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COMPARATIVE ANALYSIS OF WATER–HAMMER CALCULATION BY THE APPROXIMATE AND THE COMPLETE METHODS OF CHARACTERISTICS

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Introduction

The object of the paper is a comparative analysis of water-hammer calculation by the approximate and complete method of characteristics, i.e. taking into consideration simplified or complete differential equations describing the unsteady flows of water in pipelines. With regard to complete characteristics equations the liquid flow velocity in the pipeline and the slope of the pipe are additionally analyzed.

The paper also deals with the effect of some parameters upon the pressure head in unsteady state. Such parameters as flow velocity of liquid in the steady state conditions (v_0), absolute roughness of pipeline (k), and number of scanning sections of a pipe (N) have been taken into account.

Attention has been paid to the necessity of satisfying Courant's stability condition while choosing the appropriate sizes of the characteristic meshes.

Notation

a – velocity of pressure wave propagation [m/s];
A – cross-sectional area of pipe [m^2];
D – internal diameter of pipe [m];
E – elasticity modulus of pipe [Pa];
E_C – modulus of fluid elasticity [Pa];
f – friction coefficient;
g – acceleration of gravity [m/s^2];
H – pressure head [m];
k – absolute roughness of pipe [m];
L – length of pipe [m];
N – number of scanning sections of a pipe;
p – pressure [Pa];

s – thickness of pipe wall [m];
t – time [s];
T_z – time of valve closing [s];
v_0 – initial flow velocity of liquid [m/s];
v – mean flow velocity of liquid [m/s];
x – abscissa along the pipeline [m];
ν – kinematic viscosity coefficient of liquid [m^2/s];
ρ – density of the liquid [kg/m^3].

Solution of Water-Hammer Problem by the Characteristics Methods

In order to take into account the effect of the fluid velocity upon the pressure head during water-hammer phenomenon the fundamental differential equations describing this phenomenon take the form of [7, 8, 9, 14, 15]

$$\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} + g\frac{\mathrm{d}z}{\mathrm{d}x} + \frac{f}{2D}v|v| = 0, \tag{1}$$

$$a^2\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial t} = 0. \tag{2}$$

The first equation is an Euler's equation of motion known in hydraulics, the second one is an equation of continuity. The above equations form a pair of quasi-linear partial differential equations of hyperbolic type containing two dependent variables, i.e. velocity v and pressure p as well as two independent variables, i.e., abscissa x along the pipeline axis and time t.

Approximate Method of Characteristics

The original set of partial differential equations (1) – (2) will be replaced by two sets of ordinary differential equations [2, 3, 4, 5, 7, 13, 15, 16]

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{g}{a}\frac{\mathrm{d}H}{\mathrm{d}t} + \frac{f}{2D}v|v| = 0, \tag{3}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = +a \tag{4}$$

and

$$\frac{\mathrm{d}v}{\mathrm{d}t} - \frac{g}{a}\frac{\mathrm{d}H}{\mathrm{d}t} + \frac{f}{2D}v|v| = 0, \tag{5}$$

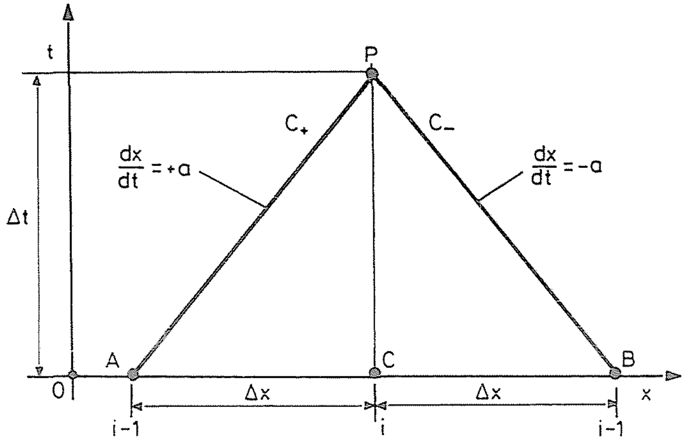

Fig. 1.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -a. \tag{6}$$

Equations (4) and (6) are equations of characteristics, while (3) and (5) are compatibility equations (of invariants on appropriate characteristic).

The characteristics equations can be presented graphically as shown in *Fig. 1.* For a constant value of pressure wave velocity a, lines AP and BP are straight lines.

