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DETERMINATION OF THE ADJUSTING CIRCLE BY MEANS OF LINEAR OBSERVATION EQUATION

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Abstract

Denoting the third nonlinear term with z_0 in the Equation (6), let us set up the linear observation equation (10). After having completed the adjustment we compute the necessary radius r on the base of equation (16). Relationship of the reliability of the adjusted radius Q_{rr} (23) and of the adjusted points Q_{kk} (28) of the circle are given, too. To prove the correctness of the algorithm worked out a programme has been compiled for the microcomputer PTA 4000+16 (SHARP 1500A) [Enclosure 1]. In the enclosure 2 some examples are presented.

Keywords: adjusting circle, linear observation equation, least square method.

Introduction

At setting out and measuring of circular sectioned objects observation equations are calculated. The observation equation is definitely determined by the x_0 and y_0 coordinates of the centre and the radius r. The computation is carried out according to the theorem of the least squares, by means of the method of adjustment of observation equations (group II). Adjusted values of the parameters x_0, y_0 and r are usually determined in two steps. First a matching (x_0), (y_0) and (r) approximate values are calculated, then the non-linear observation equations $f(x_0, y_0, r)$ are developed at these values. After having the mentioned preliminary steps carried out, the adjustment follows by means of calculating the changes $\mathrm{d}x_0$, $\mathrm{d}y_0$ and $\mathrm{d}r$. Final values are obtained by summing up the results of the first and second steps [7] :

$$x_0 = (x_0) + \mathrm{d}x_0\,;$$
$$y_0 = (y_0) + \mathrm{d}y_0\,;$$
$$r = (r) + \mathrm{d}r\,.$$

By means of the method introduced in this article the solution can be obtained in one step. As linear observation equations are used, neither the approximate values nor their changes should be determined, since the parameters of the adjusting circle can be obtained directly from the adjustment.

Notation

x_0, y_0		coordinates of the centre of the adjusting circle
r	=	radius of the adjusting circle
z_0	=	auxiliary unknown (for substituting the radius)
x_i, y_i		coordinates of the measured points
v_i	=	correction belonging to the respective points
v'_i	=	reduced correction
$\mathbb{A}$	=	coefficient matrix of the correction equation system (form matrix)
$\mathbb{l}$	=	vector of the constant terms
$\mathbb{N}$	=	coefficient matrix of the normal equation
m_0	=	standard deviation of unit of weight
m'_0	=	standard deviation of unit of weight computed from the reduced corrections
f	=	number of redundant observations
m_{x0}, m_{y0}, m_r		standard deviation of the adjusted parameters
$\mathbb{Q}_{xyz}$	=	weight-coefficient matrix of the parameters
Q_{rr}	=	weight-coefficient of the adjusting radius
Q_{xr}, Q_{yr}		connecting weight-coefficients
Q_{00}	=	reliability of the adjusted centre
Q_{kk}	=	reliability of the adjusted contour
F	=	function-matrix
f	=	functor of the function of radius
x_k, y_k		coordinates of an individual point of the contour
δ	=	bearing

Formulating the Linear Observation Equation

Equations for the contour line as a special case of the curve of second order can be written as follows — from mathematical handbooks [6, 1]:

$$(x - x_0)^2 + (y - y_0)^2 = r^2 , \tag{1}$$

$$x^2 + y^2 + Ax + By + C = 0 \tag{2}$$

and

$$x^2 + y^2 + 2mx + 2ny + q = 0 \,. \tag{3}$$

Equations (2) and (3) are the same basically, the only difference is the notation.

For coordinates x and y both equations are of second order, so the equation of the circle is that of second order.

From point of view of the parameters to be determined, the two equations are differing. *Eq.* (1) is a second degree one for parameters x_0, y_0 and r, while *Eq.* (2) is a first degree one for parameters A, B and C.

In both cases the parameters are independent from each other, they are equal in number, and parameters of the two equations can be unambiguous [6].

Parameters of *Eq.* (1) are having a direct geometric meaning, so this is utilised as an observation equation in geodetic applications [7].

Parameters of the second equation have no direct geometric meaning, but they can be directly utilised as observation equation without development.

In the following we will introduce a method to formulate an observation equation, corresponding to *Eq.* (2) from *Eq.* (1).

