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VERTICAL FITTING OF GPS MEASUREMENTS

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Abstract

The methodological analysis of the author was motivated by the fact that his team could apply its GPS techniques directly by using an existing geoid map (GAZSÓ, TARASZOVA, 1984). Regarding the Hungarian particularities, he analysed the sensitivity of the determination of elevations above sea level by applying GPS methods to the knowledge of geoid heights. He pointed out the possibility that a 7-parameter transformation of spatial coordinates can provide automatically a linear elevation fitting. To make a global approximation to the geoid in Hungary, he derived a third-degree formula with two variables by means of Chebyshev approximation.

Keywords: geoid height, coordinate transformation, GPS technology, global positioning system.

Basic Relations

Using GPS measurements, we get the elevations above an optionally chosen ellipsoid (H). Geodesy, however, uses heights above sea level (h). The difference of these two values is called geoid height (or undulation, $\underline{n}$). By definition (SOHA, 1992) let be:

$$h = H - n. \tag{1}$$

Because H depends on the orientation and size of the ellipsoid (spheroid), this $\underline{n}$ is also system-dependent. If we define formula (1) over a geometric ellipsoid, then it is expedient to distinguish cases of national ellipsoids as follows:

$$h = H' - n'. \tag{1b}$$

So there are two different ways of the vertical fitting of GPS measurements. The choice depends on which system has been used for undulations that are at our disposal. An example for a geocentric system is the WGS-84, and for a regional system the S-42 (Datum of Pulkovo 1942) (PELLINEN, DEUMLICH, 1978). Another example for a local (Hungarian) system is the HD-72 (Hungarian Datum 1972). Because of the global character of the GPS technology, a universal geocentric geoid could be fitted to each country with the highest consistency. This alternative is specified as a desirable one in the future (SOHA, 1992).

Determination of Elevation Differences

To reach the highest accuracy, geodesists use GPS for relative coordinate-determination.

Concerning height-values, we use the formulas below:

$$\begin{aligned} \Delta h &= h_2 - h_1, \\ \Delta H &= H_2 - H_1. \end{aligned} \tag{2}$$

It is important to make clear what the relation between Δh and ΔH is. On the basis of (1), we can write:

$$\begin{aligned} \Delta h &= h_2 - h_1 = H_2 - n_2 - (H_1 - n_1) \\ &= H_2 - H_1 - (n_2 - n_1) = \Delta H - \Delta n. \end{aligned} \tag{3}$$

If $\Delta n \approx 0$ (i.e. geoid heights at the two relative points are nearly the same), then we can use GPS for the determination of heights above sea level as a good approximation and as levelling.

When can this approximation be used? Consulting an older geoid map of Hungary, we can see that undulation differences between the two far-off shores of Lake Balaton amount to 1 m. Considering the accuracy of GPS and the accuracy categories of practical geodesy, if the baselines are some kilometer long, the above approximation can become one of the advantages of GPS technology.

The GPS receiver used for our practical analysis calculates and provides the ΔH values automatically in the quasi-geocentric system of the stations. It has been found that our suppositions above have been justified by test measurements. However, we warn the users against the general application of this convention ($\Delta h = \Delta H$) because there is a better solution, as it can be seen below.

The Role of Geoid Heights in Spatial Similarity Transformation

In order to clear up the basic concepts, the elevation determination has been regarded so far, intentionally as a particular application of the GPS technology. Now we analyse the common spatial positioning, which is more complex but has a better performance. The task is the following: the geocentric co-ordinates XYZ determined by GPS have to be transformed into a local φ, λ, h system. How can we attain this?

To solve the problem, we must define common points (control points) between the geocentric and local systems and we must use some kind of co-ordinate transformation. For the latter, the model of Burša–Wolf (PELLINEN, DEUMLICH, 1978) is a very good, wide-spread method for spatial transformations. Let X, Y, Z be the geocentric system and X', Y', Z' the local system. This model uses translation, rotation and scale correction. Transformation parameters are first derived by the required minima of 3 common (identical) points.

These parameters are then applied to the new points. To test our GPS receivers, we used the computer software of this model. In this paper, only those steps are discussed which are relevant to heights.

In this task, we must solve the transition in both directions between φ, λ, h ($2+1$ dimensions) and X, Y, Z (3 dimensions). In our point of view, the conversion's problematic part lies between $\underline{h}$ and $\underline{H}$.

If we substitute a plane for the geoid on a region of transformation, then we reach the following formula expressing the undulation:

$$n = a_0 + a_1\varphi + a_2\lambda. \qquad (4)$$

If we know the undulations at our control points, then we can determine a_i coefficients from the related φ, λ and $\underline{n}$ values by means of minimum 3 identical points. Let's recognize that (4) is formally a linear transformation (with translation and rotation) which – when using a theoretical proofing – could be reduced with the parameters (translation and rotation) of the similarly linear transformation of Burša–Wolf. It results in the fact that transformation without undulation is equivalent to the transformation with its linear elements. *Summarized* as a dogma: the 7-parameter transformation used on the region of common (or reference) points linearly fits the elevations above sea level. It is a better approximation than the one expressed in (3) $\Delta n \approx 0$. The question is left: what the extent of the region is, where the geoid is regarded as a plane? Our test has shown a very favourable result: there was no observable error on fitting triangles with sides of 25 kms.

This advantage of the GPS technology is well utilized in local fittings, i.e. positioning in network densification. For the purpose of measurement of longer base lines, the linear approximation of undulations is not allowed.

Higher-Order Approximation of Geoid Heights

In order to use computer-software in Hungarian network for longer baselines, author derived the third order trend of geoid for Hungary from an older geoid-map (Astro-gravimetric determination AGRG-78). (GAZSÓ, TARASZOVA, 1984; SOHA, 1990.) Its structure looks like the following:

$$\begin{aligned} n =& a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi\lambda + a_5\lambda^2 + \\ &+ a_6\varphi^3 + a_7\varphi^2\lambda + a_8\varphi\lambda^2 + a_9\lambda^3. \end{aligned} \tag{5}$$

Parameters a_i were determined by the Chebyshev even approximation. This method resulted in errors smaller by 30 % than the method of least-squares. Remainders at 14 reference points can be seen in the next table (where LNM = adjustment with least-squares, Ch = according to Chebyshev method).

Table 1

No	V_{LNM} m	V_{Ch} m
1	0.11	0.17
2	0.03	0.17
3	0.02	−0.10
4	−0.16	−0.17
5	0.28	0.17
6	−0.11	−0.17
7	0.12	0.17
8	0.12	0.17
9	0.05	0.13
10	−0.26	−0.17
11	−0.12	−0.17
12	−0.12	−0.17
13	0.00	0.04
14	0.04	0.17
Max.	**0.28**	**0.17**

Two features of Chebyshev method are conspicuous: the even approximation, and the minimization of absolute value of maximal error.

The question arises: why we do not calculate with the error of 0.17 metre that can cause the same level of errors in the determination of heights above sea level. It can happen because we assume the linear correcting effect of the Burša–Wolf model (continuously in future, too). If this is not possible, formula (5) is used on smaller regions.

These formulas and determined parameters were built into the software used at the Service (GPS Fitting software) (Paskó, Tóth, 1991). The work of the team has made possible that geodesists (surveyors) can utilize the GPS technology with success until the digital geoid (with a high resolution) in Hungary appears (Soha, 1992).

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