Equations of invariants (3) and (5) (compatibility equations) transcribed in form of ordinary differential equations can now be transformed into appropriate finite-difference equations. Thus we shall have:

$$\frac{v_p - v_A}{t_p - 0} + \frac{g}{a}\frac{H_p - H_A}{t_p - 0} + \frac{f}{2D}v_A|v_A| = 0 \tag{7}$$

and

$$\frac{v_p - v_B}{t_p - 0} + \frac{g}{a}\frac{H_p - H_B}{t_p - 0} + \frac{f}{2D}v_B|v_B| = 0. \tag{8}$$

As it is seen in *Fig. 1* the difference of ordinates in points P and zero can be substituted by Δt, and the equations will then adopt the form as below:

– C_+ compatibility equation

$$v_p - v_A + \frac{g}{a}(H_p - H_A) + \frac{f\Delta t}{2D}v_A|v_A| = 0, \tag{9}$$

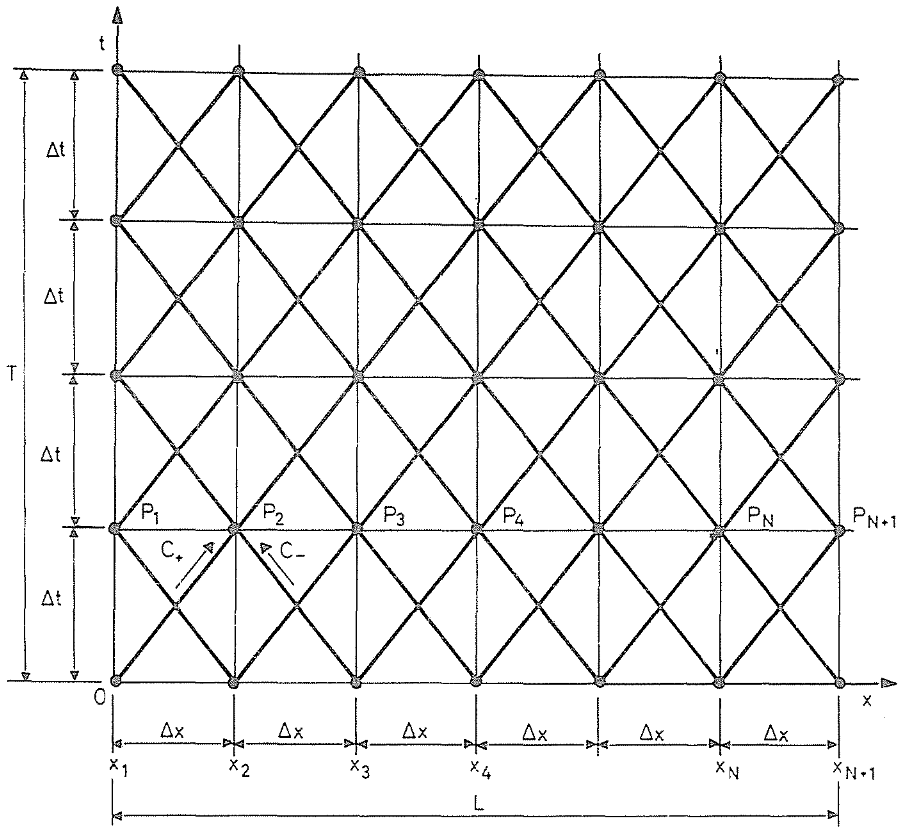

Fig. 2.

– C_- compatibility equation

$$v_p - v_B - \frac{g}{a}(H_p - H_B) + \frac{f\Delta t}{2D} v_B |v_B| = 0. \tag{10}$$

In order to solve numerically the set of equations (9) and (10) the difference method will be applied. For that purpose it is necessary to divide the real pipeline into an optional number of scanning sections of a pipe which can be an even or odd number different from zero. When dividing the pipe into N segments, the length of each segments will be: $\Delta t = L/N$.

If the computation step (trunk) Δx is a known value, one can easily find out the time step $\Delta t = \Delta x/a$. Now it is possible to draw a grid of characteristics as seen in *Fig. 2.*

The initial conditions correspond to the steady-state conditions, whereas the boundary ones will be discussed later.

Numerical Solution by Computer

To calculate values $H = H(x,t)$ and $v = v(x,t)$ in internal points of the grid of characteristics (*Fig. 2*) it is necessary to solve *Eqs.* (9) and (10) with regard to H_p and v_p:

$$H_{p_i} = 0.5[(H_{i-1} + H_{i+1}) + \frac{a}{g}(v_{i-1} - v_{i+1}) + \\ -\frac{a}{g}\frac{f\Delta t}{2D}(v_{i-1}|v_{i-1}| - v_{i+1}|v_{i+1}|)], \tag{11}$$

$$v_{p_i} = 0.5[(v_{i-1} + v_{i+1}) + \frac{g}{a}(H_{i-1} - H_{i+1}) + \\ -\frac{f\Delta t}{2D}(v_{i-1}|v_{i-1}| + v_{i+1}|v_{i+1}|)] \tag{12}$$

for $2 \leq i \leq N$.

