o = M\{(\Delta g - \mathbb{A}\mathbb{X})^2\}. \tag{8}$$

Although especially in the case of free-air gravity anomalies, a series of data reduction is often needed to reach satisfactory results because this kind of gravity anomalies contains all the information and effects which is present in the Earth's gravity field. Therefore, a large variety of trend removals is possible because the anomalies are physically (that is deterministically) interpretable due to their physical origin.

It will be shown that geological-geophysical data involved in the process of prediction may reduce significantly prediction errors even if their density and geometrical distribution is poor. Obviously, these auxiliary data should be independent from gravity data and gravity measurements at least in a regional sense.

Data Sets Used in Test Computations

One regional and four local real data sets were available to carry out the computations.

The regional data set (HGN I–II) consists of 509 measured points of the 1^{st} and 2^{nd} order national gravity network 1959 [RENNER, 1959]. The distribution of the gravity stations is nearly homogeneous (c.f. *Fig. 1*) and covers whole Hungary. The network refers to the Potsdam Gravity System, thus there is approximately a +14 mgal bias in the gravity values. The point density of the network is 1 point/180 km^2. The free-air anomalies used in this study were computed by applying the Gravity Formula 1967, because the reference surface for Hungary is the IUGG67 ellipsoid.

The local data sets are parts of the very dense gravity database of the Eötvös Loránd Geophysical Institute with an average density of 1.3 points/km^2. The point distribution is not homogeneous due to the nature of the landwide gravity survey methods. The gravity stations are placed along lines which are usually parallel with the road system of the country (c.f. *Fig. 2*). These data sets refer to the IGSN71 datum point.

Trend Models

Five different trend models were selected for test computations.

The Mean Gravity Anomaly as a Trend Model for the Prediction

Subtraction of the areal mean from the data is the simplest way to 'center' the gravity anomalies (i.e. the trend is assumed to be constant in *Eq.* (9)

$$\mathbb{A}_i \mathbb{X} = M\{\Delta g\} \quad \text{for every} \quad i \tag{9}$$

$$\text{where} \quad i = 1,\ 2,\ \ldots,\ n_p.$$

Before using this simple model one should verify and prove the stationarity of gravity data [KAULA, 1959] because the subtraction of the mean value of gravity anomalies distorts the global spectrum of the gravity field [SCHWARZ and LACHAPELLE, 1980], and it influences the form of ACF according to the Wiener-Khinchin Theorem.

For three parts of Hungary the fundamental statistical parameters are listed in *Table 1*. The parts were selected from the HGN I–II data set according to the topographical conditions (c.f. *Fig. 1a*).

Table 1

Fundamental statistical parameters of free-air gravity anomalies and elevations

area	point number	mean anomaly [mgal]	variance [$mgal^2$]	mean elev. [m]	S. D. [m]
Dunántúl	212	+30.5	179	+159	±59
Alföld	185	+26.5	46	+103	±22
EKH	84	+29.7	158	+169	±85
TOTAL	509	+28.4	124	+138	±60

Since there are no large differences in the values of parameters and both gravity field and topography can be classified smooth [PRIOVOLOS] 1988], we used the mean free-air anomaly as a trend model. The constant trend is usually, a too rough approximation and this fact is reflected in the relatively high value of C_o=124 $mgal^2$ on the total area.

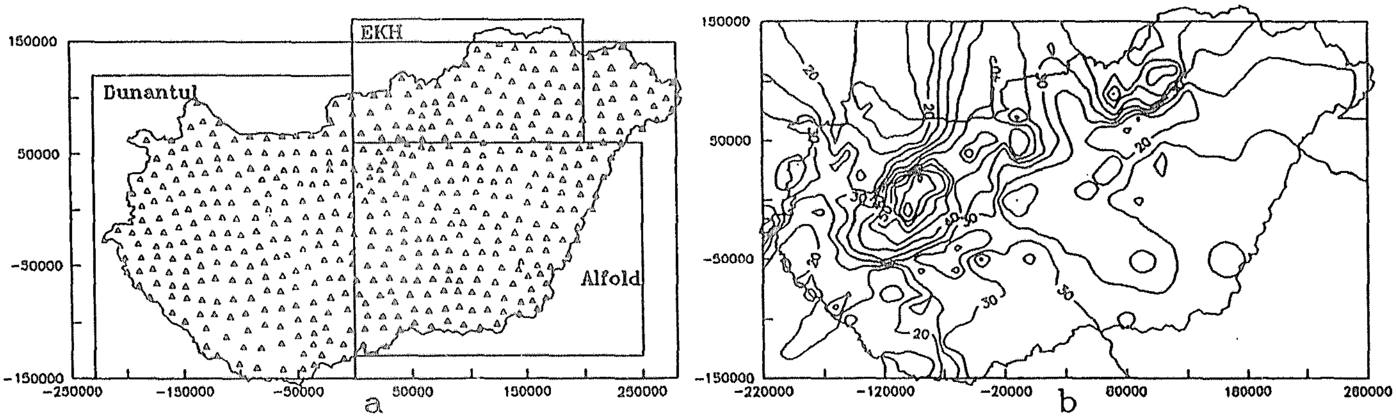

Fig. 1. a — point distribution of gravity stations in HGN I–II data set and areas (named Dunántúl, EKH, Alföld) separated for statistical test;
b — Free-air gravity anomaly map of Hungary contour int.: 5 mgal

