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ANALYTICAL AND NUMERICAL STUDY ON THE LATERAL INSTABILITY OF A PLATED BRIDGE

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Abstract

Theoretical and experimental investigation of simple supported cantilevered beam with open, mono-symmetrical cross-section is given. Approximate energy solution for the critical load, results of second-order computer analysis, model tests and actual design problem are presented. Numerical approaches were developed or applied with special attention to critical situation that may arise at free erection of continuous beams.

Keywords: lateral instability, critical load, numerical approach, experimental investigation, plated bridge.

Introduction

At the free erection of bridges especially in case of continuous beams special problems arise. Relatively long cantilever as a continuation of the erected simple supported beam in itself requires close attention. Furthermore, some technologies use cranes where the application point of the load due to the unit to be built in is relatively high above shear center as it is illustrated in *Fig. 1.*

The above mentioned circumstances may cause critical situation, especially in case of open cross-sections restrained only at intervals by cross-beams and cross-ties.

Because of the relatively few available theoretical results the paper offers three approaches for the analysis of the problem:

- energy method to obtain the critical load.
- a second-order elastic computer analysis with program STERUE (SZATMÁRI,1990);
- tests results of small scale model specimens.

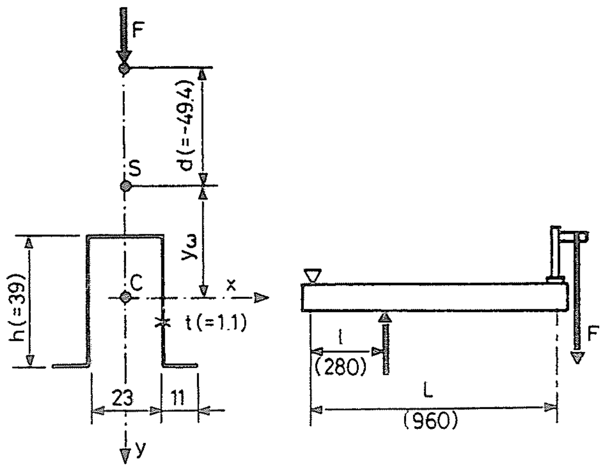


Fig. 1.

Critical Load by Energy Method

The notations are shown in *Fig. 1.* Actual dimensions refer to the test specimen discussed later. The angle of rotation γ and the lateral deflection u are assumed in the form of polynom of fourth degree:

$$\gamma = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + A z^4. \tag{1}$$

$$u = b_0 + b_1 z + +b_2 z^2 + b_3 z^3 + B z^4. \tag{2}$$

The constants a_0 through a_3 and b_0 through b_3 can be obtained by substituting the four boundary conditions for γ and u, respectively:

$u = u'' = \gamma = \gamma'' = 0$ at $z = 0$

$u = \gamma = 0$ at $z = l$

$u'' = \gamma'' = 0$ at $z = L$.

The above listed boundary conditions do not take into consideration the shear force in the plane of lateral deflection u and the torque at $z = L$. Furthermore, one free parameter per displacements γ and u allows only an upper bound solution. Leaving the estimation of the degree of approximations induced for subsequent discussion of numerical and experimental results, after substitutions the assumed functions may be written in the following form:

$$\gamma = A L^4 f, \tag{3}$$

$$u = B L^4 f, \tag{4}$$

with

$$f = b\zeta - 2\zeta^3 + \zeta^4,$$
$$b = 2\lambda^2 - \lambda^3,$$
$$\lambda = l/L,$$
$$\zeta = z/L.$$

The total potential energy of the system is expressed by the well-known equations:

$$P_1 = \frac{1}{2}\Big[EJ_y \int_0^L (\gamma'')^2 dz + 2\int_0^L M\gamma' u' dz\Big],$$

$$P_2 = \frac{1}{2}\Big[EC_w \int_0^L (u'')^2 dz + EJ \int_0^L (\gamma')^2 dz + \int_0^L M\gamma' u' dz + 2\beta_y \int_0^L (\gamma')^2 dz + Fd\frac{\gamma_L^2}{2}\Big].$$

where:

$$M = F(1 - \frac{1}{\lambda})z, \qquad 0 \le z \le l,$$
$$M = F(z - L), \qquad l < z \le L,$$
$$\beta_y = \frac{\int y(x^2 + y^2)}{J_x} - y_w$$

