A UNIFIED SOLUTION FOR COORDINATION
BY TRIGONOMETRIC METHODS

E. PAPP

Department of Geodesy
Technical University, H-1521, Budapest

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Abstract

This short essay discusses the application of elementary computation methods for a unified solution of coordination by trigonometry. It means that after computing bearings and distances for establishing rays, coordinates for unfixed point are obtained by using the polar method in the computation process.

Description of the basic concept is completed by the algorithm of a computation process being applicable to HT PTA 4000+16/SHARP PC 1500A computer written in BASIC language. The software gives a real interactive communication channel between the user and the program.

The portable computer can be combined with a KA 160/CE-516P printer. The software also understands special symbols as well as the Greek and the Hungarian alphabets. Each problem is being supplied by a flowchart of the computation process.

A unique location of a new point, relative to 2 fixed stations can be achieved by observing at least two geometrical parts. Two internal angles or two sides or one angle and one side of a triangle formed by rays connecting control stations and a new point can be the geometrical parts for fixing a point.

Various methods are known for the computation of coordinates of an unfixed station. This paper discusses a unified method that applies entirely polar coordination for fixing a point. Using this method, intersection by bearings/angles or sides and resection, double resection and other methods (Hansen, Marek) of fixing a point can be reduced to a simple computation process of polar coordination.

Notations

\( l \) = circle reading
\( \delta \) = bearing, provisional bearing
\( t \) = distance
\( z_i \) = adjusting constant
\( ZK \) = mean adjusting constant
\( \alpha \) = included angle
\( \beta \) = included angle
\( \gamma \) = included angle
\( \varepsilon \) = included angle
\[ X = \text{(total) northing} \]
\[ Y = \text{(total) easting} \]
\[ \Delta X = \text{partial northing} \]
\[ \Delta Y = \text{partial easting} \]

1. Intersection

1.1. Intersection by angles

Given \( A(Y_A; X_A); B(Y_B; X_B) \) and the readings \( (l_{AP}; l_{AB}; l_{BA}; l_{BP}) \) to forward directions observed at \( A \) and \( B \) respectively (Fig. 1).

The internal angles are obtained as follows

\[ \alpha = l_{AB} - l_{AP} \quad \text{and} \quad \beta = l_{BP} - l_{BA} \]  \hspace{1cm} (1.1)

1.2. Intersection by provisional bearings

Given \( A(Y_A; X_A); B(Y_B; X_B) \) and the readings to forward directions observed at \( A \) and \( B \). A provisional adjustment is then performed to obtain provisional bearings \( \delta_{AP} \) and \( \delta_{BP} \). The direction method of triangulation is employed in the computation process (Fig. 2).

When the final bearings \( \delta_{AP} \) and \( \delta_{BP} \) are given then intersection by bearings is performed (Fig. 3). We also have to notice that the unfixed point \( P \) is always the second of the triangle’s three points starting from \( A \), and that lettering follows the clockwise rule (\( APB \) sequence) in the computation. The arrows pointing to the unfixed station show the bearings, while distances and bearings between \( AB, AP \) and \( BP \) are denoted by the

\[ \delta_{AB} = \delta_3; \quad t_{AB} = t_3 \]
\[ \delta_{AP} = \delta_1; \quad t_{AP} = t_1 \]
\[ \delta_{BP} = \delta_2; \quad t_{BP} = t_2 \]

symbols.
Coordinates for $P$ will be computed by the polar method based on station $A$ involving $\delta_1$ and $t_1$. Then $\delta_1$ and $\delta_2$ are computed using the calculated values for $\delta_3$ and $t_3$ (Fig. 3).