It is presumed that measurement of all the points (or coordinates) can be considered of equal reliability. It is known that the adjusting circle of a group of points is a circle having a minimal value of the sum of square of distances v measured between the circle and individual points [7]. Since these distances are radial ones, they can be regarded as corrections of the radius obtained from the adjustment:

$$r_i = (r - \mathrm{v}_i)^2 = (x_i - x_0)^2 + (y_i - y_0)^2 \,, \tag{5}$$

where r = radius of the adjusting circle
v = correction belonging to individual points
x_i, y_i = measured coordinates of the points

After having raised to the second power, then disregarding v_i^2, the adjustment equation will be after rearrangement:

$$r\mathrm{v}_i = x_i x_0 + y_i y_0 - \frac{1}{2}\left(x_0^2 + y_0^2 - r^2\right) - \frac{1}{2}\left(x_i^2 + y_i^2\right) \,. \tag{6}$$

Introducing the reduced correction

$$\mathrm{v}_i' = r\mathrm{v}_i \,, \tag{7}$$

the auxiliary unknown

$$z_0 = -\frac{1}{2}\left(x_0^2 + y_0^2 - r^2\right), \tag{8}$$

and constant term

$$\mathbf{l}_i = -\frac{1}{2}\left(x_i^2 + y_i^2\right), \tag{9}$$

the observation equation can be written by the following formula

$$\mathbf{v}' = x_i x_0 + y_i y_0 + z_0 + \mathbf{l}_i . \tag{10}$$

Dimension of the original correction $\mathbf{v}_i$ is a unit of length, whilst of the reduced correction $\mathbf{v}_i'$ is a unit of length on the second power.

The above observation equation is linear for parameters x_0, y_0 and z_0.

Carrying out the Adjustment

Coefficient matrix of the observation equation system $\mathbf{A}$ (form matrix) and vector of the constant terms $\mathbf{l}$ will have the following form:

$$\begin{matrix} x_0 & y_0 & z_0 \end{matrix}$$

$$\mathbf{A} = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ & \vdots & \\ x_i & y_i & 1 \\ & \vdots & \\ x_n & y_n & 1 \end{bmatrix}, \tag{11}$$

$$\mathbf{l} = \begin{bmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ \vdots \\ x_i^2 + y_i^2 \\ \vdots \\ x_n^2 + y_n^2 \end{bmatrix}. \tag{12}$$

The following equation system will result from it by means of the adjustment of the observation equations:

$$\begin{bmatrix} \sum x^2 & \sum xy & \sum x \\ \sum xy & \sum y^2 & \sum y \\ \sum x & \sum y & n \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} \sum x\left(x^2 + y^2\right) \\ \sum y\left(x^2 + y^2\right) \\ \sum x^2 + y^2 \end{bmatrix} = 0, \tag{13}$$

where n = number of measured points.

Measurements of individual points are regarded having the same weight, the unit of weight.

After changing over to a coordinate system of centre of gravity the coefficient matrix of the normal equation can be formulated by following way (using x, y notation):

$$\begin{matrix} x_0 & y_0 & z_0 \end{matrix}$$

$$\mathbf{N} = \begin{bmatrix} \sum x^2 & \sum xy & 0 \\ \sum xy & \sum y^2 & 0 \\ 0 & 0 & n \end{bmatrix}, \tag{14}$$

since the sum of x and y coordinates relative to the centre of gravity equals zero. Therefore the normal equation system of three unknowns will fall apart to a system of two unknowns and a system of one unknown. After having the system of equations solved, the adjusted parameters can be obtained in the following form:

$$x_0 = \frac{\sum y^2 \sum x\left(x^2+y^2\right) - \sum xy \sum y\left(x^2+y^2\right)}{2\left(\sum y^2 \sum x^2 - \left(\sum xy\right)^2\right)},$$

$$y_0 = \frac{\sum x^2 \sum y\left(x^2+y^2\right) - \sum xy \sum x\left(x^2+y^2\right)}{2\left(\sum y^2 \sum x^2 - \left(\sum xy\right)^2\right)},$$

$$z_0 = \frac{\sum x^2 + y^2}{2n}. \tag{15}$$

Radius of the adjusting circle can be computed by the following equation:

$$r = \sqrt{x_0^2 + y_0^2 + 2z_0}. \tag{16}$$

Reduced correction can be determined from the following equation:

$$\mathbf{v}_i' = \mathbf{A}\mathbf{x} + \mathbf{l}_i, \tag{17}$$

i. e. they can be obtained from the following equations:

$$\mathrm{v}_i' = x_i\left(x_0 - \frac{1}{2}x_i\right) + y_i\left(y_0 - \frac{1}{2}y_i\right) + z_0. \tag{18}$$

Original correction can be obtained, based on formulae (7) as follows:

$$\mathrm{v}_i = -\frac{\mathrm{v}_i'}{r}. \tag{19}$$

Determination of Data of Reliability

Sum of squares of the corrections can be determined from the corrections obtained by *Eq.* (19), or by the generally known supplementary normal equations. In the latter case the sum of squares of reduced corrections will be obtained, which should be divided by r^2.