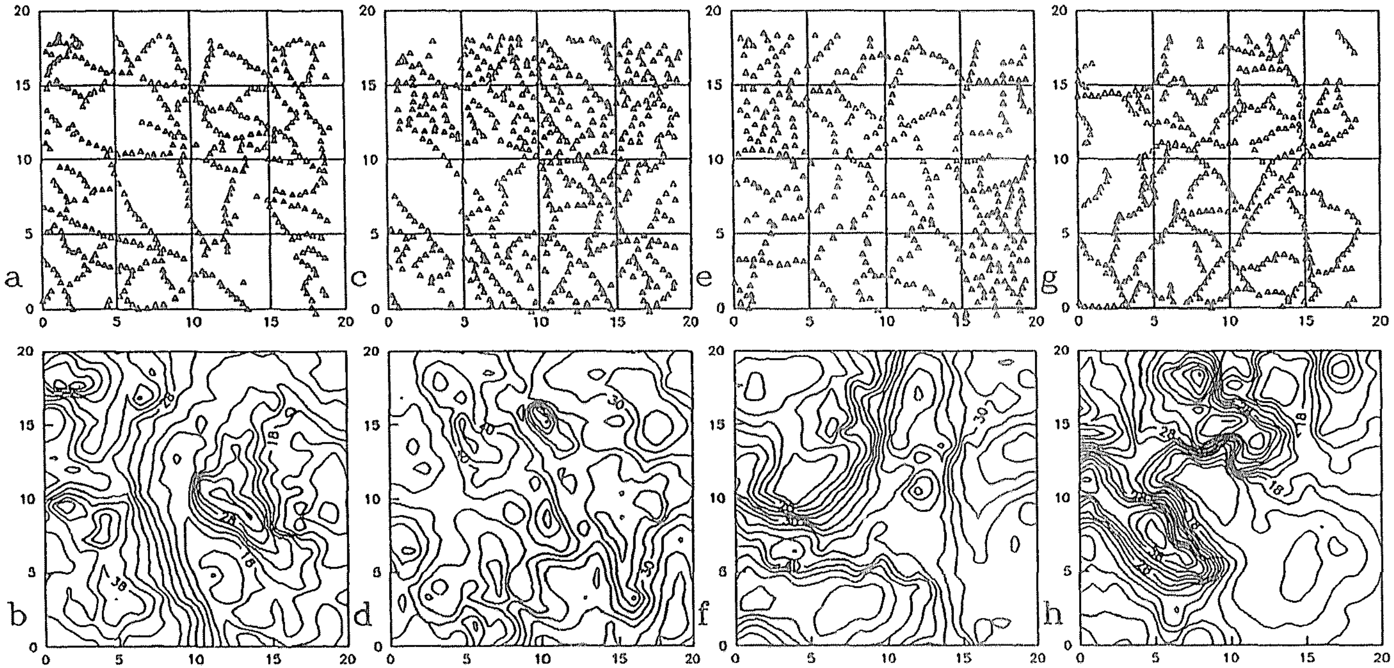

Fig. 2. a, c, e, g — point distributions of local data sets 024, 034, 136, 185, respectively; b, d, f, h — Contour lines of Free-air gravity anomalies of local data sets 024, 034, 136, 185, respectively; contour int.: 2.5 mgal