Boundary Conditions

On the left end a reservoir of constant fluid level has been assumed. Hence it is possible to put down the following equations with the negligence of the velocity head:

$$\left.\begin{aligned} H_{p_1} &= H_0 = \text{constant}, \\ v_{p_1} &= v_2 + \frac{g}{a}(H_{p_1} - H_2) - \frac{f\Delta t}{2D}v_2|v_2|. \end{aligned}\right\} \tag{13}$$

The second equation has been obtained by transforming the equation of C_- compatibility.

On the right end it is assumed that the pipeline is closed by means of a valve of linear characteristic. The boundary equations will have the following form:

$$\left.\begin{aligned} v_{P_{N+1}} &= v_0\left(1 - \frac{t}{T_z}\right) \text{ for } 0 \leq t \leq T_z, \\ v_{P_{N+1}} &= 0 \text{ for } t > T_z, \\ H_{P_{N+1}} &= H_N - \frac{a}{g}\left(v_{P_{N+1}} - v_N\right) - \frac{a}{g}\frac{f\cdot\Delta t}{2D}v_N|v_N|. \end{aligned}\right\} \tag{14}$$

Program for Computer

Program for computer has been written in FORTRAN language and can be modified by introducing different boundary conditions.

The input data are introduced by aid of NAMELIST instruction which has some advantages in comparison to other methods of the data reading. A significant advantage is the fact that it is unnecessaty to prepare the data in a suitable sequence for the data are identified by their name which prevents errors while modifying certain magnitudes, and the like. Each input parameter is defined in the COMMENT instruction of the program which allows a quick orientation in the program.

When starting the program the method of determining the pressure wave velocity a and the friction coefficient f is given. Both values can be read when we are in possession of, for example, the measurement results or the calculated results in compliance with certain formulae.

While calculating the pressure wave velocity various methods of pipe constraint condition are taken into consideration, and the friction coefficient f is calculated by means of the Colebrook — White's equation widely used in hydraulics and in water supply systems.

Complete Method of Characteristics

In this case the slope of pipeline and the velocity of water are taken into account in the characteristics equations. The original set of partial differential equations (1) – (2) will be replaced by two sets of ordinary differential equations of the form of [6, 8, 15]:

– equation of C_+ characteristics

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{g}{a}\frac{\mathrm{d}H}{\mathrm{d}t} - \frac{g}{a}v\sin\alpha + \frac{f}{2D}v|v| = 0, \tag{15}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v + a\, tag16$$

– equation of C_- characteristics

$$\frac{\mathrm{d}v}{\mathrm{d}t} - \frac{g}{a}\frac{\mathrm{d}H}{\mathrm{d}t} + \frac{g}{a}v\sin\alpha + \frac{f}{2D}v|v| = 0, \tag{17}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v - a. \tag{18}$$

In the numerical procedure it is assumed that the characteristic curves can be substituted by straight lines in each time step Δt. Such a simplification is possible since $a \gg v$.

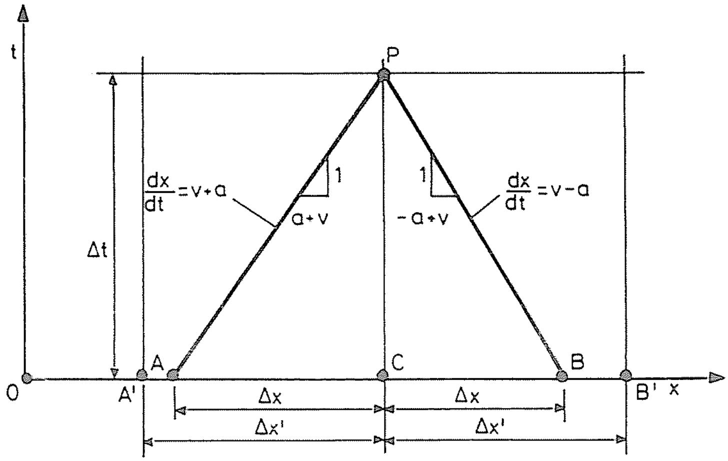

Fig. 3.

We construct a grid of mesh dimensions Δx and Δt, so that the characteristic curves crossing at point P will be straight lines (see *Fig. 3*). The slope of these lines is determined by the known velocity value of the previous time step.

It is also worth mentioning that the characteristics crossing point P do not pass through the points of grid A' and B'. However, they pass through points A and B lying upon axis of abscissae between points A' and B'.

The ordinary differential equations (15) – (18) will be replaced by finite-difference equations of the following form:

$$\frac{v_p - v_A}{\Delta t} + \frac{g}{a}\frac{H_p - H_A}{\Delta t} - \frac{g}{a}v_A \sin\alpha + \frac{f}{2D}v_A|v_A| = 0, \tag{19}$$

$$\frac{v_p - v_B}{\Delta t} - \frac{g}{a}\frac{H_p - H_B}{\Delta t} + \frac{g}{a}v_B \sin\alpha + \frac{f}{2D}v_B|v_B| = 0. \tag{20}$$

The unknown quantities H and v at points A and B can be determined by interpolation with the known values $H = H(x,t)$ and $v = v(x,t)$ at points A', B' and C of the grid of characteristics.