Number of redundant observations can be determined by

$$f = n - 3\,, \tag{20}$$

since one correction was computed for each of the points, and the number of the parameters is three.

Standard error of weight can be determined from the original $\mathbf{v}'$ corrections as well, by means of the following equations:

$$m_0 = \sqrt{\frac{\sum \mathbf{vv}}{f}} = \frac{1}{r}\sqrt{\frac{\sum \mathbf{v}'\mathbf{v}'}{f}}\,,$$

$$m_0' = \sqrt{\frac{\sum \mathbf{v}'\mathbf{v}'}{f}} = r m_0\,. \tag{21}$$

For deducing the standard error of parameters, the inverse of the coefficient matrix of normal equation system $\mathbb{N}$ belonging to coordinates of centre of gravity should be written, which is the weight-coefficient matrix of the parameters:

$$\mathbb{Q}_{xyz} = \begin{bmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{xy} & Q_{yy} & 0 \\ 0 & 0 & Q_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\sum y^2}{\mathrm{Det}} & \frac{-\sum xy}{\mathrm{Det}} & 0 \\ \frac{-\sum xy}{\mathrm{Det}} & \frac{\sum x^2}{\mathrm{Det}} & 0 \\ 0 & 0 & \frac{1}{n} \end{bmatrix}\,, \tag{22}$$

where

$$\mathrm{Det} = \sum x^2 \sum y^2 - \left(\sum xy\right)^2\,.$$

Weight-coefficient of the adjusting radius, as weight-coefficient of a function can be determined by means of the principle of error propagation. The vector f^T can be obtained as the partial derivative *Eq.* (14) according to x_0; y_0 and z_0.

$$f^T = \begin{bmatrix} \frac{x_0}{r} & \frac{y_0}{r} & \frac{1}{r} \end{bmatrix}\,.$$

Weight-coefficient of the radius:

$$Q_{rr} = f^T Q_{xyz} f = \frac{1}{r^2}\left(x_0^2 Q_{xx} + 2x_0^2 y_0 Q_{xy} + y_0^2 Q_{yy} + Q_{zz}\right) . \tag{23}$$

Standard error of the adjusted parameters could be computed from the values of weight-coefficient and by means of the previously determined m_0 values as follows:

$$m_{x0} = m_0'\sqrt{Q_{xx}}; \qquad m_{y0} = m_0'\sqrt{Q_{yy}}; \qquad m_r = m_0'\sqrt{Q_{rr}} . \tag{24}$$

Connecting coefficients between x_0 and r_0, and between y_0 and r_0 can also be determined by the general principle of error propagation by means of the following function-matrix:

$$\mathbb{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{x_0}{r} & \frac{y_0}{r} & \frac{1}{r} \end{bmatrix} .$$

From the $Q_{xyr} = \mathbb{F}^T Q_{xyz}\mathbb{F}$ matrix product it is obtained directly:

$$Q_{xr} = \frac{x_0}{r}Q_{xx} + \frac{y_0}{r}Q_{xy}$$

and

$$Q_{yr} = \frac{x_0}{r}Q_{xy} + \frac{y_0}{r}Q_{yy} . \tag{25}$$

Reliability of the adjusted centre at a δ arbitrary direction:

$$Q_{00} = Q_{xx}\cos\delta^2 + Q_{xy}\sin\delta\cos\delta + Q_{yy}\sin^2\delta , \tag{26}$$

which is the equation of the curve of nadir of the centre.

By means of well-known methods, both the minor axis and the major axis as well the alientation of the error ellipsoid can be computed [2, 4].

Reliability of adjusted contour Q_{kk} is not uniform over the points of the circle, as a matter of fact it is corresponding to the reliability of correction **v** belonging to an arbitrarily chosen δ direction.

