LOAD DISTRIBUTION CAPACITY OF RAILWAY SUPERSTRUCTURE

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Received: December 20, 1991

Abstract

Factors influencing the load distribution capacity of railway superstructure on steel bridges, the effect of railway superstructure as continuous beam on elastic supports on the influence lines of the supporting beam, as well as the possibility of approximative calculations are demonstrated in this paper.

Keywords: track on steel bridges, design of steel railway bridges.

1. Introduction

Some years ago, some members of the International Union of Railways or UIC shortened for its well-known denomination in French, performed extensive theoretical and experimental examinations on the floor system of the railway bridges with open floor with the aim of acquiring more exact information on the behaviour of their structure [1]. It was stated, as a result, that the stresses measured on the floor structure of those railway bridges were generally smaller than those calculated in a traditional way and that, as one of the causes of this phenomenon, the load distribution capacity of the railway superstructure was indicated and even the reckoning with it was recommended. A part of the railway companies (e. g. DB, SBB, BR, CFR) prescribes, or at least, allows reckoning with the load distribution capacity of the railway superstructure, while another part of them does not use this possibility.

In the former railway bridge regulations of our country, this question was not dealt with, neither was it taken into consideration in our inland design practice. Since in the future it can be expected that the prescriptions concerning the load distribution capacity with the different national railway companies will draw each other nearer, it is worth pondering the factors influencing the load distribution capacitiy of superstructures, as well as the possibilities and consequences of reckoning with it in calculations.

2. Factors Influencing the Load Distribution Capacity of Railway Superstructures

In the superstructure systems of the inland standard-gauge railways, the continuous rail is supported at certain points. If the superstructure is of such layout that the iron bridge is loaded directly by the superstructure without ballast-bed, then the load of rails is transferred to the supporting bridge structure either through cross-ties (bridge sleepers or maybe steel girders) or through rail fastenings applied directly, and rendered elastic. This rail propped up elastically at some points is considered to be a continuous beam on elastic supports. The obvious advantage of the elastic support, as contrasted to the rigid one, lies in the fact that the load over the place of support is always distributed to supports more than one: i. e. the load is distributed by the elastic support.

If the rail is considered to be a continuous beam on elastic supports, then its behaviour and — thereby its load distribution capacity, too — will depend on three factors:

— the rigidity of supports (e. g. sleepers),
— the spacing of supports (distance between the sleepers),
— the rigidity of rails (their moment of inertia).

In the following, first of all, the superstructure with cross-ties (sleepers), will be dealt with here.

2.1. Rigidity of Support, Sleepers

The rigidity of support means the vertical displacement of the support calculable for unit force. Under actual circumstances, the deformation of the sleeper is due to compression perpendicular to fibres, as well as to shear and bending. Due to the defective character of the indispensable experimental data to the calculation of deformation, under the home conditions, the calculation of deformation was based on the dominant compression perpendicular to the fibres, particularly the compression of a prism having a base area of $20 \times 20 = 400\,\text{cm}^2$ and approaching the thickness of the sleeper ($h = 20\,\text{cm}$) loaded perpendicularly to fibres was accepted as the rigidity of support. This seems to be an average approximation with respect to the variety of sleeper fastenings and the structural solution to supporting the sleepers.

The sleepers used by the MÁV (Hungarian State Railways) are of oak wood by stipulations. Though some kind of oak is grown all over the northern hemisphere everywhere, the literature data shows that at least twenty-five kinds of timber are applied in the capacity of cross-tie or sleeper, and among them, the coniferous wood (softwood) and the leaf wood (hardwood), respectively, take a share of 50 – 50 %, which — by all means — involves a warning about the danger of using the most different load-distribution prescriptions of the railway companies without due inquiries and consideration. In addition, it also should be taken into consideration that the properties of the same kind of wood can have a wide dispersion depending on the site of growth (habitat), or the purpose of growth, respectively. The rigidity of sleepers is individually influenced by the layout of bearing, the age of sleepers and the measure of wear.

With respect to the home utilization of the oak wood sleepers, as a modulus of elasticity (Young's modulus) perpendicular to the fibre, the value $E_{\perp} = 100\,\mathrm{kN/cm^2}$ was chosen on the basis of rare literature data; with softwood, the value $E_{\perp} = 50\,\mathrm{kN/cm^2}$ generally accepted for coniferous wood as a more elastic support, while a value: $E_{\perp} = 150\,\mathrm{kN/cm^2}$ accepted for beech wood as a more rigid support were reckoned with for the modulus of elasticity perpendicular to the fibre. No data were found for pseudo-acacia wood, but it is probably harder than oak. For the sake of comparison, calculations were performed also for steel cross-ties, whose rigidity was defined as the compression of a prism (web plate) with a base area of $20 \times 1\,\mathrm{cm^2}$ and a height of 20 cm.

2.2. Spacing of Bridge Sleepers (Distance between Sleepers)

The load distribution capacity of superstructures is also a function of the spacing of sleepers. According to the Railway Bridge Regulations of 1951, the prescribed spacing of sleepers is 65 cm, while the still permissible spacing is 69 cm. According to the Railway Bridge Draft Regulations of 1976, a spacing of 60 cm is prescribed in the case of superstructures having long-welded (ribbon) rails, while a spacing of 65 cm is permitted in exceptional cases. In the old-time bridge designs, spacing of bridge sleepers of 80 cm can also be encountered. A denser spacing of the sleepers occurred in individual cases in old-time prescriptions, as e. g. a spacing of 60 cm prescribed by the Prussian State Railways at the turn of the century.

