PERIODICA POLYTECHNICA SER. CIVIL ENG. VOL. 36, NO. 2, PP. 139–146 (1992)

UTILIZATION OF AUXILIARY DATA BY GPS FOR BLOCK ADJUSTMENT

GY. IVÁN and F. PÉNTEK

Department of Photogrammetry
Technical University, H–1521 Budapest

Received: July 1, 1992.

Abstract

The use of kinematic GPS positioning in aerial triangulation can make the block adjustment methods more profitable, since its utilization reduces the number of the required ground controls. The experiments in the direction of it started at the end of the 80's in the countries of Western Europe and in the USA. This paper is a summary of the foreign experiences and describes a possible solution of the GPS-aided block adjustment. The paper is an extract of a presentation delivered at the IX. Seminar of Cosmic Geodesy held in Budapest on 14-th of November in 1990.

Keywords: block adjustment, GPS, auxiliary data.

Introduction

The utilization of auxiliary data looks back on long past in the photogrammetric block adjustment methods. The results of the modern, computer-aided navigation systems (CPNS, PICS) were used like auxiliary data in the 80's in addition to the data of the conventional process (statoscope, APR) (ACKERMANN, 1986a).

The more wide-ranging application of the Global Position Systems (in the next GPS) can be considered as a revolution of the positioning in the 90's. From the middle of the 80's the experiments and researches were started out for this direction. Many kinds of excellent and promising results had been known from the area of photogrammetric application.

The first NAVSTAR/GPS receivers appeared in Hungary at the end of the 80's. Thus the experimental measurements have been started first of all on the area of geodesy.

The foreign experiments have verified the large potential of GPS from the point of view of the photogrammetric applications (DETREKŐI and LÁNGNÉ VARGA, 1990).

In the first part of the paper we would like to show the advantages which arise from the use of GPS, in the aerial triangulation. Later on, taken into account the foreign experiments, we will show a possible solution of the treatment of photogrammetric auxiliary data by GPS, slightly deviated from the conventional procedure.

Utilization of GPS in Photogrammetry

There are two kinds of photogrammetric application of GPS based on the observational technique. The static observations are generally used for the measuring of the ground control, while the kinematic observation technique is served for direct determination of element of the exterior orientation. In the following we will deal only with the application of kinematic GPS observation in photogrammetry, because the static positioning is a little irrelevant, when we're talking about aerial triangulation.

Employment of the Kinematic GPS Positioning

Depended on the required accuracy the processing methods are the next:

- differential pseudorange
- phase-smoothed pseudorange
- phase reduction

The connection between the scale of mapping and the required accuracy of position of the perspective centres is known from Ackermann's experiments (ACKERMANN, 1986b). Taken into consideration only the 'Z' coordinate of the perspective centres, since we can determine it with the least accuracy by kinematic GPS positioning, the just mentioned connection is shown on *Table 1,* where we have used the required kinematic GPS technique instead of required accuracy of the perspective centres (HEIN, 1989).

Table 1
Connection between the scale of mapping and the kinematic GPS technique

Scale of mapping	Kinematic GPS technique
1:100000 – 1:25000	Differential pseudorange
1:10000 1:5000 – 1:1000	Phase-smoothed pseudorange
Coordinate determination	Phase reduction

Connection between the Photogrammetric and the GPS Data

The GPS, one of the up-to-date positioning systems, fits to the conventional photogrammetric systems, theoretically, the most harmoniously. But, since there are differences in the time and the position of observation between the two systems, therefore we have to determine the connection between the two observational methods.

Based on the foreign experiments, these connections and differences are the next:

- difference between the time of the GPS observation and the time of exposure.
  Therefore we have to indicate the time of exposure on the time-scale of the GPS receiver. This condition can effect large changes in the structure of cameras, because the sign of exposure-time must be fixed on the time-scale of receiver with 1 msec accuracy (HEIN, 1989).
  In this theme there is a promising result. When we use a photosensor between the lense and the photo, we can fix the time of exposure on the time-scale of receiver with several milliseconds accuracy (VAN DER VEGT, 1989).
- Resulted from the mentioned, ones the determination of antenna position of the receiver, (or the perspective centre), at the time of exposure, is a large problem.
  But the experiments have shown, that the joint adjustment of GPS and the conventional photogrammetric data provides of the order of several centimeters accuracy of position for the perspective centres. This accuracy is available with the use of simple, linear connections in the block adjustment (COLOMINA, 1989).
  The exceptional advantage of the joint adjustment is the drastic ground control reducing effect, since the GPS antenna positions are the control data. But it is known, the nearer control data are to the level of the flight, the more accurate we have to know the calibration data of the camera, since the errors of the interior orientation influence the accuracy of the ground points considerably (HEIN, 1989).
  As a consequence of this, if we'd like to carry out a GPS-aided block adjustment, we'll have to use a well-calibrated camera or to apply a self-calibrating program.
- The GPS antenna and the perspective centre of the photo do not coincide, thus we have to model this near constant excentricity.
- During the photogrammetric flight there is a drift of the GPS receiver of the order of several mm/sec. Of course we have to take into consideration the drift in our block adjustment program.