Therefore the vector $\mathbf{f}^T$ important to the deduction can be obtained from *Eq.* (10) after divided by r:

$$\mathbf{f}^T = \left[\frac{x_k}{r} \quad \frac{y_k}{r} \quad \frac{1}{r}\right] . \tag{27}$$

Carrying out the multiplication $f^T \mathbf{Q}_{xyz} f = Q_{kk}$

$$Q_{kk} = \frac{1}{r^2}\left(x_k^2 Q_{xx} + 2x_k y_k Q_{xy} + y_k^2 Q_{yy} + Q_{zz}\right) , \tag{28}$$

where

$$x_k = x_0 + r\cos\delta_k \qquad \text{and} \qquad y_k = y_0 + r\sin\delta_k$$

are coordinates of a point of the curve.

Maximum and minimum of it depends on the position of the measured points. Generally two minima can be formed, since the investigation of extreme values leads to an equation of fourth power.

Description of the Program

For carrying out the computation, program was written to computer HT PTA 4000+16 (SHARP PC 1500A) in BASIC.

The 18 options secured by 6 reserve-keys were utilised to form nine Hungarian letters with accent in writing. By means of utilising these possibilities, both the presentation and printing the data could be carried out in accordance with Hungarian ortography. The computer presents — over the input data and computed quantities — the adjustment circle and its reliability relations as well. The curve corresponding to the nadir of the adjusted centre and the adjusted circle are drawn in dashed line. Outer and inner error circle, error curves $r + m_k$ and $r - m_k$ corresponding to the standard error are also drawn. For the sake of better realisation, the radial corrections are represented at a different scale.

Table 1 contains the list of program.

Examples

Four computations are shown (*Table 2*). At case 1 there are four symmetrically positioned points having the same absolute correcting values with the same sign in pairs. Due to the symmetrical positioning, the nadir curve of the centre, the inner and outer error curves are concentric with the adjusting circle.

In the second case three points were chosen relatively close to each other, so the ellipse of error is extremely deformed and the nadir curves were transformed to almost a circle. Standard errors of cylindrical points close to the points are small, but opposite to the points this value has grown more 300-times. In case of three points, the number of redundant

