RIGOROUS ADJUSTMENT OF A TRAVERSE

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Abstract

In the first part of this paper computation of traverses tied and oriented at both ends was introduced by means of direct observation, based on the principle of the least squares. In the following part formulas were shown for determining measures of accuracy for surch traverses. After the theoretical chapters, applicibility was proved by means of a numerical example.

1. Introduction

"For more than hundred years professional literature has been dealing with adjustment of traversing, and papers discussing methods of optimal adjustment of traversing, could not even be listed here" [1].

One would not be able to find a more appropriate introduction to papers on adjustment of traverses than this first sentence of the quoted work. For this reason there will no list be presented on most important professional works, only some directly used works are mentioned in the reference.

This papers deals with rigorous adjustment of traverses tied and oriented at both ends.

With introducing and spreading EDMs, utilizing traversing on greater lengths came to the front. It is obvious than in case of long traverses besides precise measurements it is important to utilize rigorous adjustments. Up-to-date computational features make possible and continually growing demends require utilizing rigorous methods, algorithms and programs based on least square methods for the purposes of geodetic computations.

After describing rigorous adjustment of traverses, determining measures of accuracy will be presented.

Intention of this paper is to form a suitable denoting and computational algorithm for computers. Some formulas will be introduced which are difficult to compute manually while adjustment of direct observations. Such formulas are standard error of co-ordinates of traversing points regarded as unknowns and measures of accuracy computed from them.

Utilizing the principle of rigorous adjustment and connecting measures of accuracy will be shown by means of an example.

Notations

$\mathbf{B}^*$ = coefficient matrix of conditions equations
$\mathbf{U}$ = vector of adjusted measurements
$\mathbf{C}$ = vector of known values originated from geometric connections
$\mathbf{l}$ = vector of constant terms
$\mathbf{v}$ = vector of adjustment corrections
$\mathbf{M}_{LL}$ = variance-covariance matrix of measurements
$\mathbf{Q}_{LL}$ = weight coefficient matrix of measurements
$\overline{m}_0^2$ = a priori value of standard error of unit weight
$\mathbf{k}$ = vector of correlates
$\mathbf{Q}_{ll}$ = coefficient matrix of normal equation system
$\mathbf{L}$ = vector of measurements
δ = bearing
Y_i = $\Big\}$ plane coordinates
X_i =
n = number of traverse stations
β_i = traverse angles
t_i = lengths of traverse legs
$b_{1i} \ldots b_{4i}$ = elements of coefficient matrix of conditions equations
$\mathbf{M}_{ll}$ = inverse of coefficient matrix normal equation system
m_0 = standard error of unit weight
$\mathbf{Q}_{VV}$ = weight coefficient matrix of corrections
$\mathbf{Q}_{UU}$ = weight coefficient matrix of the adjusted measurements
$\mathbf{M}_{UU}$ = variance-covariance matrix of adjusted measurements
$\mathbf{Q}_{\binom{Y}{X}}$ = weight coefficient matrix of coordinates of traverse stations
$\mathbf{M}_{\binom{Y}{X}}$ = variance-covariance matrix of traverse stations
$\mathbf{E}$ = matrix formed by partial differential quotients of coordinates of traverse stations to measurements
$\mathbf{N}_i$ = variance-covariance matrix of traverse stations
m_{Y_i} = $\Big\}$ standard error of coordinates of traverse stations
m_{X_i} =
$C_{Y_iX_i}$ = covariance of traverse stations
K_i = mean standard error of position of traverse stations
K_k = mean standard error of position of the traverse
$m_{\max_i}$ = largest standard error of traverse stations
$m_{\min_i}$ = smallest standard error of traverse stations
λ = eigenvalue of variance-covariance matrix of traverse station
$\delta_{\max_i}$ = bearing of the largest standard error

2. The principle of rigorous adjustment of traverses

Adjustment of traverses is practically carried out by means of the method of direct observations. Results of observations must fulfill given at the adjustment conditions. The original results of observations are usually not fulfilling these conditional equations.

