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CRACKS AS IMPORTANT CONSTITUENTS OF STRUT AND TIE MODELS

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Abstract

Construction of strut and tie model (STM) at D-regions is based on elastic stress distributions by imagining the forces as resultants of stresses. Beside 'smooth' stress-trajectories there are 'turbulent' places having a decisive influence on the cracks. Therefore, it is useful to use alternative STMs, one constructed on the base of smooth trajectories, and another which fits the turbulent stresses. Examples are given for different types of structural elements.

Keywords: strut and tie model (STM), elastic stress-trajectories, smooth and turbulent stress-trajectories, alternative STMs for one structural element.

Principles of the Strut and Tie Model

Principles of the strut and tie model, and a short review of its application is given according to SCHLAICH (1991) as follows. Internal stresses of RC structures are characterized by bending moments, axial and shear forces that are determined using well-known methods of the structural analysis. Connections between bending moments and deformations as well as distributions of stresses due to internal forces and moments are given on the base of a slightly modified elementary bending theory of bars which also takes the specific behaviour of the structural concrete into account.

However, sections or regions of an RC structure may behave differently and a linear distribution of axial strains and stresses due to the so-called Bernoulli–Navier hypothesis can only be assumed for limited parts of the structure, referred to as B-regions. (*Fig. 1*).

Sections or regions where the distribution of strains and stresses significantly differs from those obtained by the elementary bending theory of bars are called D-regions. At D-regions, e.g. near concentrated loads, frame corners (*Fig. 2*), etc. assumptions of elementary bending theory of RC beams have to be replaced by those of a strut and tie model. B-regions and D-regions can be separated using Saint-Venant's principle. The first

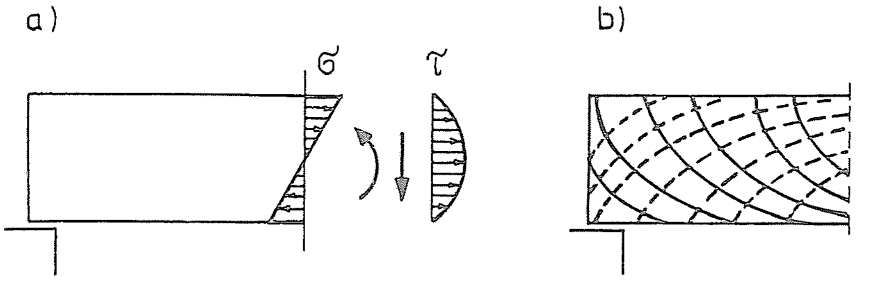

Fig. 1. Elementary bending theory for prismatic beams

step of developing a strut and tie model is the assignment of D-regions of the structure in question, then, more or less correct distributions of elastic stresses and the principal stress directions are to be determined for the D-regions (for example by FEM analysis) (*Fig. 3a*). This way more or less correct stress diagrams at the boundaries of the D-region also become known.

Assuming that pencils of trajectories of compressive and tensile principal stresses in a D-region represent 'load paths' of compressive and tensile forces and replacing these pencils by properly directioned straight struts and ties, respectively, a contiguous and rigid plane truss or space grid can be constructed (*Fig. 3b*).

This truss or grid is called the replacement strut and tie model of the D-region. Bar forces acting at the struts and ties are the resultants of compressive and tensile stresses. The directions of struts have to be taken in the average direction of the trajectories of compressive stresses and located about the central lines of the pencils. The ties should follow the tensile stresses in the same way.

Strut and tie modelling obviously provides the structural analyst with some freedom of choice that can be used to aim either at the safest or at the cheapest or at an otherwise optimised solution.

For practical reasons (e.g. to produce a simpler replacement truss or to simplify the manufacturing of the reinforcement) one usually does not closely follow the principal compressive and tensile stress directions. In this case it is necessary to consider the consequences of these deviations, that is, to check the equilibrium and to adjust the amount of reinforcement for taking into account its deviations from the principal directions (*Fig. 4*).

If modelling does not closely follow the stress-flows, it can cause incompatibilities in the corresponding strains, that means, cracks and plastic deformations have to develop. It is well known that concrete has a low tensile strength and permits limited plastic compressive deformations. To