Table 1

```
10:"C" REM COMPUTA
   TION OF THE AD
   JUSTING CIRCLE
15:REM PROGRAMMER
   DR. SZABOLCS
   CSEPREGI,
   ISTVÁN KÁDÁR,
   ERIK PAPP
20:REM LIST OF CO
   ORDINATES
30:"F":CLS:INPUT
   "NUMBER OF POI
   NTS=";N:DIM
   K(N+9,2)
40:FOR I=1 TO N :CLS
   :WAIT 0J$ =STR$
   I:PRINT"POINT
   NUMBER (";J$;")=";:
   WAIT:INPUT
   K(I,0)
50CLS:WAIT 0:
   :PRINT"Y(";J$;
   ")=";:WAIT:
   INPUT K(I,1)
60 CLS:WAIT 0 :
   PRINT"X(";J$;
   ")=";:WAIT:
   INPUT K(I,2):
   NEXT I
65:REM PRINT OF
   COORDINATES.
70:"N"CSIZE 2:TAB
   6:LPRINT"LIST
   OF":TAB 4:
   LPRINT "COORDI
   NATES":TAB4:
   LPRINT"------
   -----"
75:CSIZE 1:LPRINT
   "POINT NUMBER
   Y-COORDINATE":
   LPRINT
80:TAB 20:LPRINT
   "X-COORDINATE"
   :LF 3:CSIZE 2:
   FOR I-1 TO N
90:USING"####":
   LPRINT K(I,0);
   :USING"+#####
   ###.###":
   LPRINT K(I,1):
   TAB 4 LPRINT K
   (I,2):LPRINT:
   NEXT I
95:REM COMPUTATION
   OF COORDINATES
   FOR THE CENTRE
   OF GRAVITY
100:"S"YS=0 :XS=0:
   FOR I=1 TO N: YS
   =YS+K(I,1):XS=
   XS+K(I,2): NEXT
   I:YS=YS/N:XS=X
   S/N
105:REM REDUCED NO
   RMAL EQUATION
110:XX=0:XY=0:YY=0
   :XD=0:YD=0:D2=0
120:FOR I=1 TO N:X=
   K(I,2)-XS:Y=K(
   I,1)-YS:XX=XX+
   X*X:XY=XY+X*Y:
   YY=YY+Y*Y:T=X*
   X+Y*Y
130:XD=XD+X*T:YD=Y
   D+Y*T: D2=D2+T*
   T: NEXT I:D=(XX
   +YY)/2: XD=XD/2
   :YD=YD/2: D2=D2
   /4
135:REM SOLUTION OF
   THE REDUCED
   NORMAL EQUATION
140:DET=XX*YY-XY*X
   Y:AA=YY/DET:BB
   =XX/DET:AB=-XY
   /DET:CC=1/N
150:XO=AA*XD+AB*YD
   :YO=AB*XD+BB*Y
   D:ZO=CC*D:R=
   SQR(XO*XO+YO*
   YO+2*ZO)
160:RR=(XO*XO*AA+2
   *XO*YO*AB+YO*Y
   O*BB+CC)/R/R
170:VV=D2-XD*XO-YD
   *YO-D*ZO:MO=.0
   5: IF N-3LET MO
   =SQR (VV/(N-3))
180:MXO=MO*SQR AA:
   MYO=MO*SQR BB:
   MR=MO*SQR RR
185:REM PRINTING
   OF THE RESULTS
190:TAB 3:LPRINT
   "DATA OF THE":
   LPRINT"ADJUST
   ING CIRCLE":
   LPRINT "------
   ----------"
200:CSIZE 1 :LPRINT
   "PARAMETER
   VALUE(m)
   ":LPRINT
210:TAB 12:LPRINT
   "STANDARD DEVI
   ATION (mm)":
   CSIZE 2:LPRINT
220:USING"+######
   ##.###":LPRINT
   "Xo= ";XO+XS:
   TAB 5:LPRINT M
   X*1000:H=LEN
   STR$ INT (MX*1
   000):GOSUB 490:
   LPRINT
230:USING"+######
   ##.###":LPRINT
   "Yo= ";YO+YS:
   TAB 5:LPRINT M
   Y*1000:H=LEN
   STR$ IN (MY*1
   000):GOSUB 490:
   LPRINT
240:USING"+######
   ##.###":LPRINT
   " R=";R:TAB 5
   :LPRINT MR*10
   00:H=LEN STR$
   INT (MR*1000):
   GOSUB 490:
   LPRINT
245:LPRINT "-------
   -----------"
250:CSIZE 1 TAB 4:
   LPRINT "REFERE