2.3. Rigidity of Rail

The load distribution capacity of rails is, of course, influenced — in addition to the material of sleepers and their spacing — also by the rigidity of rails (moment of inertia). The home assortment of rails is copious enough, it covers a wide range of rigidity from the rail of system i ($I = 447$ cm^4), not existing supposedly any more on bridges, through rails of 34 kg (c, $I = 934$ cm^4), rails of 48 kg ($I = 1747$ cm^4) and rails of 54 kg ($I = 2346$ cm^4) to the rails of UIC 60 ($I = 3055$ cm^4). The wearing of reduces its rigidity consequently, a wearing of great extent involves a considerable reduction in rigidity.

3. Theoretical Models

3.1. The Rail as a Continuous Beam on Elastic Supports

If the problem emerges as to: in what measure the weight of wheels over a sleeper can be distributed by the rail, then the calculation is related to a continuous beam on elastic supports whose any arbitrary point is loaded by a single concentrated force, and in this case, the reactions should be determined. The solution to the problem, in this case, will be undoubtedly a work-consuming one. If the beam is considered to be a *regular* one only in that case then the three factors influencing its behaviour, namely: the rigidity of supports, the spacing of those and the rigidity of the beam are constant values, and it has an infinite number of supports, then for the solution to this case of the regular, continuous beam with infinite number of elastic supports, there has been available a relatively simple schedule already for some decades, which was renewed for the sake of calculating the orthotropic floor slab, too [2].

The assumption of an infinite number of supports naturally involves the fact that — exclusive of the case of an infinitely rigid support — the load over the support (sleeper) selected arbitrarily will be distributed to an infinite number of supports, however — when moving away from the position of load — the number of supports taking part in load distribution will vary depending on the accuracy required for calculations, i. e. on the number of decimal figures required for the calculation results.

Since the number of supports considered accordingly active with a continuous beam having an infinite number of supports will be finite by all means, therefore in the course of our calculations, a beam with a finite number of supports having a traditional solution was assumed, which involved — among others — the alleviation of calculations performed on beams rendered irregular due to the variation in the rigidity of sleepers. The number of supports or the spans, respectively, taken into account can be varied according to the accuracy required or the goal set before us.

3.2. The Rail as a Two-Span Beam

As the index number of load distribution capacity with superstructures can be considered the reaction force of the sleeper subject to unit load, which is smaller than the unit, the missing force required for restoring the equilibrium is transferred through other sleepers: so the load will be distributed.

From among the continuous beams on elastic support, it is the two-span one which can be examined most easily by loading it over the intermediate support with a unit load (*Fig. 1*). This is the simplest static model with respect to the analysis of the factors influencing the load distribution, namely: spacing of sleepers (a), rigidity of sleepers (e) and rigidity of rails (EI), or with respect to ranking the different systems of superstructures.

The ratio of the unit factors calculable from the deformation of rails, or the compression of sleepers, respectively, and the load factor calculable from load $P=1$ over the intermediate support will be the unknown resisting (restoring) moment of the support, whence value ΔA reducing the unit reaction force of the intermediate support can be calculated. Consequently, the reaction force considered as the index number of load distribution will be:

$$A = 1 - \Delta A\,,$$

where:

$$\Delta A = \frac{6EI \cdot e}{a^3 + 9EI \cdot e}\,.$$

Extreme cases can be estimated on the basis of formula ΔA:

— If $e \to 0$ (rigid sleeper), or a $a \to \infty$ (a very long distance between sleepers), then $\Delta A \to 0$, consequently, there is no load distribution.

— If $EI \to \infty$ (very rigid rail), or $a^3 \to 0$ (continuous support), then $\Delta A \to \frac{6}{9} = 0.67$, i. e. $A \to 0.33$, consequently all the three supports are loaded by a force of the same magnitude, and this is called the perfect load distribution.

Consequently, with respect to load distribution, a more advantageous effect will be provided by a softer and denser spacing of supports, and the more rigid rails.

The two-span rail (track) is also suitable for the numerical evaluation — though only of approximate value — of the factors influencing the measure of load distribution. The variation in the reaction values of the intermediate support as a function of sleeper spacing was tabulated in