If the angle method is applied then

$$\delta_1 = \delta_3 - \alpha \quad \text{and} \quad \delta_2 = \delta_3 + \beta$$

(1.2)

otherwise, $\delta_1$ and $\delta_2$ are known or can be computed by provisional adjustment. The distance $t_1$ will be obtained by using the sine formula (Fig. 3) as below

$$t_1 = t_3 \frac{\sin(\delta_3 - \delta_2)}{\sin(\delta_1 - \delta_2)}$$

(1.3)

The trigonometrical ratio of the compound angle for the numerator of the sine formula gives

$$\sin(\delta_3 - \delta_2) = \sin \delta_3 \cos \delta_2 - \cos \delta_3 \sin \delta_2$$

(1.4)

and substituting Eq. (1.3) for the numerator into Eq. (1.2) we obtain

$$t_1 = t_3 \frac{\sin \delta_3 \cos \delta_2 - t_3 \cos \delta_3 \sin \delta_2}{\sin(\delta_1 - \delta_2)}$$

(1.5)

As Fig. 4 shows the following equations can be set up

$$Y_B - Y_A = t_3 \sin \delta_3 \quad \text{and} \quad X_B - X_A = t_3 \cos \delta_2$$

(1.6)

Eq. (1.5) will then be rewritten into the following form

$$t_1 = \frac{(Y_B - Y_A) \cos \delta_2 - (X_B - X_A) \sin \delta_2}{\sin(\delta_1 - \delta_2)}$$

(1.7)
And coordinates for \( P \) are computed from the formulae given below

\[
\begin{align*}
Y_P &= Y_A + t_1 \sin \delta_1 \\
X_P &= X_A + t_1 \cos \delta_1
\end{align*}
\] (1.8)

There is no solution if

\[ z + \beta = W \]

where \( W = 180^\circ \) or \( W = 200^\circ \)

or \( \delta_1 = \delta_2 \) or \( t_3 = 0 \).

When provisional bearings are applied to the intersection problem then a semi-graphical provisional adjustment will be performed to the readings of the forward directions. Coordinates for the occupied stations and for the

![Fig. 5](image)

![Fig. 6](image)

reference objects are known, as well as a series of readings to the forward directions leading to the ROs and to the unfixed points (Fig. 2 and Fig. 5).

Firstly, a set of adjusting constants \((z_i)\) is created by calculating the differences between the final bearings available and the readings belonging to the same ray. Secondly, coordinate differences \((\Delta Y \text{ and } \Delta X)\) are computed from the adjusted constants and the corresponding distances. Then the mean adjusting constant is calculated as the closing leg of the traverse line (Fig. 6). This approximate method produces a mean adjusting constant that deviates from the value computed by the numerical method using the weighted mean by \(10^{-3}\) second of arc, therefore, the previous one can be applied to any task that may occur in practice. The adjusting constant then will be computed as follows

\[ z_i = \delta_{Ai} - l_{Ai} \] (1.9)

and the distance

\[ t_i = \left[ (Y_i - Y_A)^2 + (X_i - X_A)^2 \right]^{\frac{1}{2}} \] (1.10)
and the coordinate differences

\[ \Delta Y_i = t_i \sin z_i \sum_{i=1}^{n} \Delta Y_i \]

and

\[ \Delta X_i = t_i \cos z_i \sum_{i=1}^{n} \Delta X_i \]  \hspace{1cm} (1.11)

If \( n \) is the number of the adjusting constants, then the mean value is calculated as follows

\[ ZK = \arctan \left( \frac{\sum_{i=1}^{n} \Delta Y_i}{\sum_{i=1}^{n} \Delta X_i} \right) \]  \hspace{1cm} (1.12)

Finally, provisional bearings for the establishing directions are obtained as the sum of the mean adjusting constant and the readings for those rays that terminates at the new point.

Then:

\[ ZK_A + l_{AP} = \delta_{AP} \]
\[ ZK_B + l_{BP} = \delta_{BP} \]  \hspace{1cm} (1.13)

where \( ZK_A \) and \( ZK_B \) are mean adjusting constants for sets of forward directions observed at station \( A \) and \( B \), respectively.

If coordinates of \( A \) and \( B \) and readings for directions \( l_{PB} \), \( l_{PA} \) observed at \( P \) and/or \( l_{AP} \) and \( l_{AB} \) observed at \( A \); or \( l_{BA} \) and \( l_{BP} \) observed at \( B \); (Fig. 7) are known then the internal angles are as follows

\[ \beta = W - (\alpha + \gamma) \]
\[ \gamma = W - (\beta + \gamma) \]  \hspace{1cm} (1.14)

where \( W = 180^\circ \) or \( W = 200^\circ \).