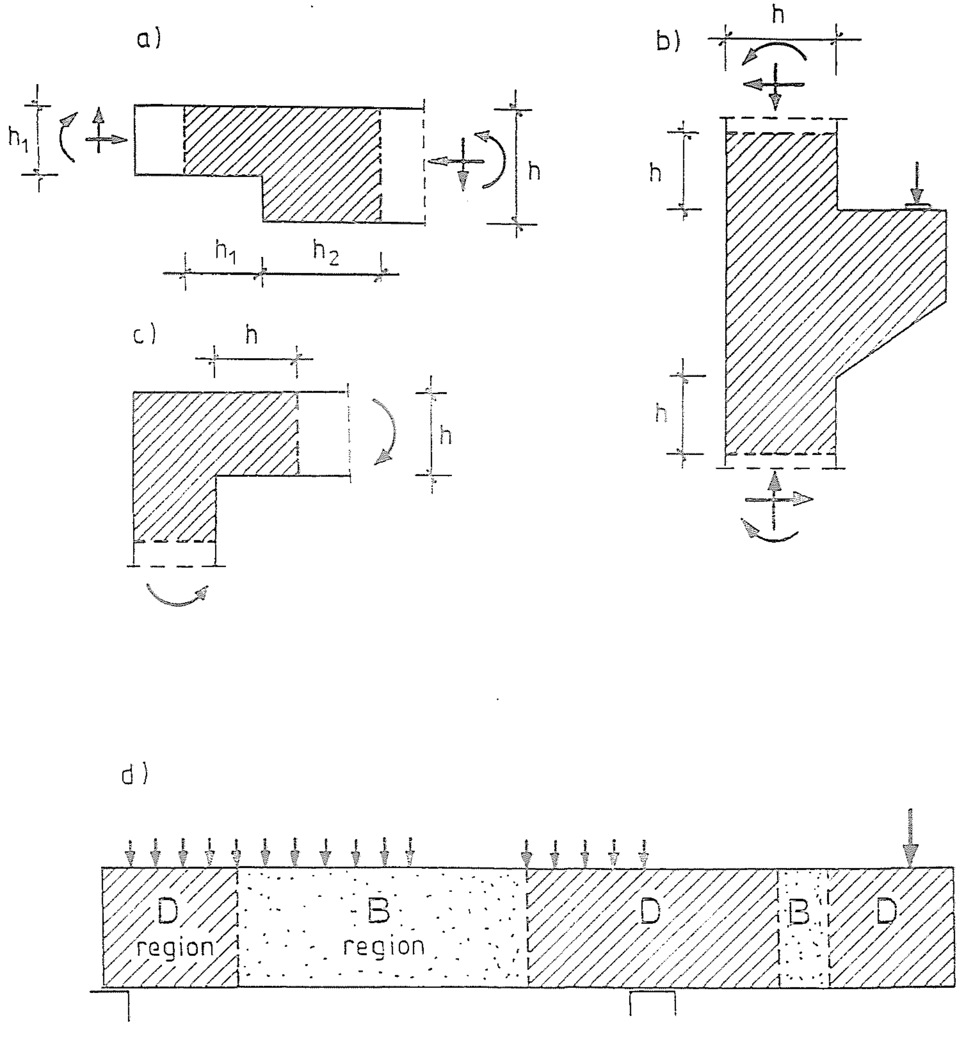

Fig. 2. Regions where there are non-linear strain distributions, separation of sections

avoid developing of wide cracks and exceeding plastic limit an additive reinforcement of two directions has to be used.

The Problems of Strut and Tie Model

The use of strut and tie models is strongly hampered by problems, not perfectly clarified so far, as follows:

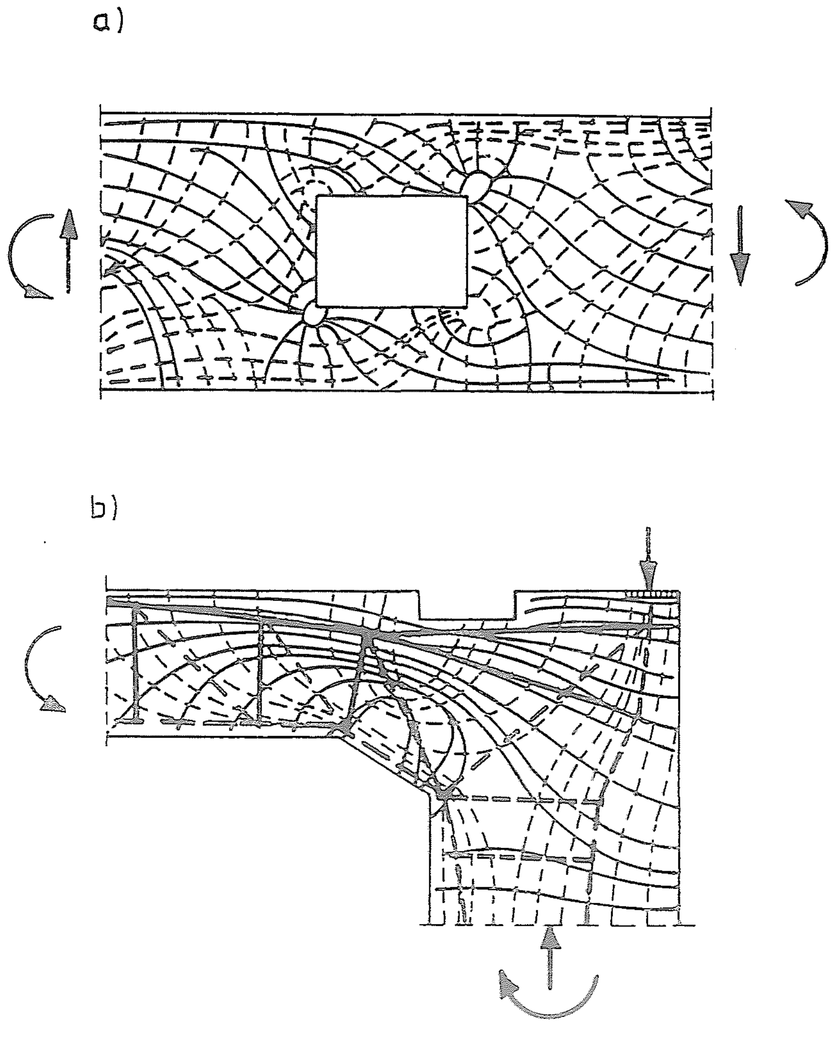

Fig. 3. Principal stresses and the image of forces

First Group of Questions

In order to determine whether principles used heretofore form a sufficient base to develop strut and tie models, which can properly model the real behaviour of structures, it is necessary to improve the adequateness of the modelling by refining the fundamental assumptions.

Questions connecting to this are as follows:

- What dense should a replacement truss be?

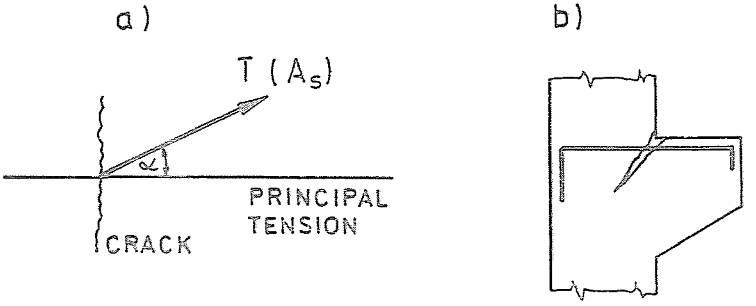

Fig. 4. Correction of reinforcement

- What are the physical limits for constructing the truss?
- How does the reinforcement influence the truss?
- When using two strut and tie models for the same structures how can the load be split into parts born by each model?
- How can the effect of the prestressing be taken into account in strut and tie modelling?
- How have statically indeterminate strut and tie models to be correctly solved ?
- How does a complicated cross-section influence the modelling?

Second Group of Questions

These are related to the accuracy of calculation. Connected to this one can ask:

- How does the deformation of strut and ties influence the action- effect of the truss elements?
- How can be compensated the neglection of the compabitility condition for the changes in length of the fictitious bars?
- What is the minimal amount of reinforcement for assuring the 'sufficient ductility'?
- What kind of safety measures have to be used to avoid erroneous dimensionings?
- What kind of results do give the comparison between the calculated and the measured values?
- How does the bond change in nodes?
- How does the deviation of the strength of the concrete influence the results obtained by STMs?