   NCE STANDARD
   DEVIATION":
   CSIZE 2
260:LPRINT USING
   "+########.###
   ":LPRINT "mo=
   ";MO:H=LEN
   STR$ INT MO:
   GOSUB 490
265:REM CALCULATIO
   N OF RESIDUALS
270:"V"LF 1 :LPRINT
   " RESIDUALS
   (mm)":LPRINT"
      ----------
   ----":PVV=0
280:FOR I=1 TO N:
   GOSUB 540
285:S$="v("+STR$ K
   (I,0)+")="
290:IF LEN S$<8 LET
   S$= " "+S$:GOTO
   290
300:LPRINT S$;:
   USING "+#####.
   ###":LPRINT V*
   1000:PVV=PVV+V
   *V:NEXT I:
   CSIZE 1:LPRINT
310:LPRINT " SUM
   OF THE RESID
   UALS SQUARES":
   CSIZE 2:LPRINT
   :GOSUB 550:
   LPRINT " vv
   = ";:LPRINT PV
   V*1E6
315:REM COMPUTATION
   OF THE ACCUR
   ACY OF THE CIR
   CLE POINTS
320:"M"CLS :INPUT
   "NUMBER OF CIR
   CLE POINTS=";M
   :LF 2:J=1
330:E=360/M:CSIZE
   1:LPRINT " RE
   LIABILITY OF THE
   CIRCLE POINTS":
   LF 2
340 LPRINT "BEARING
   STANDARD
   DEVIATION(mm) "
   :CSIZE 2
   LPRINT
350:FOR I=0 TO M-1:
   EE=E*I:C=COS E
   E:S=SIN EE:
   GOSUB 500
360:S$="("+STR$ EE
   +")="
370:IF LEN S$<8LET
   S$=" "+S$: GOTO
   370
380:LPRINT S$;:
   USING "+#####.
   ###":LPRINT QV
   :H=LEN STR$
   INT QV:GOSUB
   490
390:NEXT I:LF 3
395:REM ILLUSTRATI
   ON FOR THE ADJ
   USTING CIRCLE
400:"X"LPRINT "
   ILLUSTRATION":
   LPRINT "
   FOR THE"
405:"X"LPRINT "ADJ
   USTING CIRCLE'
   ":LF 8
410:COLOR 2:GRAPH
   :GLCURSOR (108
   ,0):SORGN :X=1
   00:Y=0:
   GLCURSOR (X,Y)
   :S=SIN 10:C=
   COS 10: FOR I=1
   TO 36:Z=X*C-Y*
   S
420:Y=X*S+Y*C:X=Z:
   LINE -(X,Y),8,
   2:NEXT I:COLOR
   I=1 TO N:GOSUB
   540
430:Y=(K(I,1)-YS-Y
   O)*F-5:X=(K(I,
   2)-XS-XO)*F-4:
   G=SQR (X*X+Y*Y
   )/600:H=V*X/G
440:K=V*Y/G:
   GRCURSOR (Y+K,
   X+H):LPRINT "o
   ":NEXT I:J=1:Z
   =100:K=10
450:C=1:S=0:GOSUB
   500:GOSUB530:
   GLCURSOR (Y,X)
   :FOR I=K TO 360
   STEP K:C=COS I
   :S=SIN I
460:GOSUB500:
   GOSUB530:LINE
   -(Y,X),0,3:
   NEXT I IF J=1
   LET J=-1:GOTO
   450
470:IF J=-1 LET J=2
   :Z=0:K=20:GOTO
   450
480:TEXT :LF 15:
   END
490:GRAPH :
   GLCURSOR (144-
   H*12, 14):
   LPRINT "-":
   GLCURSOR (0,20
   ):TEXT :LPRINT
   :RETURN
500:Q1=(XO*C*AA+XO
   *S*AB+YO*C*AB+
   YO*S*BB)*2/R
510:Q2=C*C*AA+2*S*
   C*AB+S*S*BB:Q=
   Q2:IF J<2LET Q
   =Q1+Q2+RR
520:QV=MO*SQR Q*10
   00 RETURN
530:Q=QV/10:Y=(Z+
   J*QV)*S:X=(Z+J
   *QV)*C:RETURN
540:V=((K(I,2)-XS)
   *XO+(K(I,1)-YS
   )*YO+ZO-((K(I,
   1)-YS)^2+(K(I,
   2)-XS)^2)/2)/R
   :RETURN
550:GRAPH:
   GLCURSOR (40,0
   ):ROTATE 3
   :LPRINT "M":
   GLCURSOR (41,0
   ):TEXT :RETURN
```