Fig. 7
Further steps in computation agree with those applied for the intersection by angles [also see Eq. (1.2); (1.7); (1.8)].

If coordinates for A and B and distances $t_1$ and $t_2$ measured at P are known then $\delta_3$ and $t_3$ are also computed (Fig. 8).

Coordinates for P will then be computed using polar coordinates from A station. The cosine formula gives the solution for $\alpha$ as below

$$\alpha = \arccos \frac{t_1^2 + t_2^2 - t_3^2}{2t_1t_3}$$

Therefore, the final bearing for the forward direction to P [see Eq. (1.2)] is obtained from the following formula

$$\delta_1 = \delta_3 - \alpha$$

Note: There is no solution of the problem, if

$$t_1 = t_2 < t_3$$

2. Resection

Solution of resection by the Collinns' point method can be reduced to two repeatedly performed intersections.

Given $A(Y_A; X_A); M(Y_M; X_M); B(Y_B; X_B)$ and readings to backward directions $l_{PA}; l_{PM}$ and $l_{PB}$ observed at P. The included angles are given as follows (Fig. 9)

$$\alpha = l_{PM} - l_{PA}$$

and

$$\beta = l_{PB} - l_{PM}$$

The circle drawn through A, B and P will be cut by a straight line connecting P and M at $P'$ (Fig. 10). Note that $\beta' = W - \beta$ and $\alpha' = W - \alpha$ where $W = 180^\circ$ or $W = 200^\circ$. 
Coordinates for \(A\) and \(B\), and angles \(\alpha\) and \(\beta\) are initial facts for the first intersection. After computing values of \(\delta_3\) and \(t_3\), two bearings (\(\delta_1\) and \(\delta_2\)) will be obtained from the following formulae

\[
\delta_1 = \delta_3 - \beta \quad \delta_2 = \delta_3 + \alpha .
\]  

(2.2)

The distance \(t_1\) is then computed as it is shown in Eq. (1.7) where the sine rule has been applied, and Eq. (1.8) is available for computing the coordinates of \(P'\) the same as those for the first procedure.

Firstly, the bearing \(\delta_{MP'} = \delta'_3\) and the distance \(t_{MP'} = t'_3\) secondly, \(\delta'_1\) and \(\delta'_2\) bearings are computed as follows

\[
\delta'_1 = \delta'_3 - \alpha \quad \text{and} \quad \delta'_2 = \delta'_3 + \beta .
\]  

(2.3)
Then, $t_1$ distance is calculated using the sine formula as shown below

$$ t'_1 = \frac{(Y_B - Y_A) \cos \delta'_2 - (X_B - X_A) \sin \delta'_2}{\sin (\delta'_1 - \delta'_2)} \quad (2.4) $$

Finally, coordinates of $P$ will be computed using polar coordinates

$$ Y_P = Y_A + t_1' \sin \delta'_1 $$

and

$$ X_P = X_A + t_1' \cos \delta'_1 $$

The problem can be solved if

$$ z + \beta = W \quad \text{and} \quad t_{MP'} = 0 \quad (2.6) $$

where $W = 180^\circ$ or $W = 200^\circ$

otherwise

a) In the case of $z + \beta = W$ the circle involving points $A$, $B$ and $P$ becomes a straight line and gives no solution.

The bearings $\delta_1$ and $\delta_2$ become parallel lines and cuts can not be achieved.

b) If $t_{MP'} = 0$ then $M$ falls onto $P'$ ($M = P'$ and $\delta'_3 = 0$).

3. Double resection

Double resection, and its specific versions such as the Hansen and the Marek methods can be traced back to a simple resection that involves auxiliary stations in the computation.

Given $A(Y_A; X_A); B(Y_B; X_B)$ and $C(Y_C; X_C)$ as well as readings $l_{PA}; l_{PB}; l_{PR}$ to backward directions observed at $P$ and those $l_{RP}; l_{RB}; l_{RC}$ to forward directions observed at $R$ (Fig. 11).

The included angles are as follows:

$$ x = l_{PR} - l_{PB}; \quad \beta = l_{PB} - l_{PA} \quad (3.1) $$

$$ \gamma = l_{RB} - l_{RP}; \quad \varepsilon = l_{RC} - l_{RB} $$

First, coordinates for $P'$ are computed. The circle, drawn through $A$, $B$ and $P$, is cut by the straight line joining $P$ and $R$ at $P'$. Similarly, the circle drawn through $B$, $C$ and $R$ is cut by the same line at $R'$.