The Influence of Cracks on the Strut and Tie Model

Cracks on a well-designed structure or structural element gradually appear as the intensity of the load increases, they are uniformly distributed and not concentrated in narrow strips of the structure, their widths remain moderate in the service state of the structure.

It is impossible to avoid cracks at the level of load of service state, even in case of optimal design, however, crack widths and the crack pattern can be influenced in many ways. Factors influencing the crack pattern are as follows (*Fig. 5*):

- the geometry and cross-section of the structure,
- the loads and their characteristics,
- the variance of strength of the concrete,
- the concrete covering the reinforcing bars,
- the temperature and the free motion hinder,
- the amount of reinforcement,
- the diameter of reinforcing bars,
- the distances of reinforcement,
- the direction of reinforcement (how it follows the direction of principal tensile stresses),
- the types of reinforcement (normal, prestressed or mixed),
- the bond and the anchorages.

At abrupt changes of the cross-section, so-called stress peaks develop (*Fig. 6*). Here the maximum stresses can multiply exceed the average values calculated by usual methods. Steep changes in stresses especially in tensile stresses are hardly born by materials having a limited ductility like concrete. Around stress peaks, tensile stresses quickly increase and exceed the tensile strength of the concrete. Cracks arise, large deformation of the tensile reinforcement starts and plastic zones develop while other parts of the structure are only in elastic range. The appearance of cracks signs that the equalizing of stresses has begun.

Cracks at stress peaks have a decisive influence on the load-bearing capacity of structures. These stress peaks form only small parts of the whole system of stresses and they can hardly be fit by any strut and tie model. However, disregarding them would be a bad mistake. Researches prove that unlike the other structural parts, where tensile forces can be conveniently covered by simple webs of orthogonal reinforcement, at stress peaks this method only gives a reduced-value solution both in ULS and SLS.

Researches have also shown that at places where stress peaks can develop the load bearing capacity can be increased by 20-30 % if reinforcing bars are put at right angles to the cracks. The solution can be improved