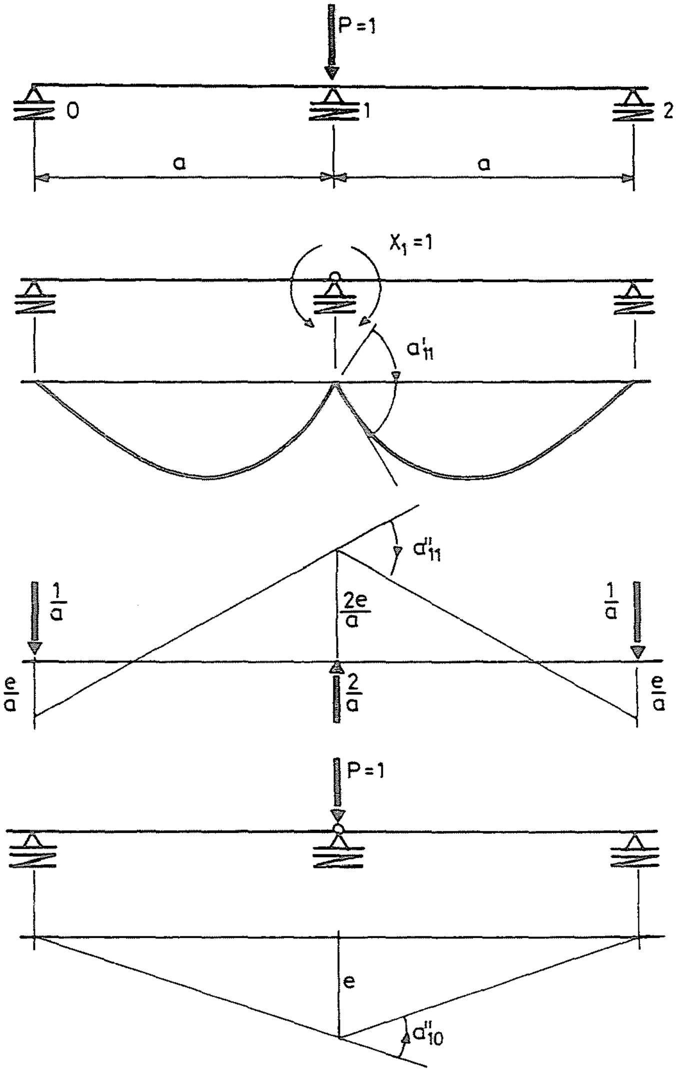

Fig. 1.

Table 1 and visualized in *Fig. 2* by forming the following combination of pairs as shown below:

- — steel cross-tie and rail of 54 kg (column 1)
- — oak sleeper and rail of 34, 48, 54 and 60 kg, respectively, (columns 2–5)
- — coniferous wooden sleeper and rail of 60 kg (column 6).

Table 1
Index-number of the load distribution capacity

Spacing of sleepers (m)	1. Steel 54	2. 34	3. Oak wood 48	4. 54	5. 60	6. Coniferous wood 60
0.20	0.518	0.389	0.364	0.357	0.351	0.342
0.40	0.835	0.614	0.520	0.483	0.454	0.400
0.60	0.941	0.807	0.711	0.662	0.619	0.515
0.62	0.946	0.820	0.727	0.679	0.635	0.528
0.65	0.953	0.838	0.750	0.702	0.658	0.548
0.68	0.958	0.854	0.771	0.724	0.681	0.568
0.70	0.961	0.864	0.782	0.738	0.695	0.582
0.80	0.974	0.902	0.838	0.799	0.760	0.647
1.00	0.982	0.946	0.906	0.879	0.851	0.756

Neither in the *Table*, nor in the *Figure* has sleeper spacing $a = 0.2 - 0.5$ m any practical significance, (i. e. other static conditions would be associated with the double-sleeper joint), sleeper spacing $a = 1.0$ m serves similarly for visualization.

As both the *Table* and the *Figure* obviously show, the load distribution capacity hardly varies within the prescribed intersleeper spacing but it is greatly influenced by the rigidity of cross-ties (their material) and the rigidity of rail, respectively; it goes so far that in the case of a direct-bearing steel cross-tie, or a rail of 34 kg (c), it is not worth dealing with load distribution.

It is probable that the wooden sleepers are not of equal rigidity originally. The effect of this can be modelled by assuming a rail of two span, i. e. in a way that in succession, coniferous wood, oak wood, coniferous wood, or oak wood, coniferous wood, and oak wood sleepers are alternately taken into consideration. The effect was controlled also by applying a multi-span beam approximating much more closely the reality, so it could be stated that *an unfavourable rigidity distribution impaired the load distribution capacity to a non-negligible extent.* Hence it may be concluded that it is the *steadily well-maintained* superstructure which should be set as an aim, and that if one or the other of the uniformly damaged sleepers is replaced by a brand new one, it will not offer any advantage with respect to load distribution.

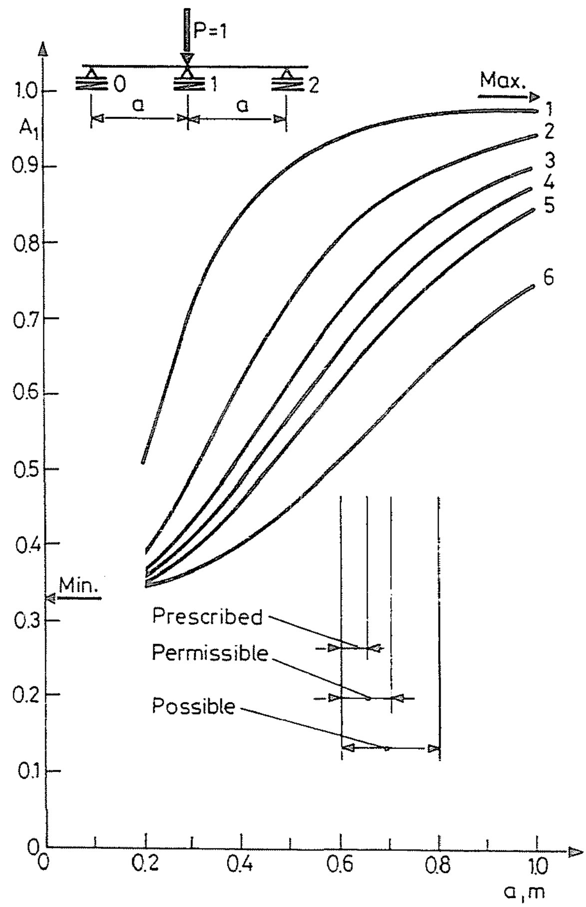

Fig. 2.

3.3. Effect of the Model Having More Than Two Spans

The two-span model is a simple one but it undoubtedly involves inaccuracies. In *Table 1*, each numerical data characteristic of load distribution could be corrected by achieving a solution to the beam of an equal number of supports, or that of an infinite number. If our aim is only the description of the load distribution capacity of the superstructure, then already by a moderate increase of the span numbers we can make sure of the fact that reaction force can be calculated from the force loading the intermediate sleeper at the same place as the index number of load distribution capacity reduced by 10%, consequently, it is stabilized on an advantageous level, however, the experience gathered from the two-span track model remains unchanged.