The initial data for the computation of $P'$ by intersection are $A(Y_A; X_A)$ and $B(Y_B; X_B)$ as well as angles $z$ and $W - (z + \beta)$. After calculating bearing $\delta_3$ and distance $t_3$, the $\delta_1$ and $\delta_2$ can be obtained as follows (Fig. 11)

$$ \delta_1 = \delta_3 - [W - (z + \beta)]; \quad \delta_2 = \delta_3 + z \quad (3.2) $$

Also note that $\delta_3 = \delta_{BA}$ and $t_3 = t_{BA}$!
The \( t_1 \) distance will then be computed by applying Eq. (1.7) while the coordinates for \( P' \) is obtained by Eq. (1.8).

The initial data for the computation of \( R' \) by intersection are \( B(Y_B; X_B) \) and \( C(Y_C; X_C) \) as well as angles \( W - (\gamma + \epsilon) \) and \( \gamma \). After computing bearing \( \delta_3 \) and distance \( t_3 \), \( \delta_1 \) and \( \delta_2 \) are calculated as follows

\[
\delta_1 = \delta_3 - \gamma; \quad \delta_2 = \delta_3 + [W - (\gamma + \epsilon)]
\]

Also note, that \( \delta_3 = \delta_{CB} \) and \( t_3 = t_{CB} \).

Coordinates for \( R' \) will then be obtained by using polar coordinates in Eq. (1.7) and Eq. (1.8).

If coordinates for \( P' \) and \( R' \) are available, then bearing \( \delta'_3 \) can be obtained and the bearing to \( P \) and \( R \) can also be computed.

Coordinates for \( P \) will be computed by intersection, where initial data are the same that have been used earlier for computing coordinates for \( P' \). The bearings applied are

\[
\delta'_1 = \delta'_3 - \pi; \quad \delta'_2 = \delta'_3 + [W - (\pi + \beta)]
\]

Since, bearing \( \delta_3 \) and distance \( t_3 \) are given (see computation of coordinates for \( P' \)), hence the bearings \( \delta'_1 \) and \( \delta'_2 \) are

\[
\delta'_1 = \delta'_3 - [W - (\gamma + \epsilon)]; \quad \delta'_2 = \delta'_3 + \gamma
\]

Knowing these bearings, \( P \) and \( R \) coordinates can be computed from \( B \) and \( C \) by the polar method (see Eq. (1.7) and Eq. (1.8)).
4. The Hansen method

Given $A(Y_A, X_A)$; $B(Y_B, X_B)$ and readings $l_{PA}$, $l_{PR}$, $l_{PB}$ for backward directions observed at $P$; and $l_{RB}$, $l_{RP}$, $l_{RA}$ observed at $R$ (Fig. 12) Then the angles are as follows

$$\begin{align*}
x &= l_{PR} - l_{PA}; \\
\beta &= l_{PB} - l_{PR} \\
\gamma &= l_{RP} - l_{RB}; \\
\epsilon &= l_{RA} - l_{RP}
\end{align*}$$

(4.1)

The circle drawn through $A$, $B$ and $P$ points is cut by a straight line joining $P$ and $R$ stations at point $P'$. The same line also cuts a circle drawn through $A$, $R$ and $B$ at $R'$.

Coordinates for $P'$ can be calculated from the given coordinates of $A$ and $B$ also involving $x$ and $\beta$ angles using the intersection method for computation (Fig. 12). The bearings are

$$\begin{align*}
\delta_1 &= \delta_3 - \beta; \\
\delta_2 &= \delta_3 + \gamma
\end{align*}$$

(4.2)

Coordinates for $P'$ will then be computed from Eq. (1.2).

To obtain coordinates for $R'$, $\gamma$ and $\epsilon$ angles and coordinates of $A$ and $B$ are used. The required bearings then

$$\begin{align*}
\delta_1 &= \delta_3 - \epsilon; \\
\bar{\delta}_2 &= \bar{\delta}_3 + \gamma
\end{align*}$$

(4.3)

And the coordinates for $R'$ can be calculated by Eq. (1.2).