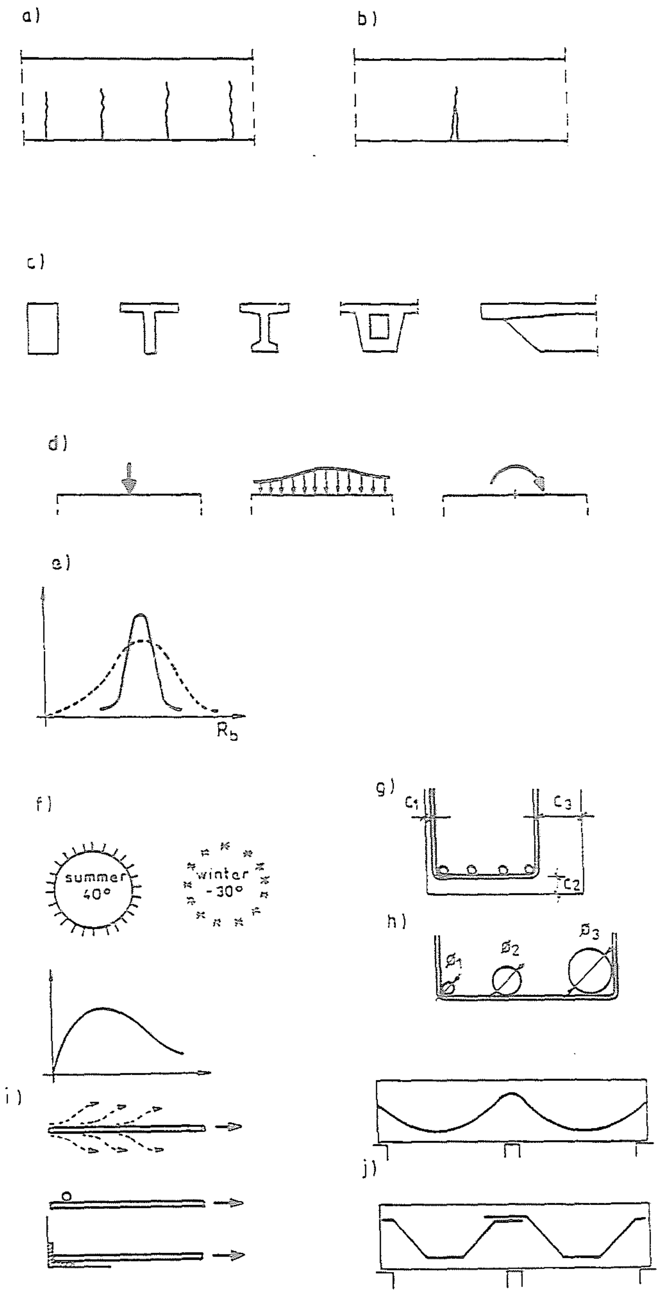

Fig. 5. Factors influencing cracks

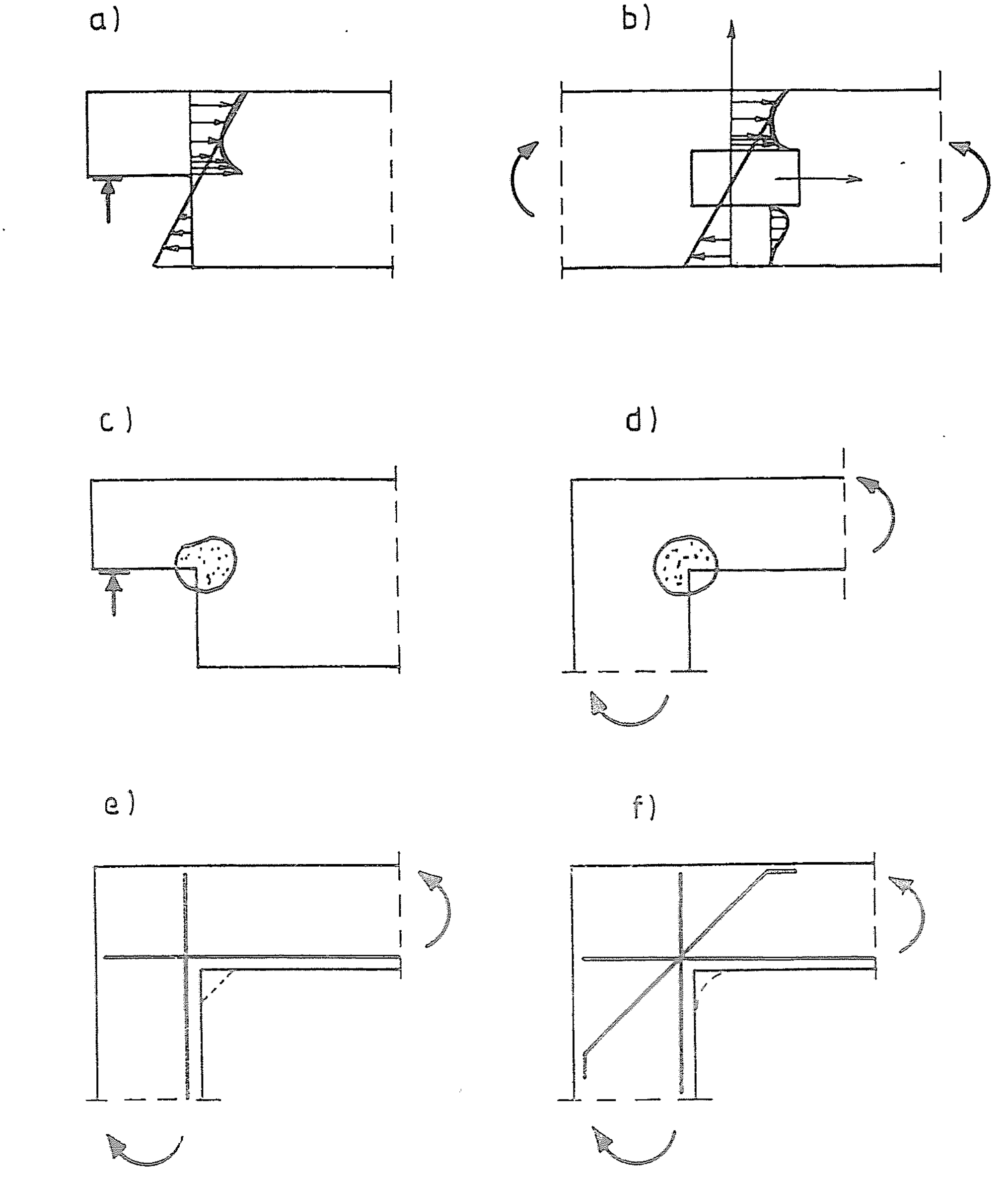

Fig. 6. Stress peaks, decisive effect on cracks

by application of wedgings, roundings up, that is, gradual and not abrupt changes of the cross-sections. These geometrical refinements also permit good possibilities to refine the reinforcement as well.

As previously mentioned, the reinforcement has a decisive effect on cracks. The correct direction of tensioned bars of strut and tie model around the stress peaks has a great importance because this gives a great influence on behaviour at SLS.

In the subsequent sections typical cracks on different structures are shown that has to be taken in consideration in constructing strut and tie models.

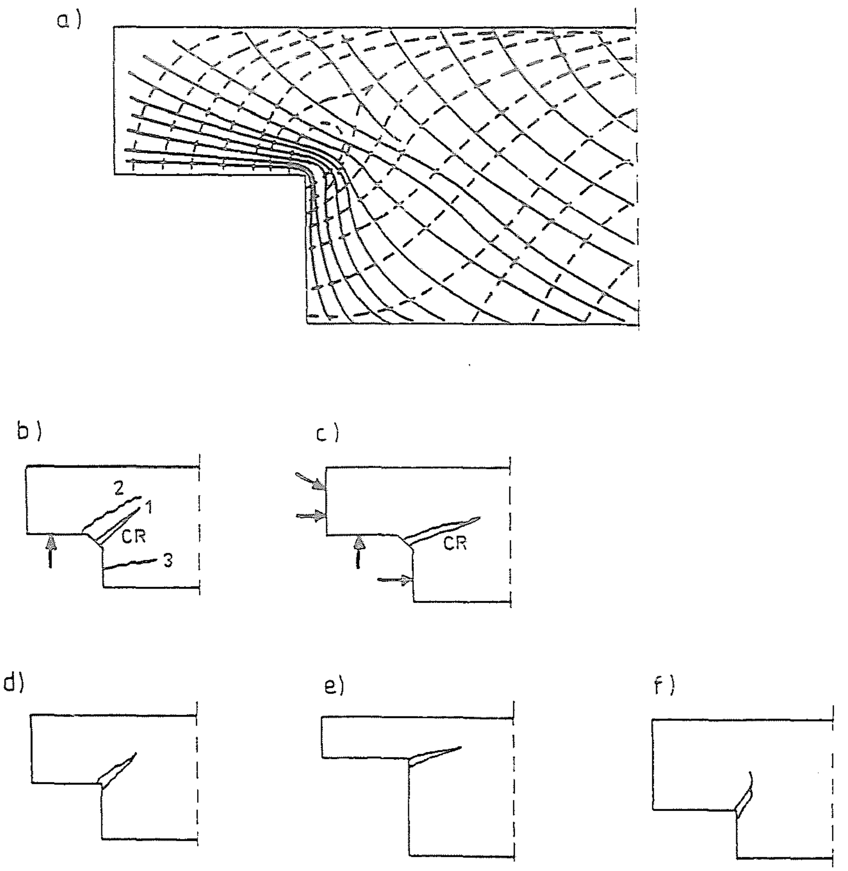

Fig. 7. Principal stresses and what influences the cracks

Beam with Dapped End

Tests on beams with dapped end show that the first crack appears at relatively low load level, (at about 25 % of the ultimate load) and it starts with nearly 45° angle at the corner (*Fig. 7*). If the beam has a correct reinforcement, the number of cracks increases to 2–3, while the length and the width of the first crack are growing. The place of the starting points and the direction of cracks are influenced by the geometrical proportion of the dapped end (such as wedging, rounding, up, etc.). Two types of the strut and tie models are recommended to use (*Fig. 8*). The load has to be split into two parts according to the proportion of stiffness of the two models.