$\gamma_L = b - 1$ (angle of rotation at $z = L$.)
Substituting the functions (3) and (4) for γ and u, putting the first variations of the total potential energies P_1 and P_2 equal to zero the determinant of the system of linear equations obtained must vanish. Finally, the critical value of F may be obtained by the following non-dimensional formula:

$$\frac{F_{CR}L^2}{\sqrt{GJEJ_y}} = C_1\Big[\sqrt{1 + \alpha(\beta^2 C_2 + C_3)} + \beta\sqrt{C_2\alpha}\Big], \tag{5}$$

where

$$C_1 = \sqrt{\frac{I_1 I_2}{I_3^2}}; \quad C_2 = \frac{I_1}{\pi^2 \varepsilon_w^2 I_1 I_3^2}; \quad C_3 = \frac{I_1}{\pi^2 I_2},$$

$$\beta = 2\varepsilon_y I_3 + \varepsilon_d \gamma_L^2,$$
$$\varepsilon_w = \frac{2}{h}\sqrt{C_w/J_y},$$

$\varepsilon_y = \beta_y/h$, (positive if application point is below shear center)

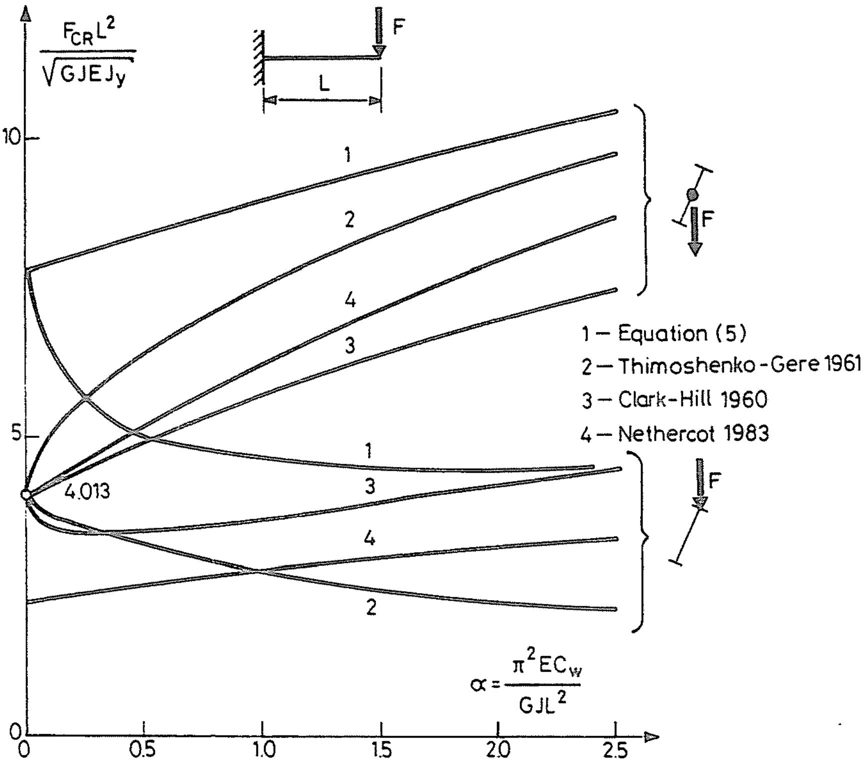


Fig. 2.

$$\varepsilon_d = d/h$$

$$I_1 = \int_0^1 (f'')^2 d\zeta;$$

$$I_2 = \int_0^1 (f')^2 d\zeta$$

$$I_3 = (1 - 1/\lambda) \int_0^\lambda (f')^2 zeta d\zeta + \int_\lambda^1 (f')^2 (\zeta - 1) d\zeta$$

To check the results *Eq.* (5) was applied to a simple cantilever beam completely fixed at the support by putting $\lambda = 0$.

The results for doubly symmetric I section ($\varepsilon_w \sim 1, \varepsilon_y = 0$) with

$$\varepsilon_d = 0 \text{ (loading at shear center)}$$
$$\varepsilon_d = -0.5 \text{ (loading on top flange)}$$

are shown in *Fig. 2*. For comparison some of the available solutions are also plotted. Due to the nature of the method applied the curves computed from *Eq.* (5) lie above the other, partly exact but partly also approximate curves. The agreement is closer in case of top flange loading at relative high value of α.

Experimental Investigations

Arrangement and dimensions of the specimens are given in *Fig. 3*.