For this purpose along C_+ characteristics advantage will be taken of linear interpolation according to the formula (see *Fig. 3*):

$$\frac{\Delta x}{\Delta x'} = \frac{v_A - v_C}{v_{A'} - v_C} = \frac{H_A - H_C}{H_{A'} - H_C},$$

where

$$\Delta x/\Delta t = a + v_{A'}.$$

Solving the above equations with regard to v_A and H_A we obtain:

$$v_A = (v_{A'} - v_C)\frac{\Delta x}{\Delta x'} + v_C,$$

$$H_A = (H_{A'} - H_C)\frac{\Delta x}{\Delta x'} + H_C.$$

Substituting $\Delta x = (a + v_A) \cdot \Delta t$ from *Eq.* (16) we obtain

$$v_A = \frac{v_C + a\frac{\Delta t}{\Delta x}(v_{A'} - v_C)}{1 - \frac{\Delta t}{\Delta x}(v_{A'} - v_C)} \tag{21}$$

and

$$H_A = H_C + \frac{\Delta t}{\Delta x}(H_{A'} - H_C)(a + v_A). \tag{22}$$

A similar analysis of C_- characteristics equations provides the following formulae:

$$v_B = \frac{v_C + a\frac{\Delta t}{\Delta X}(v_{B'} - v_C)}{1 + \frac{\Delta t}{\Delta x}(v_{B'} - v_C)} \tag{23}$$

and

$$H_B = H_C + \frac{\Delta t}{\Delta x}(H_{B'} - H_C)(a - v_B). \tag{24}$$

By neglecting small terms of higher orders in the denominator of *Eqs.* (21) and (22) we have

$$v_A = v_C + a\frac{\Delta t}{\Delta x}(v_{A'} - v_C), \tag{25}$$

$$v_B = v_C + a\frac{\Delta t}{\Delta x}(v_{B'} - v_C). \tag{26}$$

Solving *Eqs.* (19) and (20) with respect to v_p and H_p we get

$$\begin{aligned} v_p = 0.5[(v_A + v_B) + \tfrac{g}{a}(H_A - H_B) + \tfrac{g}{a}\Delta t(v_A - v_B)\sin\alpha + \\ -\tfrac{f\Delta t}{2D}(v_A|v_A| + v_B|v_B|)], \end{aligned} \tag{27}$$

$$\begin{aligned} H_p = 0.5[(H_A + H_B) + \tfrac{a}{g}(v_A - v_B) + \Delta t(v_A + v_B)\sin\alpha + \\ -\tfrac{a}{g}\tfrac{f\Delta t}{2D}(v_A|v_A| - v_B|v_B|)], \end{aligned} \tag{28}$$

where: α — slope angle of pipeline.

To ensure stability of solution it is necessary to match in an appropriate way the time step Δt so as to satisfy the condition [4, 6, 9, 11, 15]

$$t \le \frac{\Delta x}{\max|a+v|}, \tag{29}$$

where $\max|a+v|$ is the maximum expected absolute value of the sum of the pressure wave velocity and the flow speed of the fluid.

Program for Computer and Numerical Calculation

The program for computer for the characteristics method taking into account complete differential equations is similar to the one with equations in simplified form (approximate method of characteristics). The main differences appear in conditions referring to the pipeline slope and in the linear interpolation procedure.

In order to make a comparative analysis of both methods numerical calculations were carried out for a hydraulic system consisting of an upper reservoir supplying the conduit of a constant diameter and terminated with a valve of a linear closing characteristic.

Appropriate boundary conditions are presented while analyzing the approximated method (*Eqs.* (13) and (14)).

Calculation Example

Taking into account the above boundary conditions calculations of unsteady flows were carried out according to program prepared in FORTRAN language considering complete or simplified characteristics equations.

The following numerical data have been adopted:

– length of pipeline	$L = 6000.0$ m;
– internal diameter of pipe	$D = 0.80$ m;
– thickness of pipe wall	$s = 0.019$ m;
– modulus of elasticity of pipeline (cast iron)	$E = 1.0 \times 10^{11}$ Pa;
– absolute roughness	$k = 0.002$ m;
– initial fluid velocity	$v_0 = 1.5$ m/s;
– steady-state or mean pressure head – in the upper reservoir	$H_0 = 100.0$ m;
– bulk modulus of elasticity of water	$E_c = 2.07 \times 10^9$ Pa;
– water density	$\rho = 1000.0$ kg/m^3;

– water kinematic viscosity $\nu = 1.31 \times 10^{-6}\ \mathrm{m^2/s}$;
– time of valve closure $T_z = 20.0$ s

In both analyzed cases the same pipeline parameters as well as the same properties of the liquid have been assumed.