However, the rail is a multi-span one. But the nature of the regular continuous beams on elastic support involves the fact that — in the case of a model having more than two spans — the sum of the reaction forces transferred through the sleeper loaded directly and the forces transferred through the two adjacent ones. i. e. altogether through three sleepers, or in a more favourable case, through five sleepers, in all, will be greater by 6 – 7% than the load, and the equilibrium will be maintained by the negative reactions, or by negative and positive reactions, respectively, in combination acting upon the sleepers not reckoned with above. The interpretation of the negative reactions acting on the sleepers, i. e. the reactions effecting the uprise of the sleepers will be treated yet in the following. Here below, such a system of ideal beams will be assumed provisionally in which the supports and sleepers can equally bear both the positive and negative reactions.

3.4. The Beam Supporting the Railway is Not Rigid But Elastic

In the foregoing, the rail was considered to be a continuous beam on elastic supports which are borne on an iron structure and longitudinal girders assumed implicitly as rigid ones. However, the longitudinal girder of the railway bridge is not rigid but is subject to reversible deformation. The effect of neglecting this fact made on the behaviour of superstructure was estimated by two simplified computational models. In the case of the longitudinal girder having a denser spacing $l = 3.25$ m and considered to be a simple supported one, the index number of the load distribution capacity of the superstructure was reduced by about 5%, while the case of the longitudinal girder having a longer distance of spacing $l = 6.50$ involved a deviation smaller than 1% with respect to deformation. This deviation was found to be negligible. This statement results in alleviation of calculations. When the deformation of the floor structure, too, should be reckoned with in the course of determining the behaviour of the rail, then in the last analysis, a different reaction influence line, i. e. a different load distribution capacity would belong to *each sleeper* on the bridge; thus the superstructure could not be calculated independently from the supporting structure.

Among the longitudinal girders of the home railway bridges, there are girders of a shorter spacing than 3.0 m, too. In such cases, it is advisable to estimate the effect of deformation of the longitudinal girder (maybe longitudinal rib) individually without prejudices.

3.5. Reaction Influence Line of Multi-Span Rails

For the most unfavourable load of the girder supporting the railway superstructure to be calculated, it is not enough to determine the load distribution capacity of the superstructure, but the reaction influence line of the rail should also be known because only the knowledge of that enables us to calculate the values of the sleeper reaction loading the supporting girder from the wheel load of an *arbitrary position.*

With the assumption of a continuous rail, the task to be performed is to determine the reaction influence line related to the middle support of a sufficiently multi-span beam on elastic support with the demanded accuracy. When reaction influence line $\eta(A_k)$ of the middle support is to be determined, then it is advisable to use an approximation of low effect since for the calculation of reaction influence line $\eta(A_k)$, the knowledge of moment influence lines: $\eta(M_{k-1})$, $\eta(M_k)$ and $\eta(M_{k+1})$ would be required, instead — with a beam of an infinite number of spans in mind — for the calculation of reaction influence line, influence line $\eta(M_k)$ can also be used in the capacity of influence lines: $\eta(M_{k-1})$ or $\eta(M_{k+1})$, respectively, shifting it one span to the right or left.

As an way of example, such a reaction influence line can be seen in *Fig. 3* assuming a rail of 54 kg, sleeper spacing of 0.62 m and sleepers of coniferous wood. From among the 14 spans of the beam, only the influence lines of the eight intermediate spans are represented here. This influence line shows the magnitude of the load transferred from the wheel load of arbitrary position to the supporting beam through *middle sleeper k* (in this case: sleeper No 7). In the case of a sufficiently multi-span beam, it can be assumed with a good approximation that the reaction influence lines of both adjacent supports of an intermediate spacing are identical to each other and equal to calculated value $\eta(A_k)$. Thus, the ordinates of the known $\eta(A_k)$, too, can be arranged so that they show the magnitude of reaction forces loading the sleepers and arisen from the force induced above a cross-sectional area chosen arbitrarily within an intermediate spacing of sleepers.

The ordinates of the reaction influence line represented in *Fig. 3* and calculated in terms of decimal figures with respect to the sleeper spacing, and arranged according to the foregoing, were tabulated in *Table 2.* Consequently, each row of this Table shows what force is transferred by the unit load to the supporting beam through the sleepers reckoned with in succession, in the different positions of the spacing as expressed by decimal figures. *Consequently, instead of a unit load, the beam is loaded by the sleepers' reactions calculable from the unit load.*

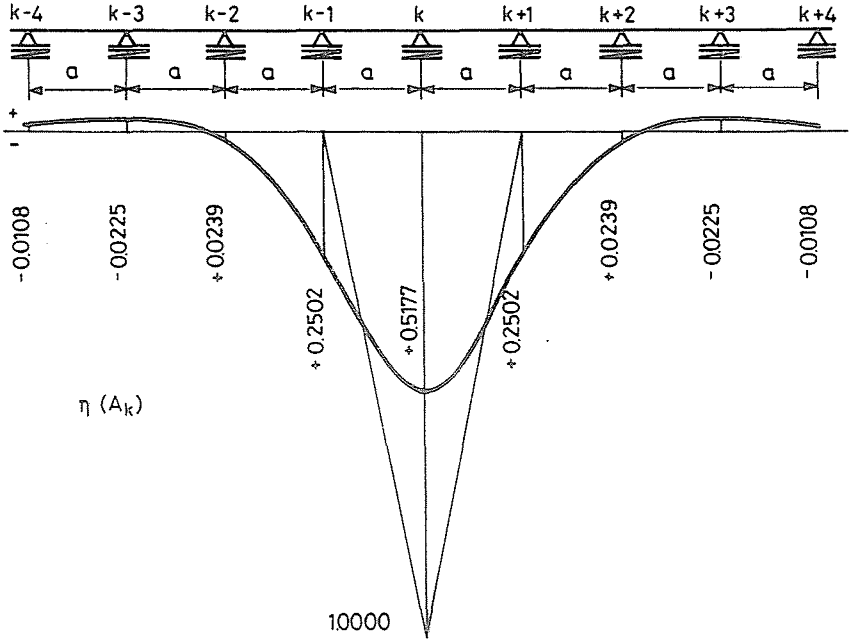

Fig. 3.