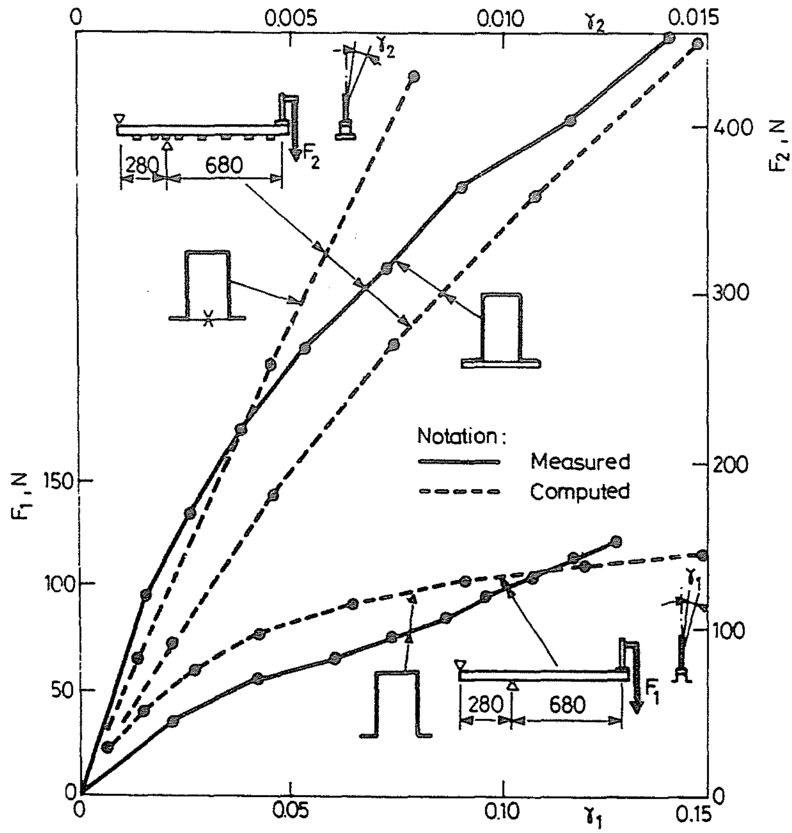


Fig. 3.

Test specimen No 1 was made with open cross-section while No 2 was restrained at intervals by flats. Equivalent thickness of the restraint on lower flange was found as 0.012 mm.

The load was applied on the attachment fastened to the end of the cantilever. The application point was adjustable in steps. Lateral deflections were measured at the top of the attachment and the bottom of the beam allowing to evaluate the angle of rotation γ and the lateral deflection of the shear center u, respectively. Tests were carried out using a small initial twist γ_0.

The plot of experimental data is given with solid line on *Fig. 3* in the form of angle of rotation γ versus load F. The ultimate loads were

$$F_{u1} = 122\text{N} \quad \text{and} \quad F_{u2} = 451\text{N}.$$

Experimental data were confronted by two numerical approaches. The results of program STERUE are also given on *Fig. 3*. The γ versus F curve of the test specimen No 2 was computed in two different ways:

— with varying cross-section due to the exact geometry and

— assuming a constant closed profile using the previously mentioned $t_{\text{equ.}}$=0.012 mm.

It can be seen that in case of the open profile the agreement of measured and computed ultimate load is quite satisfactory. On the other hand, in case of the restrained cross-section the application of the semi-opened, true numerical model results in a good match while the use of the equivalent closed profile can be accepted only as an approximation.

Critical forces for the two cases were calculated by formula (5) giving

$$F_{CR1} = 141\text{N} \quad \text{and} \quad F_{CR2} = 2500\text{N}$$

In practice it was found that a good approximation of the ultimate load F_u can be got using the following formula:

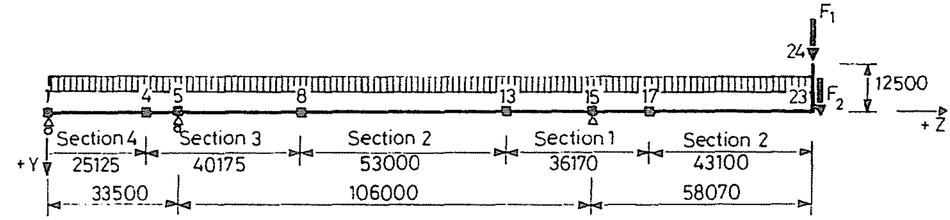
$$F_u/F_y = 0.5(\beta - \sqrt{\beta^2 - 4nF_{CR}/F_y}), \tag{6}$$

where F_y - the force causing full plastic moment (with a yield stress of 270) N/mm^2 in our case F_y=715 N), $\beta = n(1 + F_{CR}/F_y)$ and n=1.2

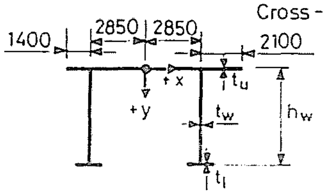
Applying the above formula to the test specimens:

$$F_{\text{u, appr,1}} = 136\text{N} \quad \text{and} \quad F_{\text{u, appr,2}} = 673\text{N}.$$

In the first case the agreement between the approximation and test result seems to be surprisingly good. The second approximation exceeds the measured ultimate load by some 50 per cent.



Cross-sectional dimensions, mm



	1	2	3	4
t_u	16	12	12	12
h_w	5000	4000	2900	2500
t_w	16	12	12	12
b_l	1200	720	720	540
t_l	50	40	40	30

Fig. 4.