Then the bearings (pointing to $P$ and $R$ stations) will be computed considering that $\delta_3 = \delta_{P,R'}$.

A repeatedly performed intersection leads to coordinates for $P$. 

Fig. 12
Coordinates and angles used in the computation are the same as the values applied to the coordinates for \( P' \). Bearings are calculated as follows

\[
\delta_1' = \delta_3' - \alpha; \quad \delta_2' = \delta_3' + \beta
\]

The \( \delta_3' \) bearing and \( l_3' \) distance as well as the angles are the same as those used in the computation of \( R' \) when coordinates are being computed for \( R \). Further bearings can be obtained from the following formulae

\[
\delta_1' = \bar{\delta}_3' - \gamma'; \quad \delta_2' = \bar{\delta}_3' + \epsilon
\]

After knowing these bearings, coordinates for \( P \) are computed based on \( A \) station while coordinates for \( R \) can be calculated from \( B \) station by the polar method [see Eq. (1.7) and Eq. (1.8)].

5. The Marek method

Given \( A(Y_A, X_A), B(Y_B, X_B), C(Y_C, X_C) \) and \( D(Y_D, X_D) \) as well as readings for backward directions \( l_{PB}, l_{PR}, l_{PA} \) and \( l_{RD}, l_{RP}, l_{RC} \) (Fig. 13).

Then the included angles are

\[
\alpha = l_{PA} - l_{PR}; \quad \beta = l_{PR} - l_{PB} \\
\gamma = l_{RC} - l_{RP}; \quad \epsilon = l_{RP} - l_{RD}
\]

The circle drawn through \( A, B \) and \( P \) is cut by a straight line joining \( P \) and \( R \) at \( P' \) while another circle drawn through \( C, D \) and \( R \) is cut by the same line at \( R' \). The computation process will then be started by calculating
coordinates for $P'$ by using the coordinates of $A$ and $B$ and $\alpha$ and $\beta$ angles (Fig. 13). It should also be noted that $\delta_3 = \delta_{AB}$ and $t_3 = t_{AB}$. Then the bearings needed for further calculations are

$$\delta_1 = \delta_3 - \beta'; \quad \delta_2 = \delta_3 + \alpha'$$  \hspace{1cm} (5.2)

where

$$\beta' = W - \beta$$

and

$$\alpha' = W - \alpha$$  \hspace{1cm} (5.3)

and

$$W = 180^\circ$$ \hspace{1cm} or \hspace{1cm} $$W = 200^\circ.$$

Equations (1.7) and (1.8) are useful for the computation of the distance $t_1$ and the coordinates of $P'$.

Similarly, coordinates for $R'$ are computed from the coordinates of $C$ and $D$ using $\gamma$ and $\varepsilon$ angles as well (Fig. 13). It should be noted that $\delta_3 = \delta_{CD}$; $t_3 = t_{CD}$.

Then, the bearings come from the following formulae

$$\tilde{\delta}_1 = \tilde{\delta}_3 - \varepsilon'; \quad \tilde{\delta}_2 = \tilde{\delta}_3 + \gamma'$$  \hspace{1cm} (5.4)

where

$$\varepsilon' = W - \varepsilon$$

and

$$\gamma' = W - \gamma$$  \hspace{1cm} (5.5)

and

$$W = 180^\circ$$ \hspace{1cm} or \hspace{1cm} $$W = 200^\circ.$$

Equations (1.7) and (1.8) will be used in the calculation and coordinates for $R'$.

Knowing the coordinates for $P'$ and $R'$, and considering that $\delta_3 = \delta_{P',R'}$, bearings belonging to the establishing directions can be obtained.

When the coordinates for $P$ are being computed, then the same initial data are used, except the bearings that follow

$$\delta_1' = \delta_3' - \alpha' \quad \delta_2' = \delta_3' + \beta'$$  \hspace{1cm} (5.6)

Similarly, the computation of the coordinates for $R$ requires the data that has been applied to the computation of $Y_{R'}$ and $X_{R'}$ coordinates. In this case, the bearings are

$$\tilde{\delta}_1' = \tilde{\delta}_3' - \gamma' \quad \tilde{\delta}_2' = \tilde{\delta}_3' + \varepsilon'$$  \hspace{1cm} (5.7)

Using the equations (5.6) and (5.7) then Eq. (1.7) and (1.6), coordinates for $P$ and $R$ can be obtained by the application of the polar method and also involving the coordinates of $B$ and $C$.