The most frequently utilized method for distributing discrepancies is distribution according to the least squares, which leads to developing a most probable and discrepancy-free system, supposing normal distribution.

The number of generally non linear condition equations are formed by means of 'n' observations, and after linearizing them one obtans:

$$\mathbf{B}^*\mathbf{U} = \mathbf{C} \tag{1}$$

where $\mathbf{B}^*$ = coefficient matrix of condition equation
$\mathbf{U}$ = vector, containing the adjusted observation results
$\mathbf{C}$ = vector of known values originated from geometric connections.

Because condition equations are satisfied by original observation results only exceptionally, generally when solving the equations, an "$\mathbf{l}$" vector of discrepancies — constant term vector — will appear, which differs from zero:

$$\mathbf{C} - \mathbf{B}^*\mathbf{L} = \mathbf{l} \neq 0 \tag{2}$$

A fulfilling the conditions can be achieved by means of "$\mathbf{v}$" adjustmental corrections.

$$\mathbf{C} - \mathbf{B}^*(\mathbf{L} + \mathbf{v}) = 0 \tag{3}$$

The above linear functional modell can be put to the following form:

$$\mathbf{C} - (\mathbf{B}^*\mathbf{L} + \mathbf{B}^*\mathbf{v}) = 0 \tag{4}$$

Reliability of the original observation results is described by a stochastic modell which can be described by means of $\mathbf{M}_{LL}$ diagonal matrix presuming independent measurements.

The $\mathbf{M}_{LL}$ variance-covariance matrix can be expressed by $\overline{m}_0^2$ a priori coefficient and $\mathbf{Q}_{LL}$ weight coefficient matrix.

$$\mathbf{M}_{LL} = \overline{m}_0^2 \, \mathbf{Q}_{LL} \tag{5}$$

Applying the least square method after solving the system of normal equations Lagrande's multiplier factor "$\mathbf{k}$".

$$\mathbf{k} = -(\mathbf{B}^* \mathbf{Q}_{LL} \mathbf{B})^{-1}\mathbf{l} = -\mathbf{Q}_{ll}^{-1} \mathbf{l} \tag{6}$$

where $\mathbf{Q}_{ll}$ is the coefficient matrix of the normal equation.

From this one can compute corrections and adjusted observations:

$$\mathbf{v} = \mathbf{Q}_{LL} \mathbf{B} \mathbf{k} = -\mathbf{Q}_{LL} \mathbf{B}(\mathbf{B}^*\mathbf{Q}_{LL}\mathbf{B})^{-1} \mathbf{l} = -\mathbf{Q}_{LL}\mathbf{B}\mathbf{Q}_{ll}^{-1} \mathbf{l} \tag{7a}$$

$$\mathbf{U} = \mathbf{L} + \mathbf{v} \tag{7b}$$

After forming the discrepancy-free system, the sought unknown co-ordinates can be computed, in our case by means of continuously polar points.

A sketch of a traverse oriented at both ends is given in Fig. 1.

There are n traversing points located between the starting point "K" and the end point "V". Number of traverse angles is $n + 2$, while number of measured distance is $n + 1$, which means that the number of observations is $2n + 3$. Number of unknown coordinates is $2n$. Number of redundant observations, i.e. that of condition equations is 3.