Joint Application of GPS and INS

The advantages of GPS/INS system can be summarized as follows:

- High frequency interpolation between the GPS positions.
- The INS can take over the positioning for a short time (approximately 50 sec), from the GPS when the receiver loses the sign.
- The determination of angles of the exterior orientation is more accurate by INS than GPS.

Joint Adjustment of GPS and Photogrammetric Data

Usually the joint adjustment program is based on a self-calibrating bundle adjustment program. But as mentioned previously, the connection between the GPS and the photogrammetric system must be modelled.

In order to determine this connection there are many algorithms in the literature. The relation between the GPS and the photogrammetric system is generally modelled by linear functions and transformations (ANDERSEN, 1989, COLOMINA, 1989).

But this modelling has a disadvantage, the large number of parameters in the adjustment damages the numerical stability of the system of the normal equations. Therefore we'd like to recommend an algorithm with few additional parameters, sketched on the next points (DETREKŐI, 1987).

A Possible Solution of the Block Adjustment with GPS Data

This algorithm is based on the utilization of additional parameters and the block adjustment with models.

Basic conditions:
We wouldn't like to deal with the method of GPS positioning in the next, thus the results of the kinematic positioning during the flight, the positions of antenna of the receiver along the flight line, and their covariance-matrix are known in some coordinate-system. If these conditions are satisfied we can perform the next.

Calculation of Antenna Position for the Time of Exposure

a. We can calculate an antenna position in between the two, known antenna positions surrounding the exposure, with simple linear interpolation, like here:

For example to the 'X' coordinate

$$X_e = \frac{X_3 - X_2}{t_3 - t_2}(t_e - t_2) + X_2,$$

here:

X_e – is the interpolated camera position,
X_2, X_3 – are the preceding and the following antenna position at the exposure,
t_2, t_3 – are the time of the preceding and the following observation at the time of exposure on the time-scale receiver,
t_e – is the time of exposure on the time-scale of the receiver.

b. Then we can calculate a weighted mean value to the position for the time of exposure. Here we take into account four antenna positions, two preceding and two succeeding the position of exposure. The weight of positions can be calculated in the next way:

$$p = \frac{C}{\Delta t}, \qquad C \text{ – is constant.}$$

here:

p – is the value of the weight,
Δt – is the time-difference between the exposure and the GPS observation.

The weight of the position, calculated with simple linear interpolation, is the mean value of the weights of the preceding and the following position.
Thus the antenna position at the time of exposure will be the next:

$$X_{gi} = \frac{X_{1i}p_{1i} + X_{2i}p_{2i} + X_{3i}p_{3i} + X_{4i}p_{4i} + X_e p_e}{p_{1i} + p_{2i} + p_{3i} + p_{4i} + p_e}$$

here:

X_{gi} – is the antenna position at the i-th exposure,
X_{2i}, X_{3i} – are the preceding and the following positions at the i-th exposure,
X_{1i}, X_{4i} – are the antenna positions preceding X_{2i}, and following X_{3i}
p_{ji} – is the weight of the j-th antenna position at the i-th exposure

These calculated antenna positions function as fictitious results during the next computation, therefore we have to know their covariance-matrix. But if we know the covariance-matrix of the GPS observations, we can compute the elements of this matrix by the error law. Since it has been one of our basic conditions, then it can be solved easily. If we'd like to take into account the errors of time-measuring we'll have to include the stochastic features of it in the matrix.