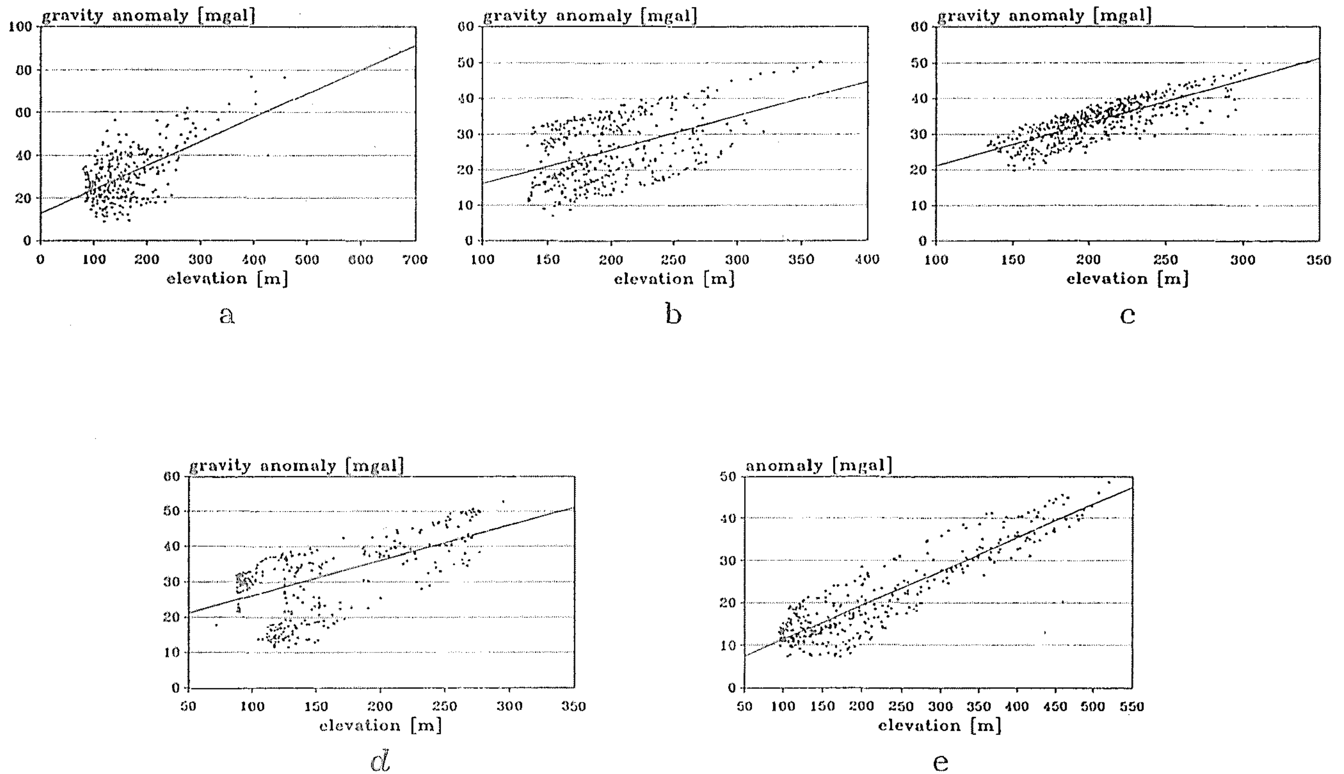

Fig. 3. Elevation dependence of Free-air gravity anomalies; a — HGN I–II data set, b — 024 data set, c — 034 data set, d — 136 data set, e — 185 data set

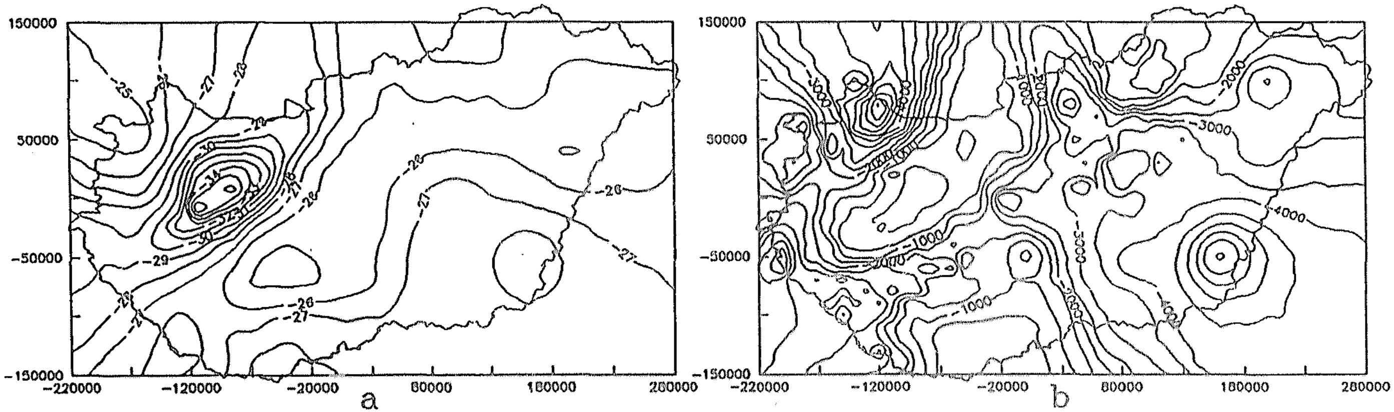

Fig. 4. a — Contour map sketch of the Moho discontinuity below Hungary. Contour int.: 1 km; b — Contour map of the pre-Tertiary basement below Hungary. Contour int.: 500 m

Table 2
Parameters of the elevation dependence of free-air gravity anomalies versus the location of data sets

area code	point number	mean Bouguer anomaly 'a' [mgal]	Bouguer coeff. 'b' [mgal/m]	density 'ρ' [kg/m^3]
024	440	+6.6	+0.0953	2274
034	520	+9.1	+0.1206	2878
136	400	+16.1	+0.0999	2384
185	440	+3.4	+0.0801	1911
HGN I–II	509	+12.99*	+0.1122	2677

* — value refers to the Potsdam Gravity System

Linear Correlation between the Free-air Gravity Anomalies and the Point Elevations as a Trend Model for the Prediction

By examining the correlation between point elevations and the corresponding free-air anomalies in *Fig. 3*, the parameters of a linear trend can be determined either by linear regression or by collocation itself [SÜNKEL 1977]. The application of linear regression computation (a special case of Least-Squares Adjustment) is more simple than the collocation because of the great number of measurements. The collocation takes, however, into account the statistical behaviour of the signal by its autocovariances in *Eq.* (3), therefore the trend parameters can be better estimated

$$\mathbb{X} = \left(\mathbb{A}^T\mathbb{C}^{-1}\mathbb{A}\right)^{-1}\mathbb{A}^T\mathbb{C}^{-1}\mathbb{x}. \tag{10}$$