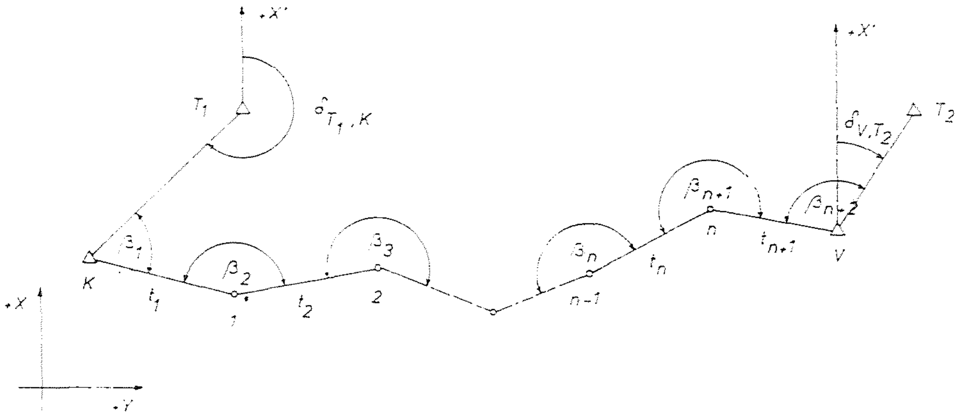

Fig. 1

The first condition equation expresses that δ_{V,T_2} computed bearing should be resulted from bearing at the starting point with utilizing error-free observations. The other two condition equations provide that sums of projections of traverse legs to the coordinate axes are equal to the corresponding co-ordinate differences between the starting and end points assuming error-free observations.

The condition equation which expresses unchangedness of bearings:

$$\delta_{V,T_2} - \delta_{T_1,K} - \sum_{j=1}^{n+2} (U_{\beta_j} - 180^\circ) = 0 \tag{8}$$

Side equations in X and Y directions:

$$Y_V - Y_K - \sum_{j=1}^{n+1} U_{t_j} \sin\left[\delta_{T_1,K} + \sum_{k=1}^{j} (U_{\beta_k} - 180^\circ)\right] = 0 \tag{9}$$

$$X_V - X_K - \sum_{j=1}^{n+1} U_{t_j} \cos\left[\delta_{T_1,K} + \sum_{k=1}^{j} (U_{\beta_k} - 180^\circ)\right] = 0 \tag{10}$$

The differences of the three conditions from 0 with introducing the original measurements from the constant term vector:

$$\mathbf{l} = \begin{bmatrix} l_\beta \\ l_Y \\ l_X \end{bmatrix} \tag{11}$$

Individual constant terms are as follows

$$l_\beta = \delta_{V_1T_2,} - \delta_{T1,K} - \sum_{j=1}^{n+2} (L_{\beta_j} - 180°) \tag{12}$$

$$l_Y = Y_V - Y_K - \sum_{j=1}^{n+1} L_{t_j} \sin\left[\delta_{T1,K} + \sum_{k=1}^{j} (L_{\beta_k} - 180°)\right] \tag{13}$$

$$l_X = X_V - X_K - \sum_{j=1}^{n+1} L_{t_j} \cos\left[\delta_{T1\,K} + \sum_{k=1}^{j} (L_{\beta_k} - 180°)\right] \tag{14}$$

The condition equation which expresses unchangedness of bearings, is linear. Coefficients of corrections of adjustment for measured traverse angles and measured lengths are as follows:

$$\frac{\partial \mathbf{1}_\beta}{\partial \beta_i} = -1 \qquad \forall i = 1(1)n + 2 \tag{15}$$

$$\frac{\partial \mathbf{1}_\beta}{\partial t_i} = 0 \qquad \forall i = 1(1)n + 1 \tag{16}$$

Partial differential quotients of side equations according to the traverse angles in Y and X directions are as follow:

$$\left.\begin{aligned} b_{1i} = \frac{\partial l_Y}{\partial \beta_i} &= -\sum_{j=1}^{n+1} L_{t_j} \cos\left[\delta_{T_1,K} + \sum_{k=1}^{j} (L_{\beta_k} - 180°)\right] \\ b_{2i} = \frac{\partial l_X}{\partial \beta_i} &= \sum_{j=1}^{n+1} L_{t_j} \sin\left[\delta_{T1,K} + \sum_{k=1}^{j} (L_{\beta_k} - 180°)\right] \end{aligned}\right\} \quad \forall i = 1(1)n + 2 \tag{17}$$

Is should be noted that

$$\frac{\partial l_Y}{\partial_{\beta_{n+2}}} = \frac{\partial l_X}{\partial_{\beta n+2}} = 0$$