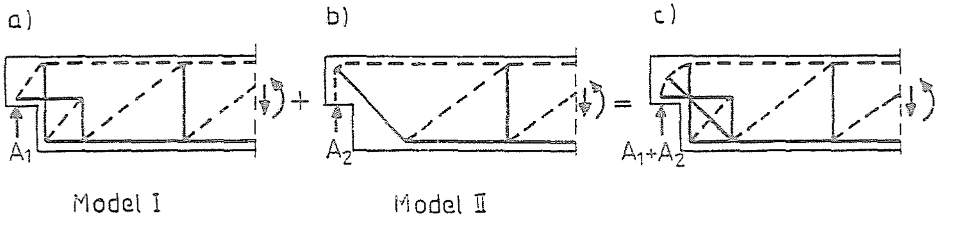

Fig. 8. Alternative models for the same structure

The Case of Corbel and Cantilever

Tests on corbels show four characteristic failures: bending, shear, diagonal splitting and crushing under the load (*Fig. 9*). In the first three cases characteristic cracks develop which feature the manner and the cause of the failure. If the bracket is connected to a column then the load has an influence on the direction of the bending cracks (*Fig. 9*).

When ratio a/d is higher than 1 then vertical stirrups are more effective than horizontal ones (*Fig. 10*). Cracks on the bracket show the importance of using oblique reinforcement. Doing so, the basic strut and tie model has to be completed with ties crossing the cracks (*Fig. 11*). Alternative solution for the struts is also given on *Fig. 11*. Again, the load has to be divided between the two models.

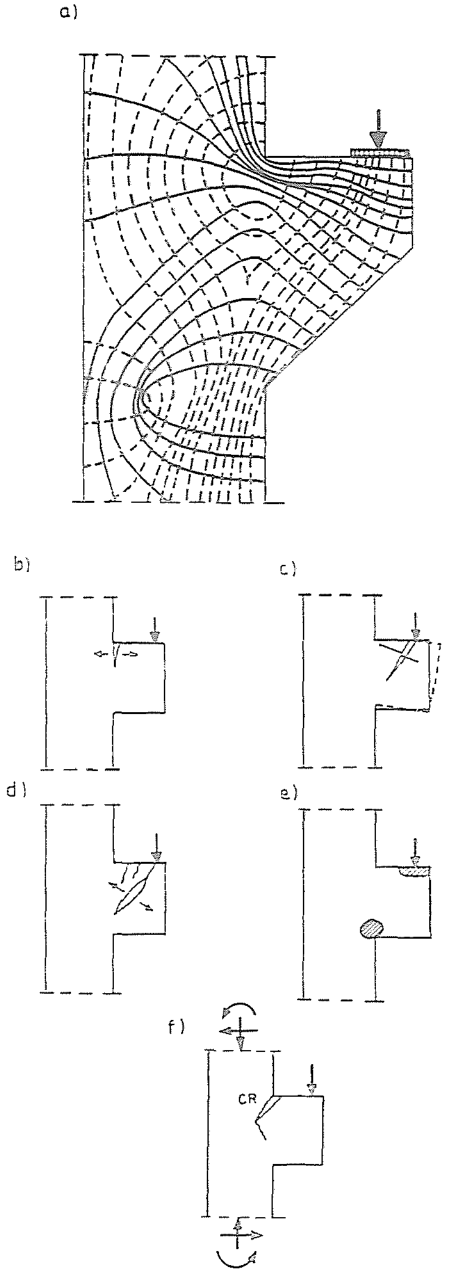

Fig. 9. Principal stresses and failure modes

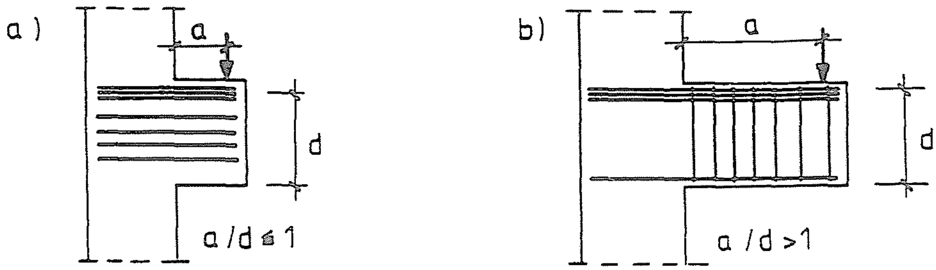

Fig. 10. Main reinforcements for corbel and cantilever

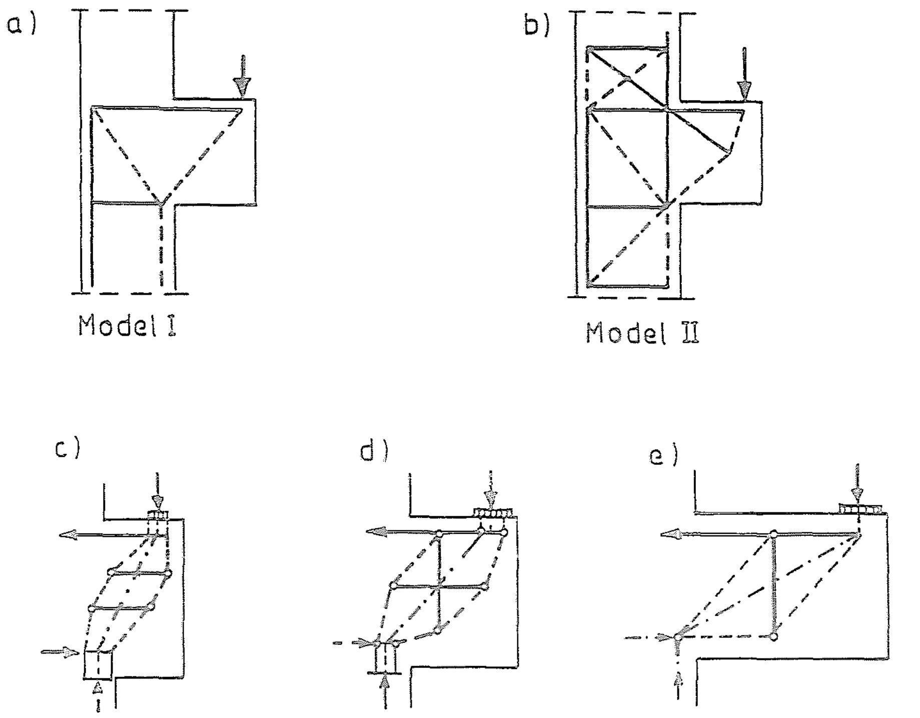

Fig. 11. Alternative models of corbel and refinements of struts

Frame Corners

The action effects arising on frame corners are very different, depending on stiffness ratio and loads. There are two characteristic cases: frame corners loaded by 'opening' moment and those loaded by 'closing' ones (*Fig. 12*).