Following the usual formalism, the linear correlation can be described by *Eq.* (11)

$$\Delta g_F^{(p)} = a + b \cdot h_p + \Delta g_B^{(p)} \tag{11}$$

where

$\Delta g_F^{(p)}$ — the free-air anomaly at point p
a — the regional/local mean Bouguer anomaly
b — the so called Bouguer coefficient
h_p — the point elevation at p
$\Delta g_B^{(p)}$ — the Bouguer anomaly at point p,

so the parameter vector has only two elements in *Eq.* (12), and the Bouguer anomaly is considered as a sum of a signal and noise components

$$\mathbb{X}^T = \{ab\}. \tag{12}$$

In *Table 2*, the parameters of linear trends can be seen as a function of the data set locations.

If we consider the approximately +14 mgal bias in the regional data set HGN I–II (i. e. we subtract it from 'a' in Table II) almost zero (slightly negative) regional Bouguer anomaly is obtained. It means that the crustal structure below Hungary reflects statistically 'random' features without significant regional density anomalies causing deterministic deviations, although locally, sometimes very significant deviations from the zero mean Bouguer anomaly can be observed. From the parameter 'b' the average density $\overline{\rho}$ of topographical masses can be computed by *Eq.* (13) where k is the gravitational constant

$$\overline{\rho} = \frac{b}{2\pi k}. \tag{13}$$

Regionally it also shows 'normal' physical conditions, however, locally the density varies between 1911–2878 kg/m^3 (c. f. *Table 2.*).

Subtracting the linear trends from the corresponding free-air gravity anomalies the variance of residual (i.e. Bouguer) anomalies decreased sometimes dramatically (c.f. *Table 3*). Therefore smaller prediction errors were expected than in the case of the area-mean trend. However, the application of elevationdependence is not a simple task, because a high resolution Digital Terrain Model (DTM) is required for the prediction of free-air gravity anomalies on the investigated area to restore this elevation dependent linear trend at the computational point.

Table 3
Statistical parameters of residual gravity anomalies

area	constant trend		linear trend	
code	mean	variance	mean	variance
	[mgal]	[mgal^2]	[mgal]	[mgal^2]
024	0.0*	78.2	−0.05	60.7
034	0.0*	27.4	−0.03	9.6
136	0.0*	99.8	+0.04	60.2
185	0.0*	93.3	−0.00	15.3
HGN I-II	0.0*	124.5	−0.02	78.9

* – values are zero by definition

Crustal Structure as Trend Model for the Prediction

Naturally, a fine and detailed 3D geological-topographic model of the investigated area could significantly reduce the variance of the signal, as it was pointed out by aothers [e.g. GEIGER et al., 1990], however more simple generalized connections can also be used between gravity anomalies and the crustal structure, as it will be shown. Due to the availability of the Moho discontinuity and basement topography data on the area of Hungary, their relation to the Bouguer anomaly field was determined. Depth data were obtained from POSGAY et al. (1981) in case of the Moho 'surface', and from the basement (pre-Tertiary) contour map of Hungary edited by Kilényi and Rumpler in 1984 (c.f. *Fig. 4*). The residual Bouguer anomalies were computed from the HGN I–II data set by *Eq.* (12) with parameters of *Table 2*. It has been assumed that the geometry of significant layers in the crust and the structure of the gravity field correlates well at least in regional sense.

In the comparison of Moho 'topography' and Bouguer anomaly a relative independency or even a slight negative correlation was observed in *Fig. 6a* caused by a local group of points from the hilly district of Transdanubia, where the Moho discontinuity reaches a depth of 37–38 km steeply falling down from the average depth of 27 km in spite of the significant positive Bouguer anomalies (c.f. *Figs. 4a–5a*). This result shows that:

- the effect of Moho topography is very small on the residual Bouguer anomalies, hence the Moho structure cannot be used in gravity field prediction as trend model,
- the slight negative correlation refers to significant density irregularities in the upper crust, since in the Airy isostatic model positive correlation is supposed between a Bouguer gravity anomalies and the geometry of Moho discontinuity,

and agrees well with MESKÓ (1988).

In *Fig. 6b* the correlation between the basement topography and the residual Bouguer anomaly is plotted. As expected, a non-linear correlation can be seen due to the significant compaction of sediments which appears in the Pannonian Basin [BIELIK, 1991]. We improved a heuristic inverse depth correlation model *Eq.* (14), which could be fitted to the point pairs $\Delta g_B^{(p)} - d_p$ successfully, but this model is valid only for the range of processed data. In *Eq.* (14) $\Delta g_B^{(p)}$ is the Bouguer anomaly corrected by the regional mean given by *Eq.* (12), and d_p is the depth of the basement at point p

$$\Delta g_B^{(p)} = \frac{A}{(d_p - \overline{d})^2} + \Delta \overline{g}_{red}. \tag{14}$$

The model parameters have the following values after the adjustment:

$$A = 112.7 \pm 61.6 \text{ mgal km}^2$$
$$\overline{d} = 2.7 \pm 0.6 \text{ km}$$
$$\Delta \overline{g}_{red} = -6.7 \pm 1.3 \text{ mgal}$$