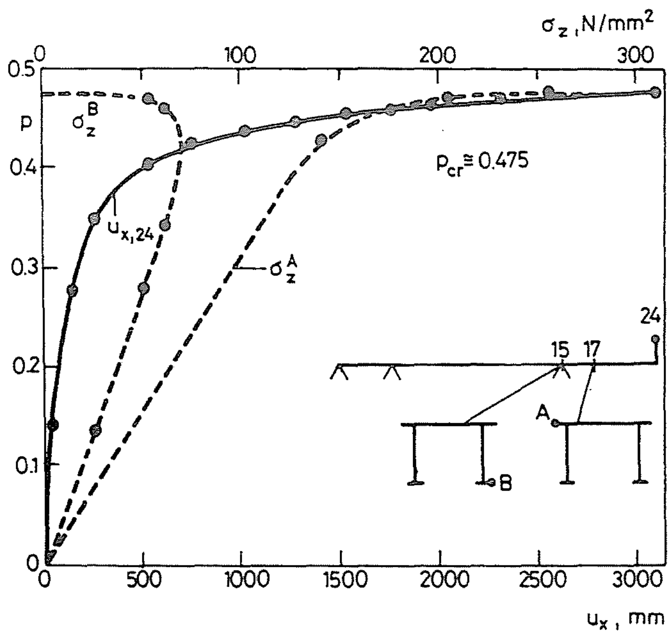


Fig. 5.

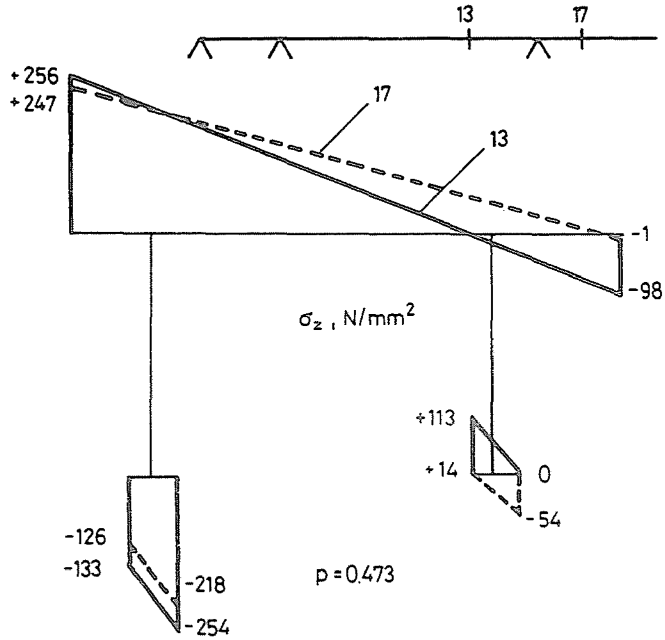


Fig. 6.

Analysis of the Erection Phase of a Highway Bridge

Having proven the efficiency of program STERUE by the comparison with the results of the model test the next example is the analysis of the lateral buckling of a highway bridge at the erection phase.

For the input of the computer program a slightly modified layout shown in *Fig. 4* was used. This was the most critical stage of the free erection from the point of view of lateral buckling: continuous beam over three supports with a 58 m long cantilever. The crane at the end of this cantilever raises the application point of the loading by 12.5 m above the traffic deck. Actual cross-sections were approximated by four typical ones (notation 1 to 4). Actual loadings are the nominal values of dead- and erection loads, the weight of the crane and the last unit to be built in, the latter magnified by a dynamic factor of μ=1.5 (see *Fig. 4*).

Displacements and stresses were computed as functions of a loading parameter p. In this way the actual numerical values of loadings at different loading levels can be obtained from the following formulae:

$$f_{eff} = pf,$$
$$F_{1,eff} = pF_1,$$
$$F_{2,eff} = pF_2,$$

that is full intensity of loading would be at $p = 1.0$. Horizontal displacement normal to the longitudinal axis of nodal point 24 (u_x) and normal stresses at cross-sectional points of special interest versus p are shown on *Fig. 5*. The distribution of normal stresses over the cross-sections 17 and 13 can be seen on *Fig. 6* at a loading level of $p = 0.473$. The results illustrated by the two figures indicate a lateral buckling at the critical value of $p_{CR} \sim 0.475$. The conclusion is that the erection of the given structure can not be carried out in the form illustrated by the scheme in *Fig 4*.

Conclusion

In case of delicate problems associated with lateral buckling where exact solutions are not available relatively simple analytical and numerical approaches give satisfactory results for the design praxis.

It should also be noted that at the free erection of bridges with open cross-section the use of temporary bracing against lateral buckling is generally unavoidable.

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