Table 1
Pressure variations in half the length of pipeline

		Pressure $H(t)$, [kPa]	
No.	Time t [s]	Approximate method of characteristics	Complete method of characteristics
1	2	3	4
1	0.0	894.0	894.0
2	2.8	894.0	894.0
3	5.6	1115.0	1113.5
4	8.4	1349.0	1346.0
5	11.2	1397.0	1376.0
6	14.0	1437.0	1426.0
7	16.8	1297.0	1299.0
8	19.5	1088.0	1073.0
9	22.3	1054.0	1062.0
10	25.1	839.0	855.5
11	27.9	770.0	764.0
12	30.7	914.0	902.0
13	33.5	947.0	932.0
14	36.3	1159.0	1145.0
15	39.1	1226.0	1254.0
16	41.9	1085.0	1098.0
17	44.7	1052.0	1041.0
18	47.5	844.0	832.5
19	50.3	778.0	763.0
20	53.1	917.0	905.0
21	55.8	949.0	935.0
22	58.6	1154.0	1142.0
23	61.4	1219.0	1225.0
24	64.2	1082.0	1094.0
25	67.0	1050.0	1042.0
26	69.8	849.0	837.5
27	72.6	784.5	772.0
28	75.4	919.0	907.0
29	78.2	950.5	942.5
30	81.0	1149.0	1137.0
31	83.8	1212.0	1226.0
32	86.6	1079.0	1086.0
33	89.3	1049.0	1035.5
34	92.1	853.0	841.0
35	94.9	791.0	778.0

Table 2
Pressure variations in the cross-section at the valve

No.	Time t [s]	Pressure $H(t)$, [kPa] Approximate method of characteristics	Pressure $H(t)$, [kPa] Complete method of characteristics
1	2	3	4
1	0.0	788.5	788.5
2	2.8	1022.0	1013.0
3	5.6	1269.0	1255.0
4	8.4	1525.0	1512.0
5	11.2	1789.0	1778.0
6	14.0	1675.0	1694.0
7	16.8	1523.0	1538.0
8	19.5	1333.0	1345.0
9	22.3	925.0	913.0
10	25.1	823.0	811.0
11	27.9	753.0	744.0
12	30.7	717.5	702.5
13	33.5	1074.0	1052.0
14	36.3	1174.0	1153.0
15	39.1	1243.0	1211.0
16	41.9	1278.0	1245.0
17	44.7	927.0	935.0
18	47.5	828.0	831.0
19	50.3	761.0	752.0
20	53.1	727.0	712.0
21	55.8	1072.0	1061.0
22	58.6	1169.0	1154.0
23	61.4	1236.0	1221.0
24	64.2	1269.0	1281.0
25	67.0	929.0	945.0
26	69.8	833.5	848.5
27	72.6	768.0	759.0
28	75.4	736.0	719.0
29	78.2	1070.0	1056.0
30	81.0	1164.0	1140.0
31	83.8	1228.0	1233.0
32	86.6	1260.0	1290.0
33	89.3	931.0	945.0
34	92.1	838.0	851.0
35	94.9	775.0	768.0

The calculation results referring to the characteristic cross sections along the pipeline are presented in tabular form (*Tables 1* and *2*).

Making an analysis of the calculation results taking complete and simplified differential equations into account it is evident that the differences between the values of pressures in appropriate cross-sections and in corresponding simulation times are insignificant since the value of the relative error is not more than 10 %.

Generally the values of the relative error amount to several per cent.

When applying pipelines made of materials much less elastic than steel (of smaller Young's modulus) the obtained results of extreme pressures also differ insignificantly.

However, the extreme pressures are definitely lower when the water-hammer phenomenon occurs in aluminium pipelines or those made of plastics (e.g. PVC) in comparison to steel or even of cast iron pipelines.

Effect of Some Parameters upon Results of Solution

Entering upon the basic calculations of the unsteady flow values for the computer, we are not always aware of the influence of certain parameters upon the results of the solution or the operation time of computer.

The analysis presented below shows effect of fluid flow velocity, absolute roughness of pipeline and number of reaches a pipeline upon the accuracy of the obtained results and the time the processor of the computer is engaged in the operation.

Effect of Flow Velocity

The numerical calculations were carried out for a simple hydraulic system with input data as in the previous example. As it is clear from the preceding considerations, the flow velocity has an immediate effect upon the friction element $\left(\frac{f}{2D}v|v|\right)$. For the calculations four different initial velocities have been taken to begin with the most often permissible one that appears in water mains ($v_0 = 1.5$ m/s), and to end with the maximum permissible velocity $v_0 = 3.0$ m/s the acceptance of which in the project requires an extra justification.