Partial differential quotients of side equations according to the lengths can be written as

$$\left.\begin{aligned} b_{3i} = \frac{\partial l_Y}{\partial t_i} &= -\sin\left[\delta_{T_1,K} + \sum_{k=1}^{j} (L_{\beta_k} - 180°)\right] \\ b_{4i} = \frac{\partial l_X}{\partial t_i} &= -\cos\left[\delta_{T_1,K} + \sum_{k=1}^{j} (L_{\beta_k} - 180°)\right] \end{aligned}\right\} \quad \forall i = 1(1)n + 1 \tag{18}$$

Reliability of the observations is given by a diagonal matrix

$$\mathbf{M}_{LL} = \langle m_{\beta_1}^2, m_{\beta_2}^2, \ldots, m_{\beta_{2+n}}^2, m_{t_1}^2, m_{t_2}^2, \ldots, m_{t_{n+1}}^2 \rangle = m_0^2 \mathbf{Q}_{LL} \tag{19}$$

Coefficient of the $\mathbf{M}_{ll} = \mathbf{B}^* \mathbf{M}_{LL} \mathbf{B}$ normal equation systems can be expressed as follows utilizing the above introduced notations.

$$\mathbf{M}_{ll} = \begin{bmatrix} \sum_{i=1}^{n+2} m_{\beta i}^2; & -\sum_{i=1}^{n+1} b_{1i}\, m_{\beta i}^2; & -\sum_{i=1}^{n+1} b_{2i\, m\beta i}^2 \\ - & ; \sum_{i=1}^{n+1} (b_{1i}^2 m_\beta + b_{3i}^2 m_{ti}^2); & \sum_{i=1}^{n+1} (b_{1i}\, b_{2i}\, m_{\beta i}^2 + b_{3i}\, b_{4i}\, m_{ti}^2) \\ - & ; \qquad - \qquad ; & \sum_{i=1}^{n+1} (b_{2i}^2\, m_{\beta i}^2 + b_{4i}^2\, m_{ti}^2) \end{bmatrix} \tag{20}$$

After solving the system of equations of a size of (3x) one obtains the values of correlates

$$k^* = (k_\beta, k_Y, k_X)^*$$

Individual corrections can be obtained from the equation (7a)

$$\left.\begin{aligned} v_{\beta i} &= (-1k_\beta + b_{1i}\, k_Y + b_{2i}\, k_X)\, m_{\beta i}^2 \\ v_{ti} &= \quad (b_{3i}\, k_Y + b_{4i}\, k_X)\, m_{ti}^2 \end{aligned}\right\} \forall\, i = 1(1)n + 1$$

$$v_{\beta n+2} = -k_\beta\, m_{\beta n+2}^2 \tag{21}$$

The adjusted measurements can be computed by means of equations (7b). Final coordinates of traverses stations are determined by means of adjusted observations

$$Y_i = Y_A + \sum_{j=1}^{i} U_{tj} \sin\left[\delta_{K,T_1} + \sum_{k=1}^{j} (U_{\beta k} - 180^\circ)\right]$$

$$X_i = X_A + \sum_{j=1}^{i} U_{tj} \cos\left[\delta_{K,T_1} + \sum_{k=1}^{j} (U_{k\beta} - 180^\circ)\right]$$

$$\forall\, i = 1(1)n + 1\,. \tag{22}$$

Of course coordinates of point $(n + 1)$ are identical with those of the end point "V".

3. Measures of accuracy of traverses

Following measures of accuracy can be determined when carrying out rigorous adjustment of traverses

a) value of weight coefficient i.e. standard errors of adjusted measurements

b) standard error of coordinates of traverse stations

c) various measures of accuracy derived from standard error of coordinates.

a) In order to develop the weight coefficient matrix of adjusted measurements one has to compute weight coefficient matrix of corrections. It can be carried out by utilizing the general law of error propogation for equations (7a) and (7b) by means of coordinates k and weight coefficient matrix $\mathbf{Q}_{LL}^{-1}$

$$\mathbf{Q}_{VV} = \mathbf{Q}_{LL}\, \mathbf{B}\, \mathbf{Q}_{ll}\, \mathbf{B}^*\, \mathbf{Q}_{LL} \tag{23}$$