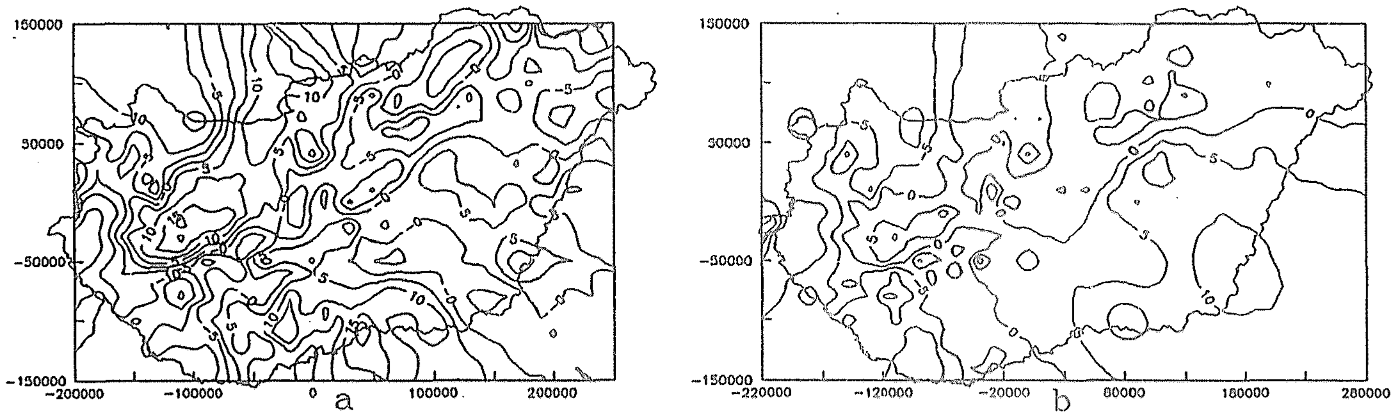

Fig. 5. a — Bouguer gravity anomalies reduced by the areal mean value. Contour int.: 5 mgal; b — Bouguer gravity anomalies reduced by the effect of pre-Tertiary basement topography. Contour Int.: 5 mgal

where A is the scaling factor of *Eq* (14), $\bar{d}$ is some shift-parameter of the model and $\Delta\bar{g}_{red}$ is the average value of gravity anomalies reduced by the depth-dependent term of *Eq.* (14).

In *Fig. 7c* the shape of the empirical ACF of the reduced anomalies shows improvement (compare it to *Figs. 7a–7b*) due to the vanishing of the slight oscillatory feature. The correlation length did not change considerably. The variance of reduced gravity anomalies is 57.8 mgal^2, which is less by 27 p.c. than the variance of Bouguer anomalies computed by the linear trend. This variance is still relatively high, because the total decrease of variance ($C_o = 124$ mgal^2) is only ≈ 50 p.c. This result implies that:

- when excluding the constant and very long wavelength terms (approx. $\lambda > 1000$ km), the main part (≈ 85 p.c.) of gravity anomaly field comes from the upper 7 km of the Earth crust and from the topography
- almost 50 p.c. of the power of gravity field ($\lambda < 1000$ km) comes from local and shallow density variations causing very short wavelength (approx. $\lambda < 100$ km) anomalies in *Fig. 5b*
- due to the lack of a detailed density model (no further signal reduction is possible) the residuals can be considered as realizations of a stochastic process although this is not a theoretical statement.

The OSU89b Gravity Field as a Trend Model for the Prediction

Since the present maximum degree and order of Global Geopotential Models (GGM) approaches 100 km resolution in wavelength, therefore their use as a reference-trend field seems to be obvious in gravity field prediction. ÁDÁM's paper (1990) has shown that gravity field computed up to $n, m = 360$ from the coefficients of OSU89b GGM fits sufficiently to the regional features of the free-air gravity field in Hungary. After removing this trend from the free-air gravity anomalies, the variance of residuals decreased from 124 to 72 mgal^2, which is almost equal to the variance of Bouguer gravity anomalies. The shape of the empirical ACF on *Fig. 7d*, however, shows some systematic features because of its oscillatory lobes. The estimated wavelength (or the average wavelength) of the dominant period (or periods PAPP, 1992]) induced by the trend removal is approximately 110 km, which corresponds to $\approx 1°$ on the sphere of the Earth. This is exactly equal to the wavelength referring to the maximum degree and order of the used model, so it may indicate some spectral problems. It is supposed that the power beyond the Nyquist frequency (f_N) is folded over into the close frequencies below f_N, therefore the high degree coefficients of the OSU model are distorted (overestimated) in magnitude and this distortion results in false periodicity of the model gravity field which is not present in the real gravity field. This folding effect is a consequence of the sampling theorem [BRIGHAM, 1974]. For the computation of OSU89b coefficients, the gravity anomalies were averaged on $0.5° \times 0.5°$ blocks (RAPP and PAVLIS, 1990), so the sampling density Δ was $0.5°$. Thus from *Eq.* (15) the wavelength λ_N of the Nyquist frequency is exactly $1°$

$$\lambda_N = 2\Delta. \tag{15}$$

Obviously, there can be other dominant wavelengths in the processed data set, but in this case, when the 1D and 2D autocovariance functions of the Free-air, Bouguer, reduced Bouguer and residual Free-air ($\Delta g_{FA} - \Delta g_{OSU89b}$) gravity anomalies in *Fig. 8a–8d* respectively, are considered and compared these components have no wavelengths significantly differing from λ_N.