4. Influence Lines of a Girder with the Load Distribution Capacity of Superstructure Taken into Consideration

The beams supporting the superstructure are loaded directly according to the traditional statics classification, so their influence lines are known. If we are to reckon with the load distribution effect of the continuous rail, then the unit load moving along the beam can similarly be carried on the rail just as in the case of a traditionally directly loaded beam, however, this unit load in each position will bring about a sleeper reaction corresponding to the continuous rail and, consequently, the load applied to the supporting beam, i. e. *the ordinate of the load influence line belonging to the load position will be the sum of the individual products of ordinates related to the load distributed to the individual sleepers, and those related to the influence lines of the direct load under the sleepers.*

The effect of the load distribution capacity of superstructure will be demonstrated by simple examples as follows. The span of the supporting beam is $l = 3.10\,\mathrm{m}$, the characteristic data of the superstructure are: rail of 54 kg, spacing of sleepers at a distance of 0.62 m, and coniferous-wooden cross-ties; *Fig. 3* and *Table 2* are based on the same data.

Table 2

Values of sleeper-reaction as a function of load position

Application point of force	Reactions of sleepers								
	A_{k-4}	A_{k-3}	A_{k-2}	A_{k-1}	A_k	A_{k+1}	A_{k+2}	A_{k+3}	A_{k+4}
k	−0.0108	−0.0225	+0.0239	+0.2502	+0.5177	+0.2502	+0.0239	−0.0225	−0.0108
$k-0.1a$	−0.0121	−0.0223	+0.0368	+0.2853	+0.5133	+0.2173	+0.0129	−0.0229	−0.0094
$k-0.2a$	−0.0136	−0.0215	+0.0518	+0.3216	+0.5007	+0.1869	+0.0038	−0.0222	−0.0081
$k-0.3a$	−0.0153	−0.0200	+0.0689	+0.3582	+0.4814	+0.1588	−0.0037	−0.0212	−0.0069
$k-0.4a$	−0.0167	−0.0176	+0.0881	+0.3934	+0.4562	+0.1330	−0.0096	−0.0198	−0.0057
$k-0.5a$	−0.0184	−0.0142	+0.1094	+0.4262	+0.4265	+0.1094	−0.0142	−0.0184	−0.0047
$k-0.6a$	−0.0196	−0.0096	+0.1330	+0.4562	+0.3934	+0.0881	−0.0176	−0.0167	−0.0043
$k-0.7a$	−0.0212	−0.0037	+0.1588	+0.4814	+0.3582	+0.0689	−0.0200	−0.0153	−0.0030
$k-0.8a$	−0.0222	+0.0038	+0.1869	+0.5007	+0.3216	+0.0518	−0.0215	−0.0136	−0.0022
$k-0.9a$	−0.0229	+0.0129	+0.2173	+0.5133	+0.2853	+0.0368	−0.0223	−0.0121	−0.0016
$k-1$	−0.0225	+0.0239	+0.2502	+0.5177	+0.2502	+0.0239	−0.0225	−0.0108	−0.0010

4.1. Reaction Influence Line of a Simple Supported Beam

The well-known reaction influence line of a simple supported beam with a span of $l = 3.10$ m is plotted in *Fig. 4* in thin line. The beam was divided into ten equal sections with respect to its function. In the case of a sleeper spacing $a = 0.62$ m, the sleepers are located either on the cross-sections indicated by even numbers (*Fig. 4a*), or on the cross-sections indicated by odd numbers (*Fig. 4b*). The reaction influence line taking into consideration the load distribution effect of the superstructure is traced out in thick line.