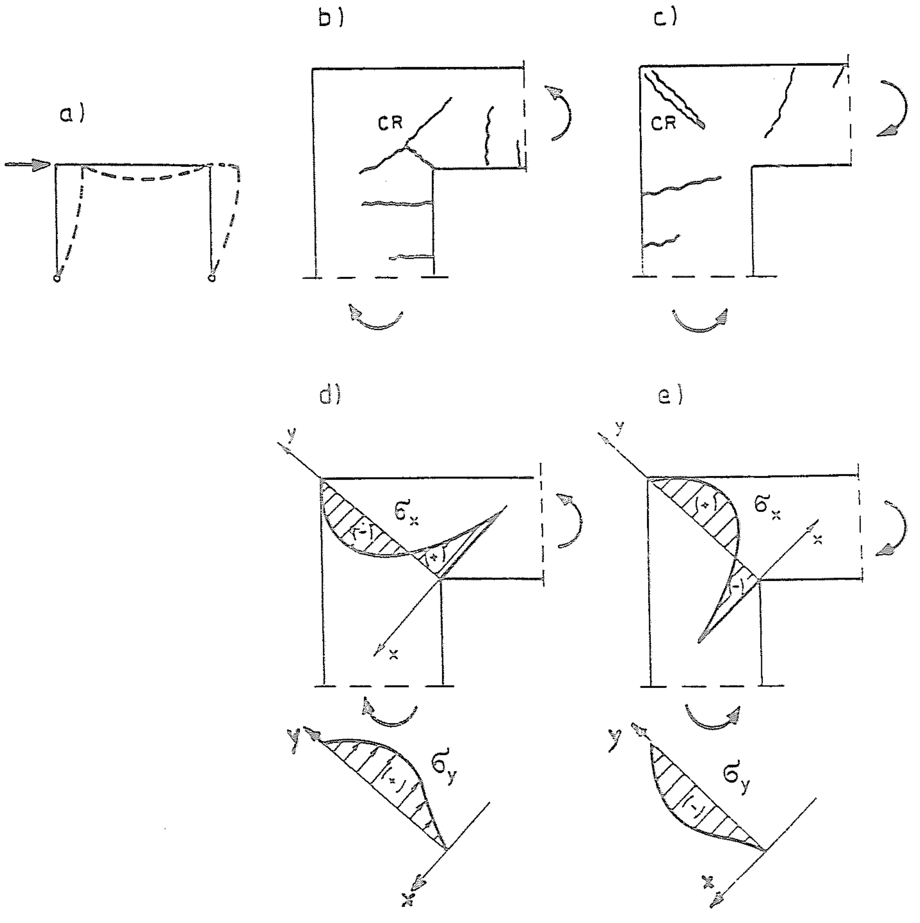

Fig. 12. Characteristic cracks and stresses at frame, corners

The cracks and their forms depend on the stresses of the used reinforcement type.

The diagonal cracks are deep on the frame corner if they are loaded by a 'closing' bending moment, therefore, taking off the tensile forces we need more ties than one. Example is given in *Fig. 13.*

At frame corners loaded by 'opening' bending moment, the crack starts from the inside corner and after a short way, it splits into two directions perpendicular to the original direction (*Fig. 12*). This again makes necessary to use two strut and tie models. The basic model and the alternative version is shown in *Fig. 14.*

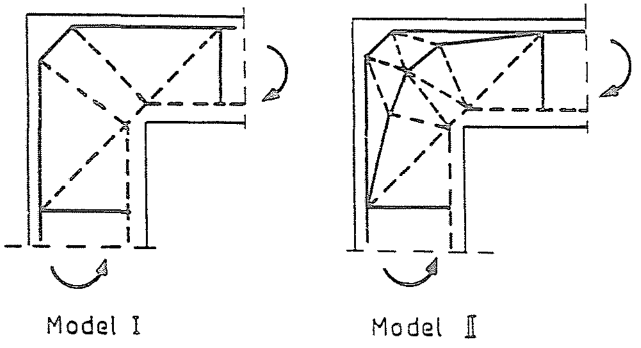

Fig. 13. Alternative models for corners with 'closing' bending moment

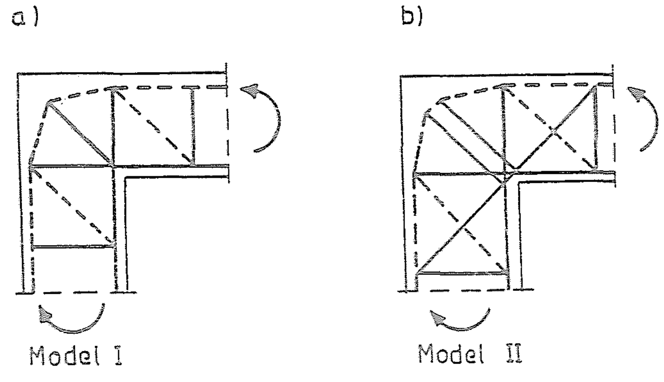

Fig. 14. Alternative models for corners with 'opening' bending moment

Openings on Beams

One of the most uncomfortable tasks for a designer is to make openings on structural parts having great statical importance. Nevertheless, the special difficulties arising in these cases have to be solved in a safe way.

Openings in homogeneous stress fields cause singularities in the stresses and make rise special cracks in well determinated places (*Fig. 16)*.

Possible strut and tie models for these cases are given in *Fig. 17.* In these cases it is also useful to use the combinations of two strut and tie models or to use a refined model.

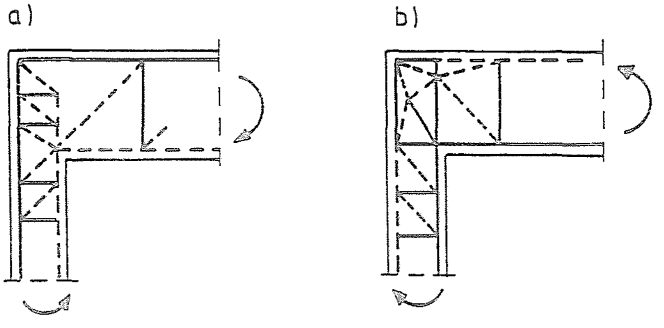

Fig. 15. Influence of geometry proportion on the model

Deep Beams

If a structure bears large loads, deep beams have to be used. A survey of the literature shows that a lot of different recommendations have been worked out to control the stresses and to design the reinforcement in deep beams.