Four sections in various directions (c.f. *Fig. 8d*) were created from the 2D ACF of residual Free-air anomalies (c.f. *Fig. 9*). Two periods are really significant in the data. One of the dominant wavelengths ($\lambda_1 \cong 1.1° \cong 120$ km) is slightly longer than λ_N, and the other one ($\lambda_2 \cong 0.8° \cong 90$ km) is slightly shorter than that.

These disadvantageous oscillations negatively influence the quality of prediction as it will be demonstrated in the discussion of practical results. Therefore, according to the recommendations by Rapp and PAVLIS (1990) every individual case in which GGM-s are planned to be used as reference gravity field should be examined carefully.

The Process of Prediction and Error Estimation

As a first step, a Hirvonen-type analytical plane ACF *Eq.* (16) was chosen as based on earlier investigations (e.g. KRAIGER, 1988). In the selected ACF model, C_o is the variance of gravity anomalies, B and p influence the curvature and correlation length parameters, r_{ij} is the distance between two point gravity anomalies Δg_i and Δg_j [MORITZ, 1980].

$$C(r_{ij}) = C_o \frac{1}{(1 - B^2 r_{ij}^2)^p}, \quad \text{where} \quad p = 0.5. \tag{16}$$

Parameters C_o and B of the ACF were fitted to the empirically determined 1D ACF-s of the different sets of gravity anomalies. As a second step all the point gravity anomaly data were predicted to themselves, so in this way point errors could have been computed by *Eq.* (17) for every gravity station. Naturally, the actual computation point

$$\delta_i = \Delta g_i^{\text{known}} - \Delta g_i^{\text{predicted}}, \tag{17}$$

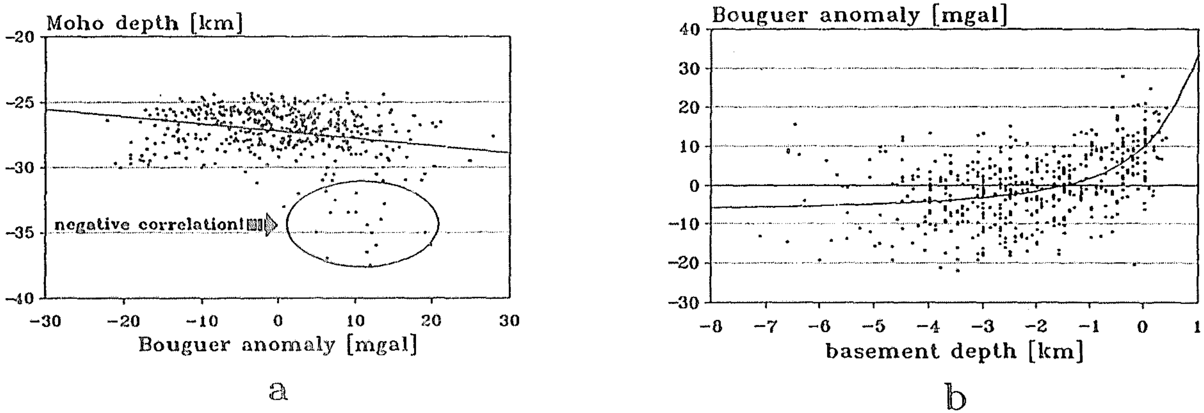

Fig. 6. a — Moho depth dependence of Bouguer gravity anomalies, b — Pre-Tertiary basement depth dependence of Bouguer gravity anomalies