The extreme pressure values are presented in graphic form in *Fig. 4* and *Table 3*. As one can easily notice while increasing the flow velocity of the fluid by 0.5 m/s, the pressure head rises on the average by approximately 19 %. In an extreme case in the valve cross-section an increase of speed from $v = 1.5 - 3.0$ m/s will be accompanied by an increment of pressure head by 67.5 %.

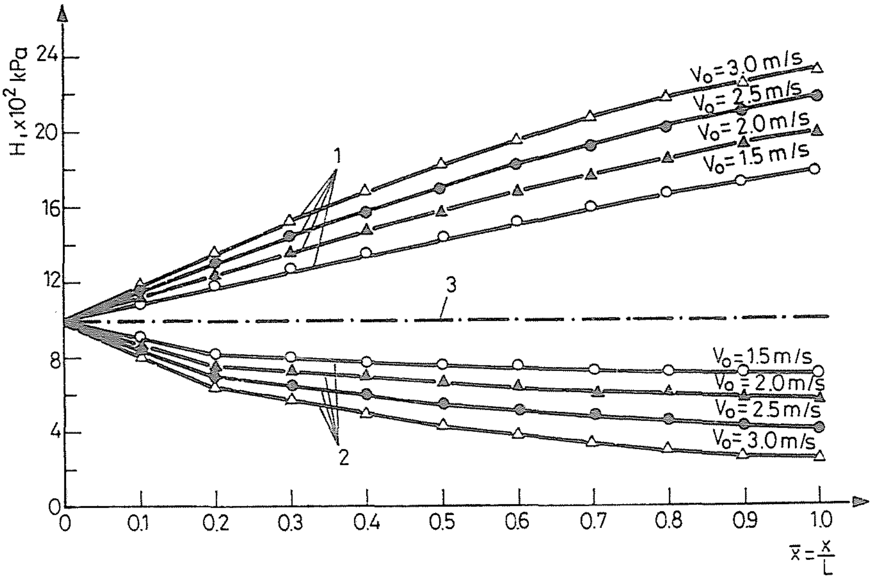

Fig. 4. Extremal pressure variations along pipeline depending on flow velocity: 1 — maximum pressures, 2 — minimum pressures, 3 — hydrostatic pressure in the initial reservior

Effect of Absolute Roughness

The absolute roughness affects the friction element value in the same way as flow velocity. Some numerical results for a system similar to the above examples but with various values of absolute roughness k will be presented.

For calculations the following values of k have been assumed: $k_1 = 0.002$ m; $k_2 = 0.003$ m; $k_3 = 0.004$ m and $k_4 = 0.005$ m. The results of the calculations are illustrated in *Table 4*.

As it is evident from *Table 4* the effect of the absolute roughness upon the pressure head is of much smaller importance than appropriate variations of flow speed. In an extreme case for terminal cross-section the pressure increment will amount to approximately 5 % in relation to the value of the lowered pressure.

In the example under consideration the values of the absolute roughness have been changed each by 0.001 m, but even changes of values of 10 times greater do not cause significant variations in the pressure rise in the unsteady flows.

Table 3
Extreme pressure values along the pipeline depending on fluid flow velocity v_0

		Pressures $H(\bar{x},t)$ [kPa]							
		$v_0 = 1.5$ m/s		$v_0 = 2.0$ m/s		$v_0 = 2.5$ m/s		$v_0 = 3.0$ m/s	
No.	Abscissa $\bar{x} = x/L$	Max	Min	Max	Min	Max	Min	Max	Min
1	2	3	4	5	6	7	8	9	10
1	0.0	1000.0	1000.0	1000.0	1000.0	1000.0	1000.0	1000.0	1000.0
2	0.1	1093.0	908.0	1124.0	878.0	1155.0	849.0	1186.0	821.0
3	0.2	1183.0	820.0	1244.0	762.0	1304.0	706.0	1362.0	635.5
4	0.3	1271.0	798.0	1359.0	724.0	1445.0	650.0	1527.0	577.0
5	0.4	1356.0	778.0	1469.0	690.0	1577.0	599.0	1680.0	506.0
6	0.5	1437.0	760.0	1573.0	660.0	1701.0	553.5	1821.0	442.0
7	0.6	1515.0	746.0	1671.0	635.0	1816.0	514.0	1950.0	386.0
8	0.7	1590.0	734.0	1762.0	613.0	1921.0	481.0	2065.0	338.0
9	0.8	1660.0	725.0	1847.0	597.0	2017.0	455.0	2166.5	301.0
10	0.9	1727.0	718.0	1926.0	585.0	2102.0	436.0	2255.0	273.0
11	1.0	1789.0	715.0	1997.5	579.0	2178.0	425.0	2333.0	256.0

Table 4
Extremal pressure values along the pipeline depending on the absolute roughness coefficient k