Tests show four characteristic failure modes (*Fig. 18*). Up to now methods mainly concentrated on avoiding bending failure but did not properly discuss the other cases, e.g. the large local stress peaks caused by large concentrated forces. In the case of normal beams, second, third, etc. cracks often rise in slightly different directions, while in the case of deep beams this divergence cannot be observed.

To approach correct force flows we again need two combinations of strut and tie models (*Fig. 9*).

Tests also show that the cracks can be essentially influenced by the proportion of horizontal and vertical reinforcement. As a rule, a larger load bearing capacity can be achieved by an enlarged vertical proportion of reinforcement. This also supports the usefulness of the application of a second model as shown in *Fig. 19.*

Conclusions

Construction of strut and tie models is based on elastic stress distributions, by imagining the strut and tie forces as resultants of stresses.

Abrupt changes in geometry at D-regions of structures cause singular stresses or turbulent places which disturb the generally smooth trajectories of principal stresses. Unlike the places where the trajectories can be easily

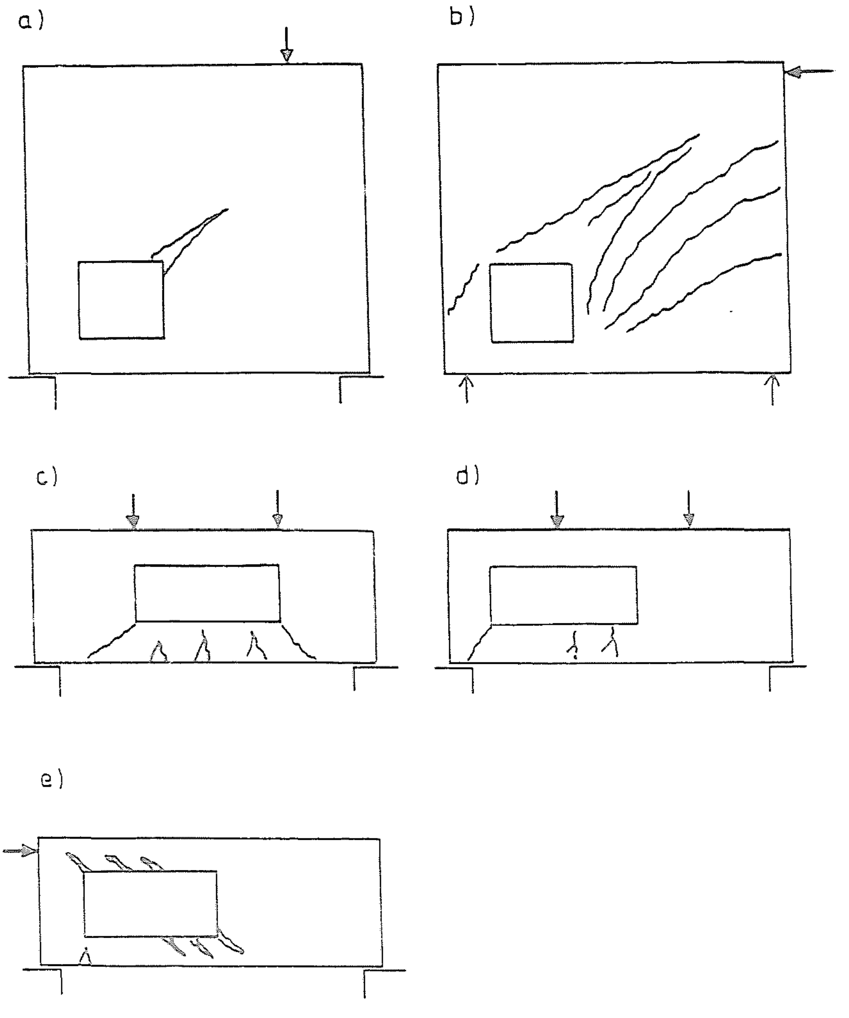

Fig. 16. Cracks around openings

described by a strut and tie model, forces which arise at the places of singularities cannot be fit in a simple way by commonly used strut and tie models. If we force to fit these forces by the model, we easily lose the simplicity of the calculation.