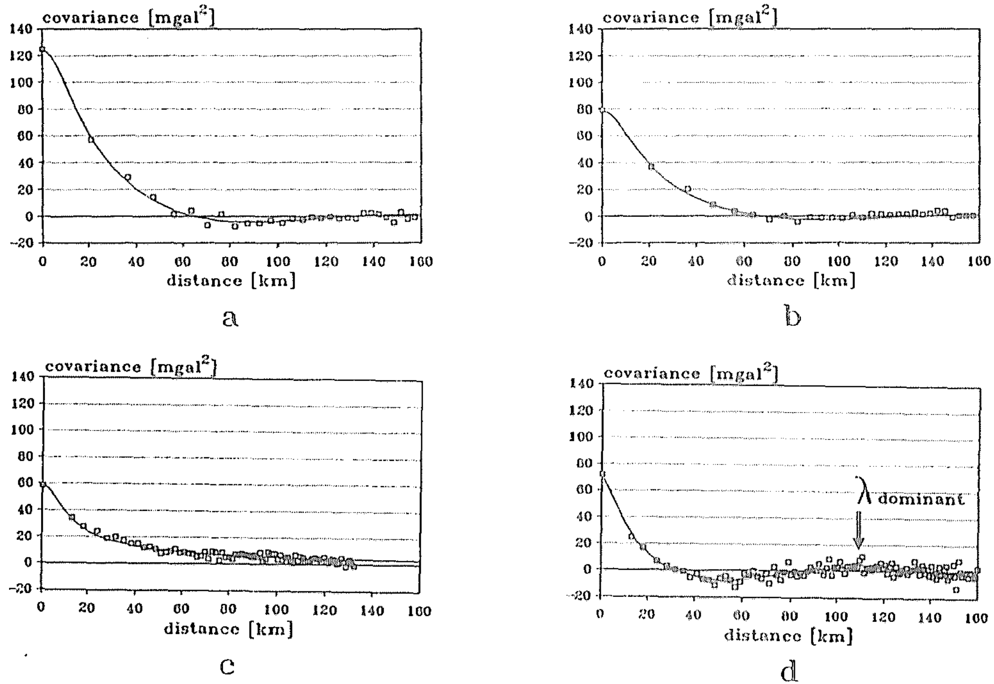

Fig. 7. Empirical autocovariance functions computed from the HGN I–II data set:
a — Free-air gravity anomalies,
b — Bouguer gravity anomalies,
c — Bouguer gravity anomalies reduced by the gravity effect of the pre-Tertiary basement,
d — Free-air gravity anomalies reduced by the gravity field of OSU89b geopotential model

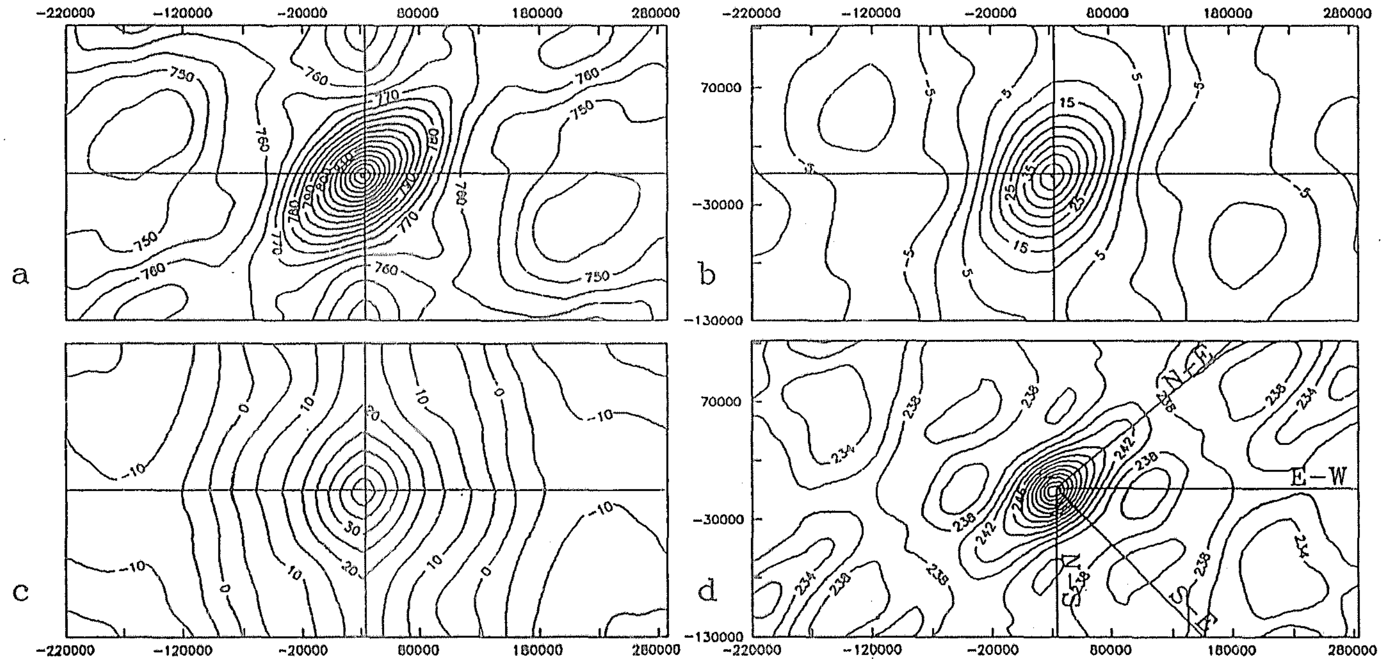

Fig. 8. 2D autocovariance functions computed from HGN I–II data set:
a — Free-air gravity anomalies,
b — Bouguer gravity anomalies,
c — Bouguer gravity anomalies reduced by the gravity effect of the pre-Tertiary basement,
d — Free-air gravity anomalies reduced by the gravity field of OSU89b geopotentcial model and section locations in the selected directions