From the above one can obtain the weight coefficient matrix of adjusted measurements

$$\mathbf{Q}_{UU} = \mathbf{Q}_{LL} - \mathbf{Q}_{VV} \tag{24}$$

Variance-covariance matrix $\mathbf{M}_{UU}$ for standard error of adjusted measurements

$$\mathbf{M}_{UU} = m_0^2\, \mathbf{Q}_{UU} \tag{25}$$

where $$m_0^2 = f^{-1}\, \mathbf{v}^*\, \mathbf{Q}_{LL}\, \mathbf{v}$$

b) In order to determine standard error of coordinates of traverse stations one must produce X and Y coordinates as functions of adjusted measurements

$$\begin{vmatrix} Y \\ X \end{vmatrix} = F(U) \tag{26}$$

This connection can be found in equation (22).

By utilizing the general law of error propagation one obtains the weight coefficient matrix of coordinates of traverse stations as follows:

$$\mathbf{Q}_{\binom{Y}{X}} = \mathbf{F}\mathbf{Q}_{UU}\, \mathbf{F}^* \tag{27}$$

where $\mathbf{F}$ is a matrix formed by partial differential quotients of eqn (22) according to the measurements.
Variance-covariance matrix $\mathbf{M}_{\binom{Y}{X}}$ which is necessary for standard error of traverse points, can be computed by the following formulae:

$$\mathbf{M}_{\binom{Y}{X}} = m_0^2\, \mathbf{Q}_{\binom{Y}{X}} \tag{28}$$

c) Several measures of accuracy can be deduced from standard error of coordinates of traverse stations.

Reliability of a point can be described by submatrix $\mathbf{N}_i$: which will be deduced with purposeful regronping of the variance-covariance matrix

$$\mathbf{N}_i = \begin{bmatrix} m_{Yi}^2 & c_{YiXi} \\ c_{YiXi} & m_{Xi}^2 \end{bmatrix} \tag{29}$$

In the principal diagonal $\mathbf{N}_i$ are the squares of standard errors of a point's coordinates. Besides the principal diagonal covariances are located too which are characteristic to the connection of the coordinates.

K_i standard error of position is frequently used for rating points of a net:

$$\mathbb{K}_i = \sqrt{\frac{m_{Yi}^2 + m_{Xi}^2}{2}} \tag{30}$$

The whole net — a traverse — can be rated by means of their quadratic mean ($\mathbb{K}_k$)

$$\mathbb{K}_k = \sqrt{\frac{\mathbb{K}^* \ \mathbb{K}}{n}} \tag{31}$$

where $\mathbb{K}$ is a vector containing standard error of position and n is the number of traverse stations.

For characterizing accuracies of nets — and positions of nets — error ellipse are widely used. For determining error ellipses their elements should be known. These elements are the maximum $m_{\max_i}$ and minimum $m_{\min_i}$ of standard error of a given point with the corresponding bearings. Greatest and smallest variances are the eigenvalues of the $\mathbf{N}_i$ matrix, while their bearings are the eigenvectors.

Eigenvalues of the $\mathbf{N}_i$ matrix are the roots of the following equation:

$$\begin{bmatrix} m_{Yi}^2 - \lambda & c_{YiXi} \\ c_{YiXi} & m_{Xi}^2 - \lambda \end{bmatrix} = 0 \tag{32}$$

The equation after development of the determinant:

$$\lambda^2 - \lambda(m_{Yi}^2 + m_{Xi}^2) - c_{YiXi}^2 + m_{Yi}^2 m_{Xi}^2 = 0 \tag{33}$$

By solving the equation one obtains the greatest and smallest values of variance.

$$m_{\max_i}^2 = \frac{m_{Yi}^2 + m_{Xi}^2}{2} + \frac{1}{2} \sqrt{(m_{Xi}^2 - m_{Yi}^2)^2 + 4c_{YiXi}^2} \tag{34}$$

$$m_{\min_i}^2 = \frac{m_{Yi}^2 + m_{Xi}^2}{2} - \frac{1}{2} \sqrt{(m_{Xi}^2 - m_{Yi}^2)^2 + 4c_{YiXi}^2} \tag{35}$$