		Pressures $H(\bar{x},t)$ [kPa]							
		$k = 0.002$ [m]		$k = 0.003$ [m]		$k = 0.004$ [m]		$k = 0.005$ [m]	
No.	Abscissa $\bar{x} = x/L$	Max	Min	Max	Min	Max	Min	Max	Min
1	2	3	4	5	6	7	8	9	10
1	0.0	1000.0	1000.0	1000.0	1000.0	1000.0	1000.0	1000.0	1000.0
2	0.1	1093.0	908.5	1093.0	908.5	1093.0	909.0	1093.0	909.0
3	0.2	1183.0	820.0	1183.0	820.0	1183.0	821.0	1183.0	821.0
4	0.3	1271.0	798.0	1271.0	796.0	1270.0	794.0	1269.0	793.5
5	0.4	1356.0	778.0	1354.0	774.0	1353.0	771.0	1352.0	768.5
6	0.5	1437.0	760.5	1435.0	755.0	1432.0	750.0	1430.0	746.0
7	0.6	1515.0	746.0	1511.0	738.0	1507.0	732.0	1504.0	727.0
8	0.7	1590.0	734.0	1583.0	725.0	1578.0	718.0	1573.0	712.0
9	0.8	1660.0	725.0	1651.0	715.0	1644.0	706.0	1638.0	699.5
10	0.9	1727.0	718.0	1715.0	707.0	1705.5	699.0	1691.0	691.0
11	1.0	1789.0	715.0	1744.0	703.0	1762.0	694.0	1751.0	686.0

Effect of Number of Scanning Sections of a Pipeline

To prove the effect of the number of pipe reaches upon accuracy of the obtained results pressure calculations in time of unsteady flows were carried out for a system as in the previous instances assuming a division into 5, 10, 20 and 40 computation reaches.

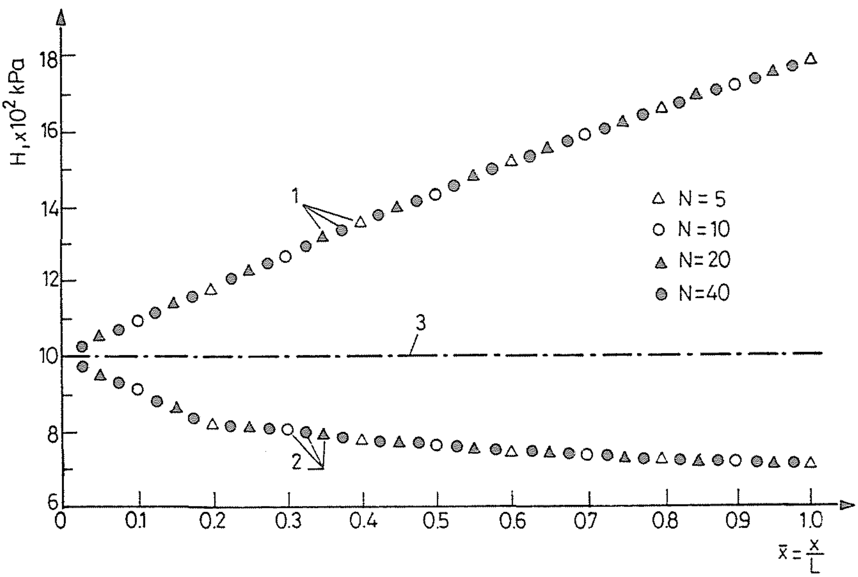

Fig. 5. Extremal pressure variations along pipeline depending on number of scannong sections:
1 — maximum pressures, 2 — minimum pressures, 3 — hydrostatic pressure in the initial reservior

The numerical calculation results of the extreme pressure values are presented in tabular form in *Table 5* and graphically in *Fig. 5.*

Analyzing appropriate values of pressure one can easily note that the value of the relative error, with respect to the extreme pressure values, varies from 0.5 – 3 %, which is not of a great significance from the practical point of view. And thus a greater number of pipeline reaches does not effect an improvement of the calculation accuracy and therefore it is recommended to devide the pipeline into 5 computation reaches as an optimum in the case of a pipe of constant diameter.