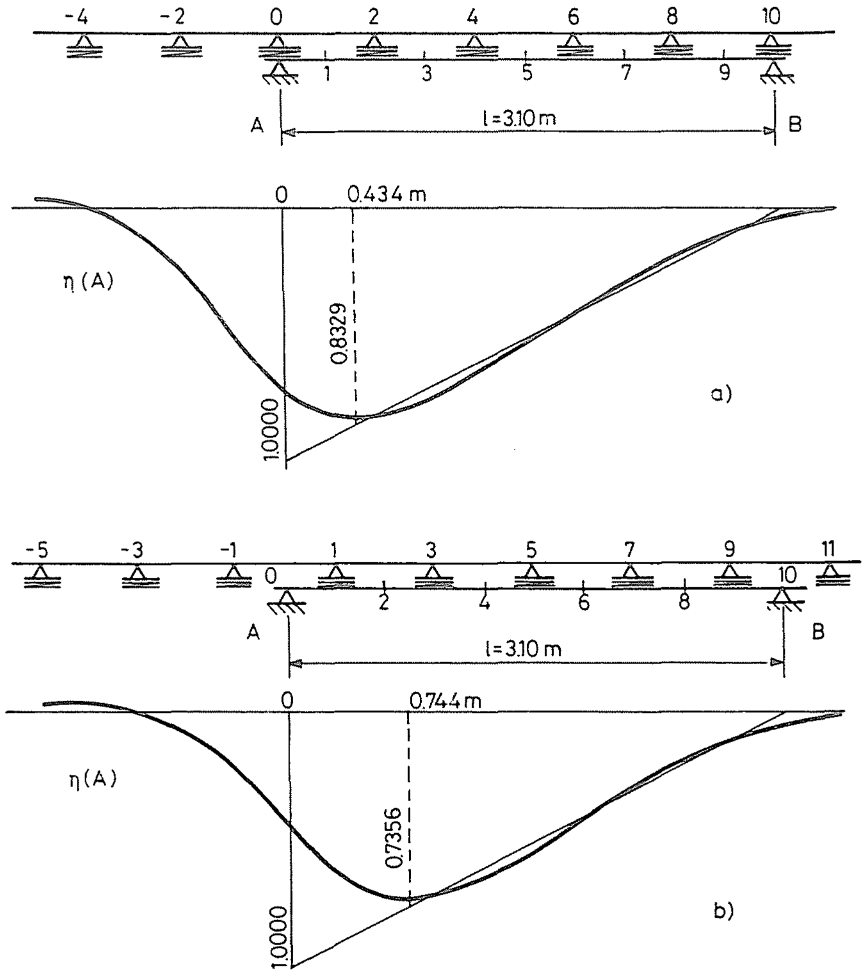

Fig. 4.

If the rail is considered to be a beam on supports of infinite number, then even a unit load applied in the infinity can cause a sleeper reaction and, in this way, the reaction influence line taking into consideration the effect of the superstructure has an influence line ordinate different from zero even in the infinity. Consequently, the reaction influence line is infinitely long. With the continuous rail regarded as a sufficiently multi-span one, the number of supports is finite, however, the total length of spacings will be added to the original length of the influence line. In *Figs. 4a* and *4b*, the small ordinates of the outer sleeper spacings are no longer represented, however, the deviation of the influence lines of direct loading and those of the superstructure from each other, as well as the effect of load positioning are remarkable. It can be verified that *the sum of the ordinates of the superstructure influence lines under all sleepers reckoned with coincides with the sum of the ordinates of the influence lines under the sleepers loaded directly.*

In the present case, this value is 3.0 in the case of sleepers of even-number position (*Fig. 4a*), while it is 2.5 in the case of sleepers of odd-number position (*Fig. 4b*). Since the influence line of the superstructure is longer, therefore the greatest ordinates are smaller than those of a directly loaded beam, and besides — apart from the symmetric influence lines — the locus of the greatest ordinate will change, too.

The *reaction influence line of the train* shown in *Fig. 5* is very instructive, which contains the variation of the reaction force by the effect of three unit loads proceeding from left to right at a distance of 1.55 m from each other. The effect of the superstructure is conspicuous, the value of the greatest reaction is only 72% of that calculated traditionally, and the characteristic peak loads are missing, the load alteration is much smoother, and the point of maximum load application is also shifted. (Similar deviations can be expected in the case of shear influence lines, too.)

The question indispensably emerges: *Why do not the connections of the longitudinal girders, or the riveting of chords, respectively, of the smaller bridges do not suffer failure in certain cases in spite of their low nominal reliability,* as well as the question: *How incorrect can the life estimation of the bridges be due to a computational model chosen improperly.*

4.2. Moment Influence Lines of Simple Supported Beams

The data of the beam and the superstructure used here are the same as the ones introduced in point 4.1, however, with a view to the reduction in the number of calculations, from among the reactions of sleepers only the

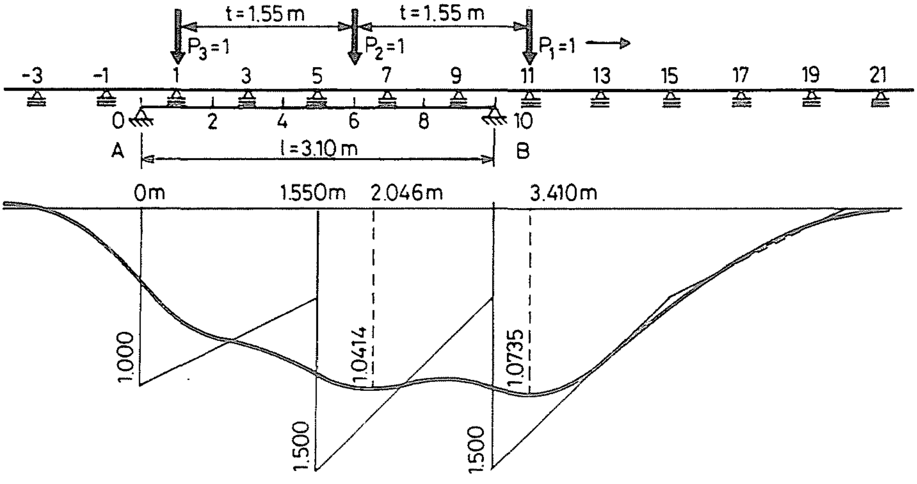

Fig. 5.

intermediate, positive ones dominating in load distribution were taken into consideration with their sum reduced to the unit (See also point 5).

In *Fig. 6* the moment influence line of intermediate cross-section No. 5 can be seen, where the deviation is more moderate than with the reaction influence line (cf. *Fig. 4*), though the greatest ordinate (0.6008) is essentially smaller here, too, than that of the directly loaded beam (0.7750). It is conspicuous that a considerable value (0.1092) is encountered in the cross-sectional area of supports, too. Consequently, if the beam is loaded by three forces of unit magnitude at a distance of 1.55 m from each other, as it was assumed in pont 4.1, too, then $M=0.1092+0.6008+0.1092=0.8192$, and this is greater than the value which would be calculated from the same load in the intermediate cross-section of a two-support beam loaded directly (0.7750). However, it is true that when this beam *is dimensioned,* then not three but only two axles are reckoned with in cross-section $x = 1.1625$ m, while $M = 0.8719$, and this is greater than value 0.8192 with the superstructure.