Value of bearing, belonging to the greatest δ_{max_i} standard error can be determined by means of the following equation:

$$\delta_{\max_i} = \frac{1}{2} \operatorname{arc\,tan} \frac{2c_{YiXi}}{m_{Xi}^2 - m_{Yi}^2} \tag{36}$$

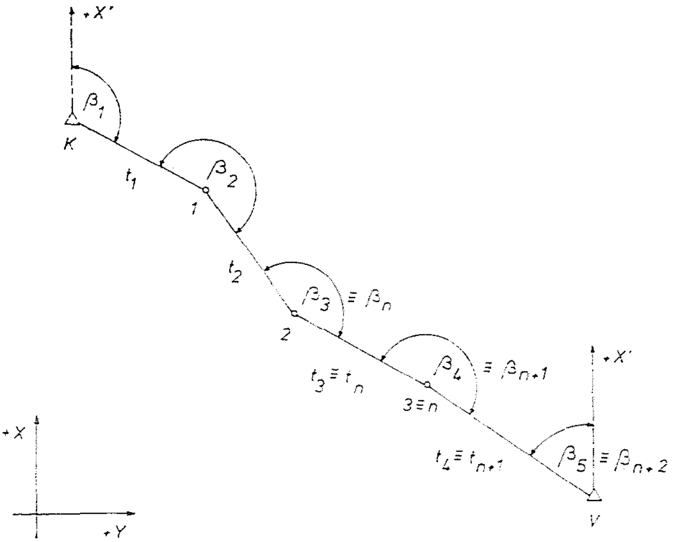

Fig. 2

4. An example for rigorous adjustment of a traverse with determining measures of accuracy

A computer program was written in TURBO PASCAL for an IBM PC/AT personal computer for solving the task.

Adjustment of traverse oriented and tied at both ends were carried out. The traverse is shown in the Fig. 2.

Number of traverse stations was	$n = 3$
Number of traverse angles was	$n + 2 = 5$
Number of distance measurement was	$n + 1 = 4$

Given data:

a) Coordinates:

$Y_K = + 5\,402{,}181$ m $\qquad X_K = +1\,194{,}769$ m

$Y_V = + 5\,783{,}332$ m $\qquad X_V = +\ \ 601{,}258$ m

b) Bearings and traverse angles:

$$\delta_{T_1,K} = 180°00'00''$$
$$L_{\beta 1} = 147°47'25''$$
$$L_{\beta 2} = 182°23'10''$$
$$L_{\beta 3} = L_{\beta n} = 174°23'38''$$
$$L_{\beta 4} = L_{\beta n+1} = 181°27'57''$$
$$L_{\beta 5} = L_{\beta n+2} = 33°58'25''$$
$$\delta_{V,T_2} = 0°00'00''$$

c) Distances:

$$L_{t1} = 183.941 \text{ m}$$
$$L_{t2} = 197.479 \text{ m}$$
$$L_{t3} = L_{tn} = 169.062 \text{ m}$$
$$L_{t4} = L_{tn+1} = 155.288 \text{ m}$$

d) Applied standard errors:

$$m_{\beta i} = 15'' \quad (n+2) \qquad m_{ti} = 15 \text{ mm} \quad (n+1)$$
$$\underset{(2n+3,\, 2n+3)}{\mathbf{M}_{LL}} = \; < 225, 225, \ldots, 225, \ldots, 225 >$$

Vector of constant terms:

$$\mathbf{l} = \begin{bmatrix} l_\beta \\ l_Y \\ l_X \end{bmatrix} = \begin{bmatrix} -35'' \\ +134 \text{ mm} \\ -3 \text{ mm} \end{bmatrix}$$

Correlates were computed by means of eqn (6) while solving the normal equation system.

$$k_\beta = -\;4.442 \text{ (arc sec)}^{-1}$$
$$k_Y = -16.081 \text{ mm}^{-1}$$
$$k_X = +11.965 \text{ mm}^{-1}$$