Use of the Calculated Antenna Positions in the Block Adjustment

So the calculated antenna positions and their covariance-matrix is available from the preceding computations. But our purpose is to create a possible connection between the antenna positions and the perspective centres. In order to determine it we have to take the following steps:

- If the relative position of the antenna and the camera wouldn't change during the flight, then the perspective centre places on a sphere with 'R' radius around the centre of the antenna. This excentricity can be measured with large accuracy under ground circumstances. But this condition is assured during flight conditions. This changes are generally modelled by time-dependent parameters. In order to model the changes of excentricity we have followed the next way in our algorithm:

The changes of excentricity have been modelled by changes of the radius of the above mentioned sphere. If the excentricity at the i-th exposure is 'R_i', then it can be written in this way:

$$R_i^2 = [R_0 + X_{gi}(a_s + a) + Y_{gi}(b_s + b) + Z_{gi}(c_s + c)]^2 \ , \qquad (1)$$

here:

X_{gi}, Y_{gi}, Z_{gi}	–	is the calculated antenna position,
a_s, b_s, c_s	–	are the parameters of a row,
a, b, c	–	are the parameters of the block,
R_0	–	is the distance between the antenna and the perspective centre measured under ground circumstances.

The row parameters indicate the changes of excentricity inside a row, while the block parameters play the same role in the block. It seems, that the excentricity depends on the antenna positions, as well as this mathematical model isn't attached as closely to the GPS technique as the models used abroad. In the future the kinematic GPS observations in Hungary can be considered as a base of refining of this simple model.

We can take into the block adjustment the *Eq.* (1) with a simple equation of condition:

$$(X_{gi} - X_{0i})^2 + (Y_{gi} - Y_{0i})^2 + (Z_{gi} - Z_{0i})^2 - R_i^2 = 0\,, \qquad (2)$$

here:

X_{gi}, Y_{gi}, Z_{gi} – are the coordinates of the i-th calculated antenna position,

X_{0i}, Y_{0i}, Z_{0i} – are the coordinates of the i-th perspective centre.

But, it seems, that this algorithm is undetermined. The assuring of the perspective position of the photos provides the determination of the algorithm written in the next point.

Assuring of the Perspective Position of the Photos

It is known from the photogrammetry that the intersection of two homologous rays is provided, when the two rays are complanar. In the following we'll use this condition of complanarity in order to provide the perspective position of the photos. We use homogeneous coordinates in the complanarity equation, because they are more treatable in our connections. Thus the condition of complanarity is the next:

$$\det \begin{vmatrix} X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 \\ X_{01} & Y_{01} & Z_{01} & 1 \\ X_{02} & Y_{02} & Z_{02} & 1 \end{vmatrix} = 0\,, \qquad (3)$$

here:

X_1, Y_1, Z_1 – the coordinates of a homologous point on the left image,

X_2, Y_2, Z_2 – the coordinates of the same point on the right image,

X_{01}, Y_{01}, Z_{01} – the coordinates of the perspective centre of the left image.

X_{02}, Y_{02}, Z_{02} – the coordinates of the perspective centre of the right image.

As seems, the *Eq.* (3) can act as an equation of condition in an adjustment method. But the four positions must be given in the same coordinate system.

If we give them in the coordinate system of left image, we'll attain the well-known block adjustment with independent models.

If we give them in that coordinate system where we had determined the GPS positions, we'll attain a bundle adjustment method. We have followed the second way as it is presented in the next section.

Joint Adjustment

After previous calculations we have two kinds of equation of condition, one for the connection between the GPS and the photogrammetric system (2) and one for the complanarity (3). In the adjustment the measured results are the calculated, fictitious antenna positions and the conventional image coordinates. The parameters are the elements of the exterior orientation of the photos and the row and block parameters.

The approximation values for the exterior orientation can be got from GPS or GPS/INS or conventional observations.

The approximation values for the row and block parameters are probably small, thus their approximation value can be zero.

About the weighting:

The weight-matrix of the image-coordinates a unit-matrix. Of course, there is no correlation between the GPS antenna positions and the image coordinates. The weight-matrix of the calculated antenna positions can be computed in the next way (DETREKŐI, 1987):

$$\mathbb{P}_g = m_{0f}^2 \mathbb{M}_g^{-1} ,$$

here:

$\mathbb{P}$ – is the weight-matrix of the calculated antenna positions,
m – is the r. m. s. value of the photogrammetric weight unit,
$\mathbb{M}$ – is the covariance-matrix of the calculated antenna positions.

The number of the equation of condition is the number of all possible homologous points plus the number of the photos.