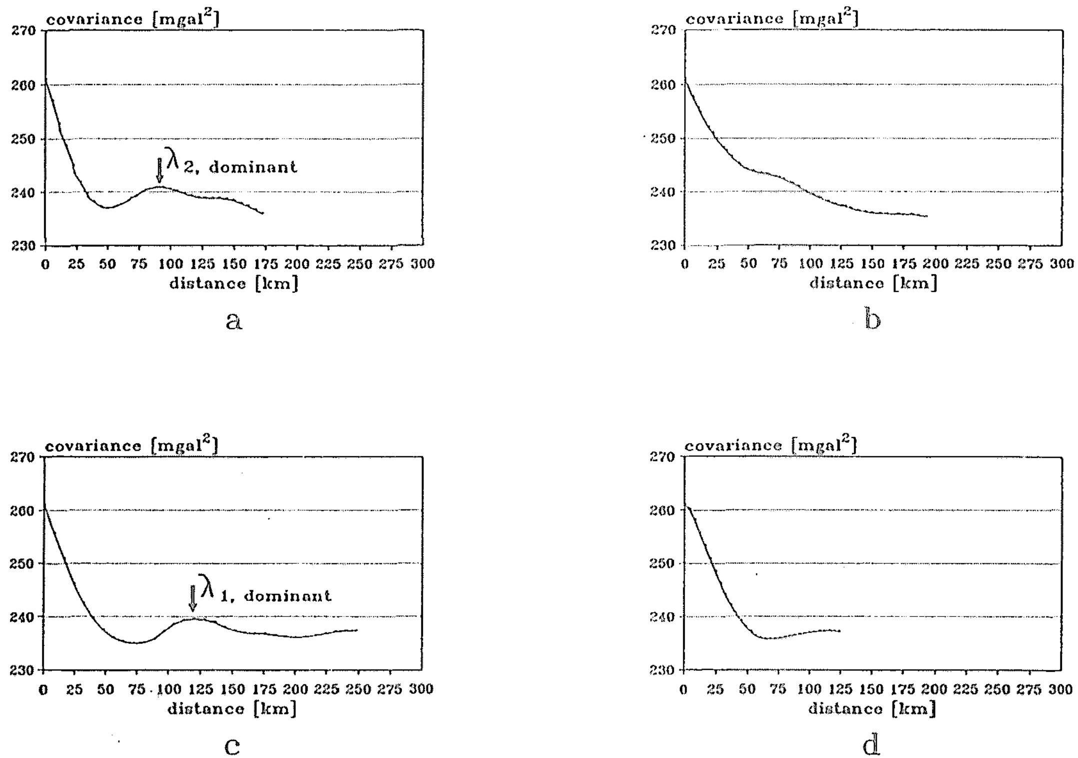

Fig. 9. Sections from the 2D ACF of Free-air gravity anomalies reduced by the gravity field of OSU89b geopotential model;
a — S — E section,
b — N — E section,
c — E — W section,
d — N — S section

where prediction was performed, was closed out from the set of available data which represented the possibly known points in the prediction. This method was used in order to

- avoid the reduction of the number of available points by selecting certain check points from the data sets,
- to increase the number of samples for the determination of statistical parameters,
- to avoid a modification of the point distribution and geometry; to let all the advantageous and disadvantageous effects act on the process of prediction (e.g. points along the edge of the area).

Since an earlier investigation [PAPP, 1992] has shown that the statistical distribution of the prediction errors is not a Gaussian (normal) but a systematically Laplacean, therefore the M. A. D. (Mean Absolute Deviation, *Eq.* (18) of residuals δ_i are summarized in *Table 4* as a function of the considered area

$$\sigma_{\mathrm{M.A.D.}} = \frac{1}{n} \cdot \sum_{i=1}^{n} |\delta_i - \bar{\delta}| \tag{18}$$

and the trend model applied in the process of prediction. In *Eq.* (18), $\bar{\delta}$ is the median and n is the number of point prediction errors derived by *Eq.* (17).

Table 4
M. A. D. values of prediction residuals in [mgal]

area	trend models			
code	1	2	3	4
024	±1.2	±0.5	–	–
034	±1.2	±0.5	–	–
136	±0.8	±0.4	–	–
185	±1.0	±0.5	–	–
HGN I–II	±4.9	±4.0	±3.5	±5.3

trend models:
1. areal mean of free-air gravity anomalies
2. elevation-dependence of free-air gravity anomalies
3. basement depth-dependence of Bouguer gravity anomalies
4. OSU89b model gravity field

From *Table 4*, the positive effect of proper trend removal is obvious although M. A. D. numbers given for the local data sets 024, 034, 136, 185 are smaller approximately by 30 p.c. than the realistic numbers of deviations, for geometrical reasons [PAPP, 1992].

Conclusions

The efficiency of Least-Squares prediction of gravity anomalies was demonstrated by practical examples. High quality prediction can be obtained by this method if a careful a priori investigation of the physical-statistical content of data is performed and a series of data reduction is applied to decrease signal variance and improve statistical conditions. The accuracy of prediction heavily depends on the resolution of the used trend model as well as on the geometrical distribution of point gravity anomalies. Therefore, it is a difficult task to produce high quality prediction ($\sigma < \pm 1$ mgal) even if the point density is extremely high (≈ 1 point/km^2) because the point distribution is usually far from the 'ideal one' and there are relatively large data gaps between the given points. These gaps can be efficiently 'filled up' by applying some additional deterministic information, therefore it is recommended to use available geodetical and geophysical data in the process of prediction which have physical relation to the anomalous gravity field. The relation can be formulated by simple and generalized, rather regional than global correlation models and these models can be used efficiently in the process of prediction.

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