Adjustment correction of measurements and their adjusted values from eqns (7) and (8) respectively.

$$\begin{aligned}
v_{\beta 1} &= -20.0'' & U_{\beta 1} &= 147°47'05.0'' \\
v_{\beta 2} &= -13.3'' & U_{\beta 2} &= 182°22'56.7'' \\
v_{\beta 3} &= -\;5.6'' & U_{\beta 3} &= 174°23'32.4'' \\
v_{\beta 4} &= -\;0.6'' & U_{\beta 4} &= 181°27'56.4'' \\
v_{\beta 5} &= +\;4.4'' & U_{\beta 5} &= 33°58'29.4'' \\
v_{t1} &= +18.7'' & U_{t1} &= 183.9597 \text{ m} \\
v_{t2} &= +18.4'' & U_{t2} &= 197.4974 \text{ m} \\
v_{t3} &= +19.1'' & U_{t3} &= 169.0811 \text{ m} \\
v_{t4} &= +18.9'' & U_{t4} &= 155.3069 \text{ m}
\end{aligned}$$

Final coordinates of traverse stations were computed from adjusted measurements by means of eqn (22)

$$\begin{aligned}
Y_1 &= +5\,500.2503 \text{ m} & X_1 &= +\;1\,039.1297 \text{ m} \\
Y_2 &= +5\,598.4996 \text{ m} & X_2 &= +\;867.8046 \text{ m} \\
Y_3 &= +5\,696.5426 \text{ m} & X_3 &= +\;730.0512 \text{ m.}
\end{aligned}$$

Some more important measures of accuracy of traverse stations are showed. Standard error of unit weight is $m_0 = 1.73$.

Standard errors of adjusted measurements, based on eqns (24) and (25) respectively, are:

$$\begin{aligned}
m_{\beta 1} &= -10.83'' & m_{t1} &= +4.27 \text{ mm} \\
m_{\beta 2} &= -10.33'' & m_{t2} &= +3.81 \text{ mm} \\
m_{\beta 3} &= -\;9.22'' & m_{t3} &= +4.83 \text{ mm} \\
m_{\beta 4} &= -\;6.89'' & m_{t4} &= +4.57 \text{ mm} \\
m_{\beta 5} &= -\;3.46''
\end{aligned}$$