In the adjustment we can take into account the possible ground controls, if we know their position in that coordinate system, where our GPS observations have been given. If it isn't realized, we have to take the parameters of the datum, between the two coordinate systems, into the adjustment. The ground controls can be taken into consideration by conditional and/or force conditional equations. But the use of force conditional equations is critical, because a false force is able to degrade the accuracy of the aerial triangulation and we have enough control data from the kinematic GPS observations. Moreover we are able to carry out the aerial triangulation without ground control.

Advantages of the Method

The photogrammetric data are separated as considerably as possible from the control data, thus the controls break the homogeneous unity of the photogrammetric data with little degree. Of course we must strive to reduce the ground control, keeping the requirements of accuracy in view, because of the economical application and the above mentioned considerations.

Disadvantages of the Method

The adjustment isn't able to give coordinates directly for the ground points we have measured on the photos. However, after the adjustment, as we'll see in the following, we can get them from a simple calculation. The stochastical features of the ground points can be derived from the use of the errors law as appropriate.

In order to assure the perspective position of the photos, it is suitable to measure homologous points as many as possible. Thus the probability of the gross errors increases, but their account is easier too, therefore the block adjustment program must possess filtering procedure of the gross errors, or instead of the least squares method we use a robust estimation procedure , for example the Danish method.

The Computation of Coordinates of the Ground Points

After the adjustment the homologous rays are complanar, thus their point of intersection will give the convenient ground point. The coordinates of this point can be calculated in the next way:

$$\begin{aligned} X &= (X_{01} - X_1)(X_2Y_{02} - Y_2X_{02}) - (X_{02} - X_2)(X_1Y_{01} - Y_1X_{01}), \\ Y &= (Y_2 - Y_{02})(X_1Y_{01} - Y_1X_{01}) - (Y_1 - Y_{01})(X_2Y_{02} - Y_2X_{02}), \\ Z &= (Z_2 - Z_{02})(X_1Z_{01} - Z_1X_{01}) - (Z_1 - Z_{01})(X_2Z_{02} - Z_2X_{02}), \\ W &= (Y_1 - Y_{01})(X_{02} - X_2) - (Y_2 - Y_{02})(X_{01} - X_1). \end{aligned}$$

The marks are shown in the *Eq.* (3).

Of course these are projective coordinates. If we would like to get the Euclidean coordinates we have to follow the next way. The connection between the Euclidean and the projective coordinates is the following:

$$X_e = \frac{X}{W}; \qquad Y_e = \frac{Y}{W}; \qquad Z_e = \frac{Z}{W}, \qquad W \neq 0;$$

here:

X_e, Y_e, Z_e – are the Euclidean coordinates.

The coordinates are defined in that coordinate system, where our GPS observations are given. The covariance-matrix of the ground points can be derived from the errors law as appropriate.

Conclusions

The block adjustment method, described in the previous point, fully depends on the auxiliary data derived from the GPS observations. The experiments have shown that the use of the GPS for the aerial triangulation leads to a drastic reduction of ground control. The most important advantage of this method is that the photogrammetric data is separated as considerably as possible from the control data, thus this method breaks the homogeneous unity of photogrammetry to a lesser degree.

References

ACKERMANN, F. (1986a): Utilization of Navigation Data for Aerial Triangulation. *ISPRS Congress,* Rio de Janeiro, 1986, Commission III.

ACKERMANN, F. (1986b): Use of Camera Orientation Data in Photogrammetry – A Review. *ISPRS Comm. I Symposium,* Stuttgart.

ANDERSEN, O. (1989): Experience with Kinematic GPS during Aerial Photography in Norway. *42-nd Photogrammetric Week,* Stuttgart, 1989.

COLOMINA, I. (1989): Combined Adjustment of Photogrammetric and GPS data. *42-nd Photogrammetric Week,* Stuttgart, 1989.

DETREKŐI, Á. – LÁNGNÉ VARGA, M. (1990): A GPS felhasználása a fotogrammetriában. (Utilization of GPS in Photogrammetry) (in Hungarian). Publication of the Institute of Geodesy of the Technical University of Budapest, 1990.

HEIN, G. W. (1989): Precise Kinematic GPS-INS Positioning. A Discussion on the Applications in Aerophotogrammetry. *42-nd Photogrammetric Week,* Stuttgart, 1989.

VAN DER VEGT, H. J. W. (1989) : GPS Test Flight Flevoland. *42-nd Photogrammetric Week,* Stuttgart, 1989.

Address:

Gyula IVÁN and Ferenc PÉNTEK
Department of Photogrammetry
Technical University of Budapest
H–1521 Budapest