In *Fig. 7* on a two-support beam, the moment influence line of the train in cross-section $x = 1.625$ m delivering the maximum moment used for the group of axles was represented. With the knowledge of those described above, it is not surprising that it is not this cross-section which is the most unfavourable one with the superstructure (for two axles $M = 0.7180$, for three axles $M = 0.7366$), and the most unfavourable load position resulting in the maximum load application will also deviate. The moment of the superstructure beam could not reach that of the beam dimensioned tradi-

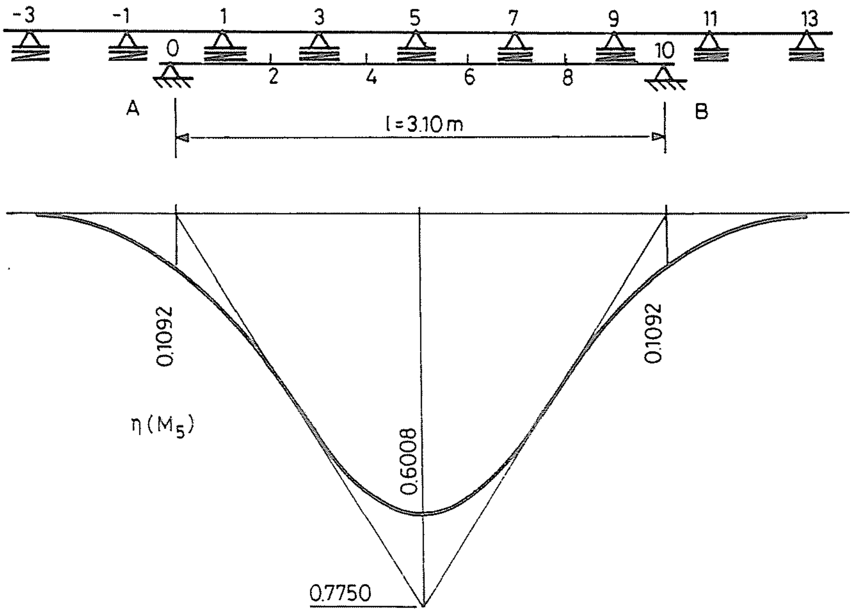

Fig. 6.

tionally, and with our experience generalized, it can be stated: *our static reflexes based on a directly loaded beam prove to be a failure.*

5. Some Corollaries

In the foregoing, it was assumed that sleepers could take also negative reactions acting in upward direction, though this is not guaranteed structurally. In spite of this fact, in case of a continuous rail, this computational model is regarded as a relatively well-approximated one, since the negative reaction — due to the moderate wheel load — is counterbalanced by the dead weight of the superstructure estimated to be about 1.0 – 1.2 kN for each support, though — when the wheel load is greater — the uprise of one or two sleepers can result in the change of both the static model and the behaviour, all the same this fact will not involve a considerable modification in the magnitude of the intermediate sleepers' reaction exerting a decisive influence on load distribution.

The multi-support model requires a considerable amount of calculation work. As a possibility of approximation, only the positive reaction values in the vicinity of the intermediate sleeper are taken into consideration.

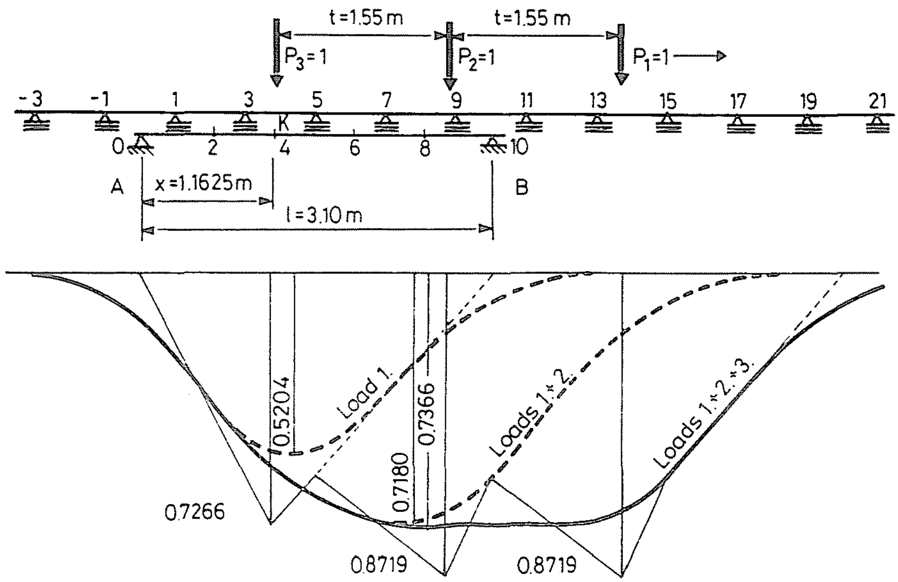

Fig. 7.

However, the sum of those is greater by 6–7% than the load, consequently, the sum of sleeper reactions is greater than the unit in the case of using influence lines. This apparent contradiction can be resolved arbitrarily if the reactions of sleepers are reduced proportionally so that their sum is equal to one: however, this procedure — depending on the influence line or the group of loads, respectively — can result in neglection in individual cases at the expense of safety. The influence line shown in *Fig. 6* was calculated on the basis of positive sleeper reactions reduced to unit. This involved a neglection of about 6% for the case of maximum ordinate value, while a neglection of about 2% was involved in the value of the maximum moment calculated from a given group of axles at the expense of safety.