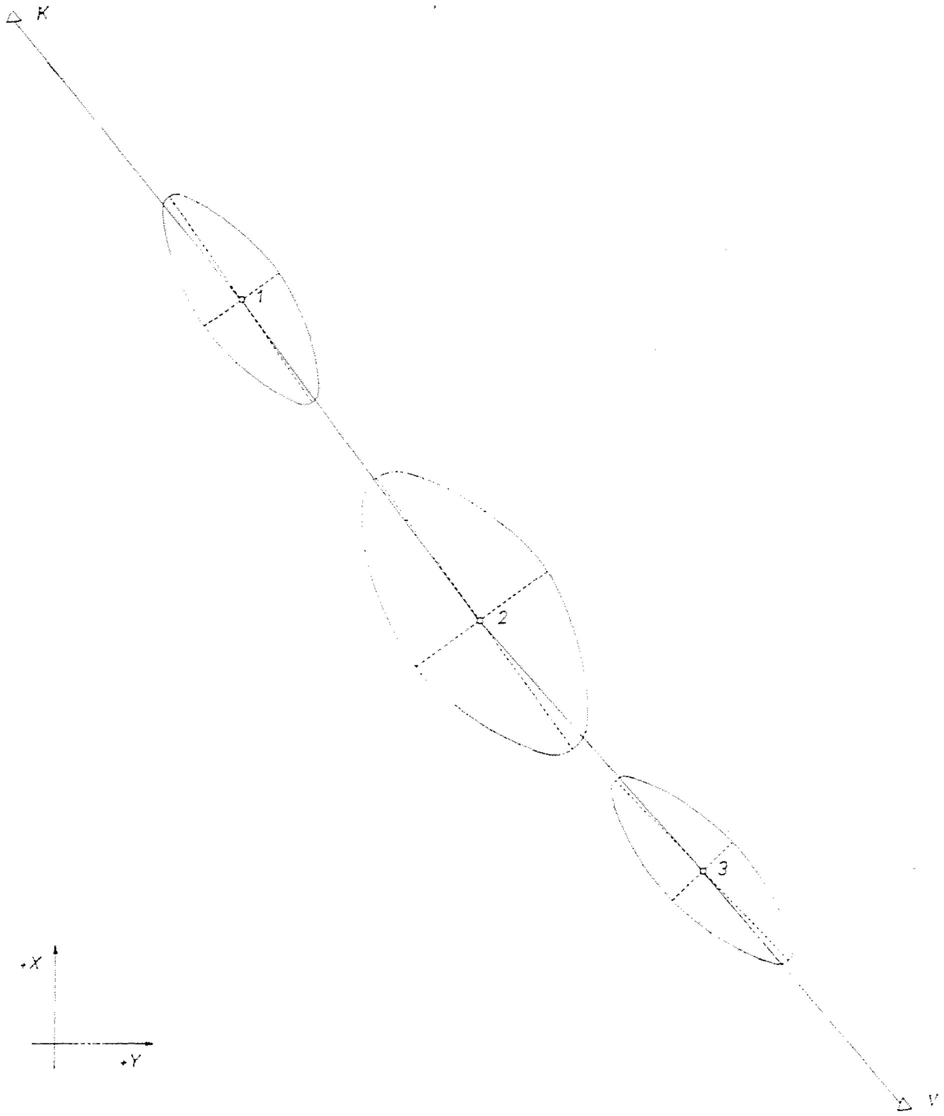

Fig. 3

Standard errors of coordinates, determined according to eqns (27) and (28), are:

$$m_{Y1} = +17.0 \text{ mm} \qquad m_{X1} = +20.5 \text{ mm}$$
$$m_{Y2} = +20.9 \text{ mm} \qquad m_{X2} = +23.9 \text{ mm}$$
$$m_{Y3} = +16.5 \text{ mm} \qquad m_{X3} = +19.9 \text{ mm}$$

Mean standard errors of position computed according to eqn (30) are:

$$K_1 = 14.5 \text{ mm} \qquad K_2 = 22.5 \text{ mm}$$
$$K_3 = 18.3 \text{ mm}.$$

Quadratic mean of standard errors of position from eqn (31):

$$K_k = 18.72 \text{ mm}$$

Elements of error ellipses of traverse points were determined according to eqns (34), (35) and (36) respectively. The error ellipse are shown in Fig. 3. Data of error ellipses computed by means of eqns (34), (35) and (36) are as follow:

δ_1	=	147°57'45″	δ_2	=	146°47'14″	δ_3	=	143°45'21″
$m_{\max_1}$	=	5.05 cm	$m_{\max_2}$	=	6.72 cm	$m_{\max_3}$	=	5.03 cm
$m_{\min_1}$	=	2.05 cm	$m_{\min_2}$	=	3.36 cm	$m_{\min_3}$	=	1.65 cm

References

1. FIALOVSZKY, L.: A sokszögelés kiegyenlítésének módszerei és azok értékelése. A Magyar Tudományos Akadémia Műszaki Tudományos Osztályának Közleményei, Budapest, 1962.
2. HAZAY, I.: Kiegyenlítő számítások. Tankönyvkiadó, Budapest, 1966.
3. HOMORÓDI, L.: Felsőgeodézia. Tankönyvkiadó, Budapest, 1966.
4. BILL, R.: Strenge Ausgleichung von Polygonzügen mit Suche grosser Fehler. AVN Karlsruhe, 1983/1.
5. HOVÁNYI, L.: Bányamérés. Műszaki Könyvkiadó, Budapest, 1968.
6. DETREKŐI, Á.: Kiegyenlítő számítások. Tankönyvkiadó, Budapest, 1973. Kézirat.

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