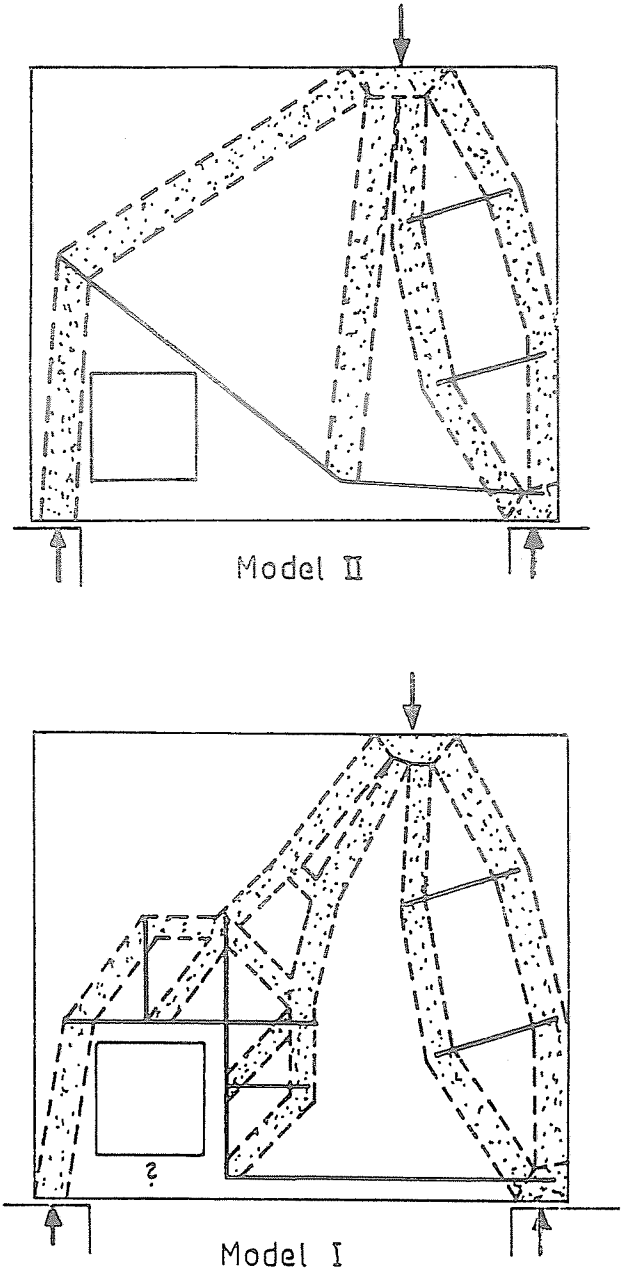

Fig. 17. Alternative models for a deep beam with opening

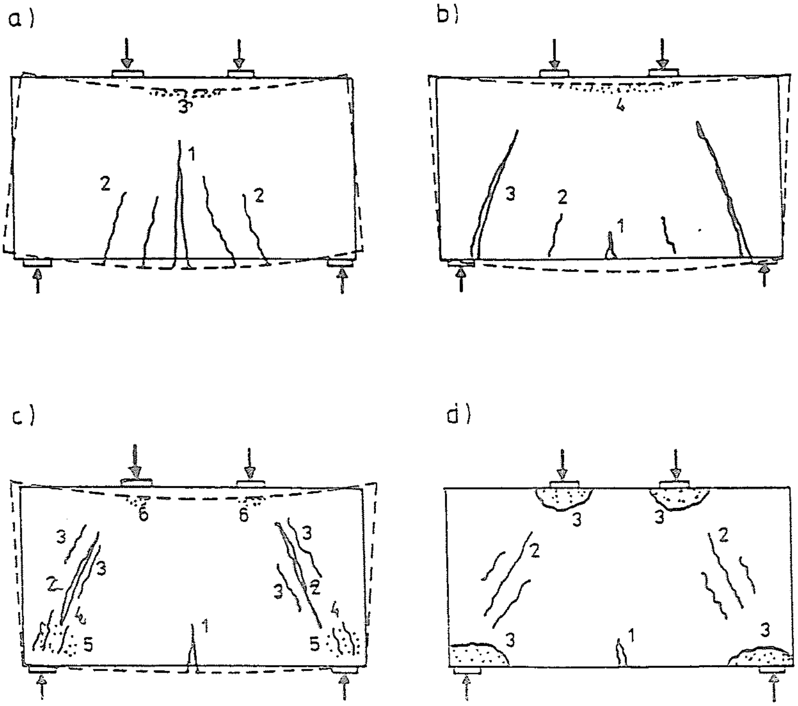

Fig. 18. Characteristic cracks and failures of deep beams

However, these turbulent places have decisive influence on the crack pattern at the D-regions and their existence must not be disregarded in the calculations. Taking into account tensile forces at these places needs a special treatment. It is necessary to construct alternative models to control the first cracks appearing at singular places. This can be done by the following way: beside of the strut and tie model, constructed on the base of smooth stress trajectories, also a second model has to be constructed the ties (or tensile forces) of which are passing through these places and make the stress peaks to distribute. The main part of this article shows such models for different structures.

The load distribution between two replacing models can be divided – in lack of a better solution – according to the proportion of stiffness of the two models. It also has to be taken into consideration that codes require more than one load case or load combination.

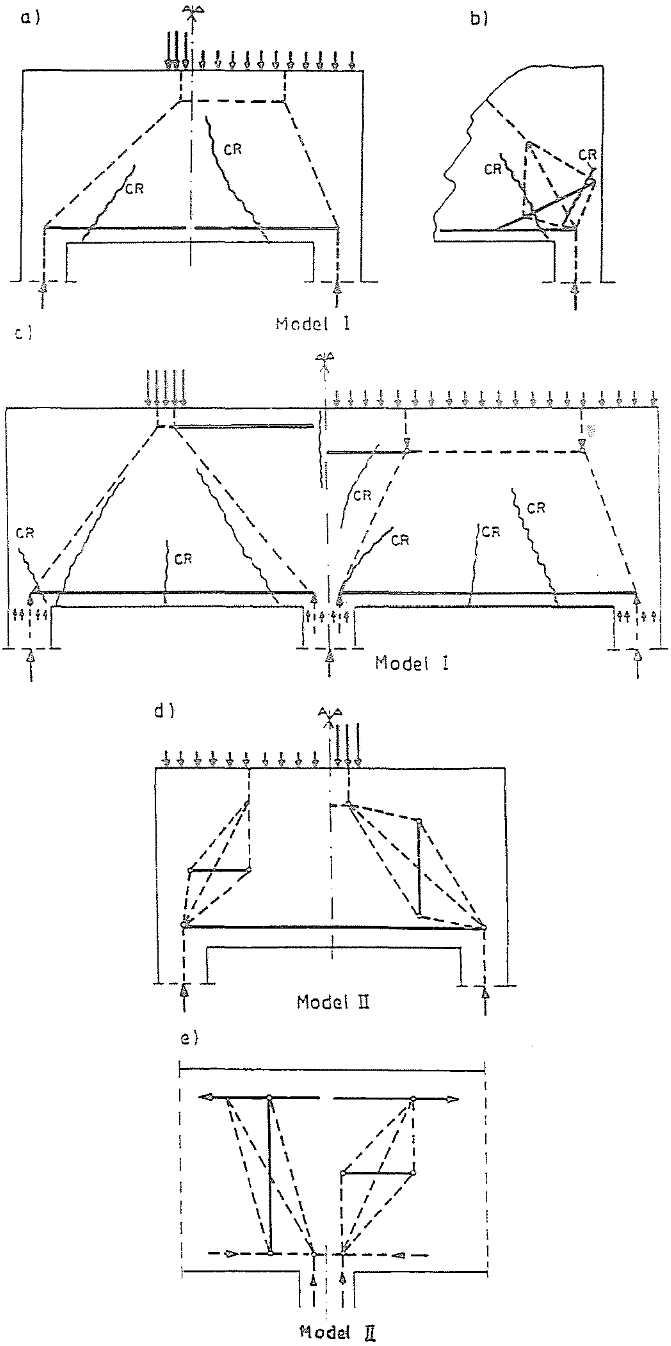

Fig. 19. Alternative models for deep beams

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