Table 2

LIST OF COORDINATES

POINT NUMBER	Y-COORDINATE X-COORDINATE
1	+100.050 +0.000
2	+0.000 +99.950
3	-100.050 +0.000
4	+0.000 -99.950

DATA OF THE ADJUSTING CIRCLE

PARAMETER	VALUE (m) STANDARD DEVIATION (mm)
Xo=	+0.000 ±70.781
Yo=	+0.000 ±70.710
R=	+100.000 ±50.024

REFERENCE STANDARD DEVIATION

mo= ±10.004

RESIDUALS (mm)

v(1)= -49.999
v(2)= +49.999
v(3)= -49.999
v(4)= +49.999

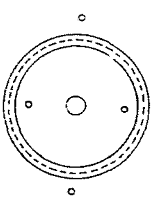
SUM OF THE RESIDUALS SQUARES

Σvv = +9999.997

RELIABILITY OF THE CIRCLE POINTS BEARINGSTANDARD DEVIATION

(0)= ±86.674
(45)= ±86.645
(90)= ±56.616
(135)= ±86.645
(180)= ±86.674
(225)= ±86.645
(270)= ±86.616
(315)= ±86.645

ILLUSTRATION FOR THE ADJUSTING CIRCLE



LIST OF COORDINATES

POINT NUMBER	Y-COORDINATE X-COORDINATE
1	-12.186 +99.254
2	+0.000 100.000
3	+12.186 +99.254

DATA OF THE ADJUSTING CIRCLE

PARAMETER	VALUE (m) STANDARD DEVIATION
Xo=	+0.097 ±82.087
Yo=	+0.000 ±2.901
R=	+99.902 ±81.679

REFERENCE STANDARD DEVIATION

mo= ±0.050

RESIDUALS (mm)

v(1)= +0.000
v(2)= -0.000
v(3)= +0.000

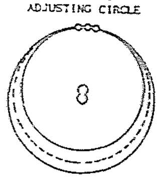
SUM OF THE RESIDUALS SQUARES

Σvv = +0.000

RELIABILITY OF THE CIRCLE POINTS BEARINGSTANDARD DEVIATION

(0)= ±0.500
(45)= ±23.724
(90)= ±81.730
(135)= ±139.738
(180)= ±163.766
(225)= ±139.738
(270)= ±81.730
(315)= ±23.724

ILLUSTRATION FOR THE ADJUSTING CIRCLE



LIST OF COORDINATES

POINT NUMBER	Y-COORDINATE X-COORDINATE
1	-17.365 +98.481
2	-8.716 +99.619
3	+0.000 +100.000
4	+8.716 +99.619
5	+17.365 +98.481

DATA OF THE ADJUSTING CIRCLE

PARAMETER	VALUE (m) STANDARD DEVIATION (mm)
Xo=	-0.035 ±23.632
Yo=	-0.000 ±1.221
R=	+100.034 ±23.453

REFERENCE STANDARD DEVIATION

mo= ±0.033

RESIDUALS (mm)

v(1)= -0.056
v(2)= +0.226
v(3)= -0.338
v(4)= +0.226
v(5)= -0.056

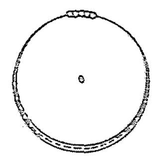
SUM OF THE RESIDUALS SQUARES

Σvv = +0.223

RELIABILITY OF THE CIRCLE POINTS BEARINGSTANDARD DEVIATION

(0)= ±0.233
(45)= ±6.799
(90)= ±23.485
(135)= ±40.174
(180)= ±47.086
(225)= ±40.174
(270)= ±23.485
(315)= ±6.799

ILLUSTRATION FOR THE ADJUSTING CIRCLE



LIST OF COORDINATES

POINT NUMBER	Y-COORDINATE X-COORDINATE
12	+23.200 +59.400
56	+25.100 +58.200
36	+27.600 +54.800
456	+27.000 +48.100
595	+18.500 +44.100

DATA OF THE ADJUSTING CIRCLE

PARAMETER	VALUE (m) STANDARD DEVIATION
Xo=	+52.013 ±29.967
Yo=	+20.001 ±54.025
R=	+8.046 ±33.890

REFERENCE STANDARD DEVIATION

mo= ±0.357

RESIDUALS (mm)

v(12)= -2.411
v(56)= +30.117
v(36)= -46.676
v(456)= +27.919
v(595)= -8.948

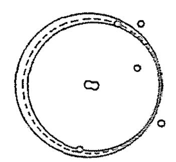
SUM OF THE RESIDUALS SQUARES

Σvv = +3951.161

RELIABILITY OF THE CIRCLE POINTS BEARING STANDARD DEVIATION

(0)= ±50.811
(45)= ±25.645
(90)= ±33.462
(135)= ±35.885
(180)= ±38.877
(225)= ±63.791
(270)= ±83.754
(315)= ±78.823

ILLUSTRATION FOR THE ADJUSTING CIRCLE



observation is zero, denominator of *Eq.* (21) is also zero. The program utilises an $m_0 = 0.05$ value.

The third example indicates that increase of number of points yields an improvement, even if the points are close to each other. Example four shows a generally positioned, non equally distributed point-arrangement. There are two maxima (120° and 280°) and two minima (50° and 150°) in standard error of cylindrical points. Optimal arrangement of points was examined, but our examples are backing the result of [8], which indicates that symmetrical positioning is the most advantageous case.

Conclusion

In this paper utilisation of linear observation equations was introduced in case of adjustment circle. By choosing suitably, it could be reached that the originally non-linear equation turns to linear one. The geodetic meaning of the parameters is not requested, computation of the important quantities — after the adjustment — is enough.

This solution was generally used at the well-known *Helmert* transformation, where a and b transformation parameters are computed instead of rotating angle and scale.

The solution introduced in this paper is suitable for cases of adjusting spheres — by means of use of the following auxiliary parameter —

$$s_0 = \frac{1}{2}\left(r^2 - x_0^2 - y_0^2 - z_0^2\right),$$

which gives a solution to determine the deformation of spherical containers.

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