However, while analyzing the water-hammer phenomenon one should take into consideration both in the simple systems and the complex ones

Table 5
Pressure values in the downstream end of pipeline for different number of scanning sections N

		Pressures values at the valve [kPa]			
No.	Time t [s]	$N = 5$	$N = 10$	$N = 20$	$N = 40$
1	2	3	4	5	6
1	0.0	788.5	788.5	788.5	788.5
2	3.4	1070.7	1070.7	1070.7	1070.7
3	6.7	1370.0	1370.0	1370.0	1370.0
4	10.1	1682.5	1682.5	1682.5	1682.5
5	13.4	1700.9	1700.9	1700.9	1700.9
6	16.8	1523.2	1523.2	1523.2	1523.2
7	20.1	1290.0	1290.1	1290.1	1290.1
8	23.5	880.5	880.5	880.5	880.5
9	26.8	776.7	776.7	776.7	776.7
10	30.2	721.6	721.5	721.5	721.5
11	33.5	1074.0	1074.0	1074.0	1074.0
12	36.9	1190.7	1190.7	1190.7	1190.7
13	40.2	1261.3	1261.4	1261.4	1261.4
14	43.6	1106.2	1106.2	1106.2	1106.2
15	46.9	845.8	845.8	845.7	845.7
16	50.3	760.7	760.7	760.7	760.7
17	53.6	724.5	724.4	724.3	724.3
18	57.0	1114.2	1114.2	1114.2	1114.2
19	60.3	1212.8	1212.8	1212.9	1212.9
20	63.7	1264.8	1264.9	1264.9	1264.9
21	67.0	929.3	929.3	929.2	929.2
22	70.4	817.9	817.9	817.9	817.9
23	73.7	751.1	751.0	751.0	751.0
24	77.1	898.4	898.4	898.4	898.4
25	80.4	1147.5	1147.5	1147.5	1147.5
26	83.8	1228.2	1228.4	1228.4	1228.4
27	87.1	1262.4	1262.5	1262.6	1262.6
28	90.5	890.7	890.7	890.7	890.7
29	93.8	796.8	796.6	796.6	796.6
30	97.2	747.6	747.4	747.4	747.4
31	100.5	1067.7	1067.8	1067.8	1067.8
32	103.9	1174.1	1174.3	1174.3	1174.3
33	107.2	1237.5	1237.7	1237.8	1237.8
34	110.6	1097.3	1097.4	1097.5	1097.5
35	113.9	858.7	858.7	858.6	858.6
36	117.3	781.9	781.7	781.6	781.6
37	120.6	749.6	749.5	749.3	749.3

that also the kind of the boundary conditions and the flow velocity affect the number of scanning section of a pipe. Making a selection of the mesh

Table 6
Comparison of the time of water-hammer calculations on the personal computer of type of IBM PC/AT depending on number of scanning sections of pipeline N

Scanning sections	Time of calculations [s]	Relation of the actual time to the t_5
1	2	3
$N = 5$	$t_5 = 27$	1
$N = 10$	$t_{10} = 95$	3.5
$N = 20$	$t_{20} = 360$	13.3
$N = 40$	$t_{40} = 1395$	51.7

size, and in the same way the length of steps Δx it is necessary to make sure that Courant stability requirement (condition) is fulfilled (*Eq.* (29)).

By increasing the number of steps Δx we considerably increase the calculation time for the number of the network points changes according to N^2. *Table 6* shows times of water-hammer calculations on the personal computer of the IBM PC/AT type in relation to the number of scanning sections of a pipe. As it follows from the above comparison with the division into 40 scanning sections the time of calculation amounts to 1395 s in comparison to 27 s with the division into 5 scanning sections. Such an arrangement results in an increase of the calculation time by about 52 times, which in consequence has some effect upon the cost of calculations.

5. Summary and Final Conclusions

1. In the paper a solution of water-hammer problem is presented taking into account complete or approximate characteristics methods. A comparative analysis of both methods was made on the basis of a calculation example of a hydraulic system which consisted of an initial upper reservoir supplying a conduit of a constant diameter with a closing valve at the end.
2. In consequence of the calculations carried out it has been proved that the differences between pressure values calculated according to both methods are insignificant because in extreme cases they reach several per cent. Thus it is possible to take advantage of the approximate characteristics method for calculations.
3. In the paper the effect of certain parameters upon the results of the solution was also subject to investigations. It has been proved that variations of the flow velocity have a great effect upon pressure head

in the unsteady state. An increase of velocity from 1.5 to 3.0 m/s will result in a rise of pressure head by about 67 %. Therefore while designing water mains it is necessary to adopt flow velocities near the lower boundary, i.e. $v_0 = 1.5$ m/s.

4. The influence of absolute roughness coefficient k is of much smaller importance than insignificant even velocity variations, although both parameters appear in the friction elements. In an extreme case a change of roughness from 0.002 m to 0.005 m was followed by a pressure drop by as low as 20 kPa which is a drop of 1 %. With regard to lowered pressure a drop of respective absolute values of about 5 % occurred.
5. An increase of the number of scanning sections of pipeline (number of steps Δx) does not effect an improvement of the calculation accuracy. For an optimal division it is recommended to apply 5 scanning sections in the case of a pipeline of constant diameter (uniform pipeline). If the number of steps Δx is increased then the time of the calculations will be considerably greater which consequently will raise the cost.
6. In making the choice of the number of scanning sections one should each time take the kind of dynamic or non-dynamic boundary conditions into consideration and comply with the stability condition of the solution.

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