6. UIC Recommendation

In the course of calculations of sleeper reactions, introduced by way of example, a softwood cross-tie was assumed though oak wood is adopted for this purpose. There are several reasons for that, among others, that this rigidity results in load distribution conditions similar to those known as the recommendations of the UIC. Since certain railway companies under the auspices of the UIC prescribe, or at least permit the assumption of the fact that the 0.5-fold value of load P over the cross-tie be transferred directly through the loaded cross-tie, while the 0.25 – 0.25-fold values be transferred through the two adjacent cross-ties.

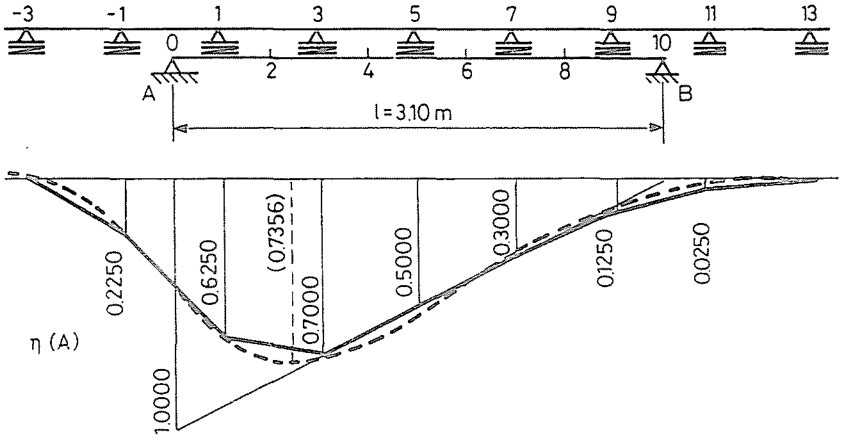

Fig. 8.

There is no continuous rail of such load distribution but it can be reckoned with in the case of a two-span rail (of point 3.2) or a continuous rail on elastic support, if e. g. the reaction values shown in the first row of *Table 2* are rounded up to 0.05. However, those contained in the recommendation are not sufficient alone, the designer would keep guessing for the lack of additional instructions, and further on, he is not protected even against making serious mistakes in the design (e. g. the designer could calculate, as well, with a value of 0.6008 corresponding to the example shown in *Fig. 6* instead of values 0.775 or 0.8192). The recommendation of the UIC may be used by a designer calculating on the basis of the influence line, for reducing separately the individual ordinates of the influence line in the cross-section of beams supporting the sleepers: however, the loads are acting also within the intersleeper space, and no instruction is contained in the recommendations of the UIC. Such recommendation would be useful, e. g. in the case of a load acting within the intersleeper space, the distribution of the load acting on two adjacent sleepers as the reaction of a two-support beam, as well as the distribution of the reaction loading the two sleepers according to the load distribution (0.25 – 0.50 – 0.25). In this way, the load is distributed to four adjacent sleepers, and the distribution can be tabulated (*Table 3*) in case we insist on calculation. This solution yields the same result which was obtained when the ordinates of the adjacent influence lines as reduced according to the distribution, and existing independently in the centre-line of sleepers are connected together by a straight line. Such a linearized reaction influence line of the superstructure can be seen in *Fig. 8* traced out in thick line, where the thin line is associated with the directly loaded beam, while the dashed line takes into consideration also the effect of the rail on elastic support. It is only obvious that the actual spacing of sleepers should be taken into consideration (cf. the difference between *Figs. 4a* and *4b*).

Table 3
Reactions of sleepers with linear approximation

Application point of force	Reactions of sleepers A_{k-2}	A_{k-1}	A_k	A_{k+1}
k	—	0.250	0.500	0.250
$k-0.1a$	0.025	0.275	0.475	0.225
$k-0.2a$	0.050	0.300	0.450	0.200
$k-0.3a$	0.075	0.325	0.425	0.175
$k-0.4a$	0.100	0.350	0.400	0.150
$k-0.5a$	0.125	0.375	0.375	0.125
$k-0.6a$	0.150	0.400	0.350	0.100
$k-0.7a$	0.175	0.425	0.325	0.075
$k-0.8a$	0.200	0.450	0.300	0.050
$k-0.9a$	0.225	0.475	0.275	0.025
$k-1$	0.250	0.500	0.250	—

According to another UIC recommendation, in addition to the distribution, even the effect of the load becoming distributed under the sleeper could be reckoned with, however, it is unnecessary with the load-distribution effect of superstructure taken into consideration.

If the statical model is based on oak wood instead of a softwood crosstie, then a load distribution $0.2-0.6-0.2$ will be obtained, however, there is no great deviation from the former one.

7. Last Remarks

When dimensioning railway bridges, the beams supporting the superstructure are dimensioned as directly loaded beams, though it is sure that the load is distributed along a continuous rail. On the basis of the example demonstrated in the foregoing, the consequences of reckoning with the effect of load distribution may be estimated. The effect of superstructure should not be neglected during the supervision of the condition of existing bridges, the selection of measuring stations, the evaluation of the result measured or the experience gained. When new bridges are designed, the fact should be noted that a more accurate calculation can involve unforseen difficulties, and the invested extra work will not result in the reduction of load in each case, however, in spite of this — since there is no railway bridge without superstructure — we should get accustomed sooner or later to the consideration of this effect.

Acknowledgement

Author considers his kind duty to thank his sometime employer, the Bridge Department of the MÁV, for the stimulation to deal with the topic, as well as the Department of Steel Structure for the possibility of performing experimental examinations.

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