6. Program description

The unified solution of coordination is being supported by a software program which is written in BASIC language and applicable to HT PTA 4000 + 16/SHARP PC 1500 A portable computers.
COORDINATION BY TRIGONOMETRIC METHODS

Fig. 14
Fig. 15
The user of the program is supplied with a real interactive software that makes usage of the program easy. As the flowchart of the program indicates, after entering the name of the program and starting it, the computer will ask for the input data. In the case of options, these might be the answers to the questions required by the program.

First, the option is selected according to the circle reading made on the theodolite, as shown below

---

**Fig. 16**
Secondly, the method of coordination is being chosen as below

$I_1$ $I_2$ $R$ $TI$ $DR$ $H$ $M$ ?

Notations

$I_1$ = intersection (angles/provisional bearing)
$I_2$ = intersection (special version)
$R$ = resection
$TI$ = intersection by length
$DR$ = double resection
$H$ = Hansen method
$M$ = Marek method

Fig. 17
COORDINATION BY TRIGONOMETRIC METHODS

Fig. 18a
DOUBLE RESECTION

A P number S mark:
X(A) = ?
Y(A) = ?
B P number S mark:
X(B) = ?
Y(B) = ?
C P number S mark:
X(C) = ?
Y(C) = ?

Reading/ Angle ?

P P number S mark:
(PA) =
(PB) =
(PR) =
R P number S mark:
(RP) =
(RB) =
(RC) =

P P number S mark:
X
Y
R P number S mark:
X
Y

Continue Y/N ?

Fig. 18b
COORDINATION BY TRIGONOMETRIC METHODS

MAREK METHOD

A P.number S.mark:
X(A)=?
Y(A)P?
B P.number S.mark:
X(B)P?
Y(B)P?
C P.number S.mark:
X(C)P?
Y(C)P?
D P.number S.mark:
X(D)P?
Y(D)P?

Reading/Angle?

P P.number S.mark:
X(PB)=
Y(PB)=
R P.number S.mark:
X(RP)=
Y(RP)=

P P.number S.mark:
X(PA)=
Y(PA)=
R P.number S.mark:
X(RP)=
Y(RP)=

Continue Y/N?

A

P P.number S.mark:
X=?
Y(?)=
R P.number S.mark:
X=?
Y(?)=

Continue Y/N?

N

Fig. 19
10: "C" REM
15: REM Program "COORDINATION"
20: CLEAR:CLS
25: INPUT "DEGREE";G
30: IF G=180 THEN 38
35: IF G=90 THEN 39
40: IF G=45 THEN 41
45: IF G=30 THEN 42
50: IF G=15 THEN 43
55: IF G=10.5 THEN 44
60: IF G=10 THEN 45
65: IF G=5 THEN 46
70: IF G=4.5 THEN 47
75: IF G=4 THEN 48
80: IF G=3.5 THEN 81
85: IF G=3 THEN 82
90: SIZE 3;LPRINT "RESECTION"
95: LPRINT "----";CLS;INPUT "Continue Y/N ?";K
100: A$(1)="A"; A$(2)="B"
105: N=3;GOSUB 15
110: GOSUB 1590: IF (K$="Y")+(K$="N")<>1 THEN
115: IF K$="Y" AND U$="G" THEN 180
120: IF K$="Y" AND U$="A" THEN :LPRINT "------":CLS "INTERSECTION"
125: IF U$="A" THEN 200
130: IF U$="G" THEN 180
135: IF U$="A" THEN 220
140: IF U$="G" THEN 240
145: IF U$="A" THEN 255
150: IF U$="G" THEN 260
155: IF U$="A" THEN 270
160: IF U$="G" THEN 280
165: IF U$="A" THEN 290
170: IF U$="G" THEN 300
175: IF U$="A" THEN 310
180: IF U$="G" THEN 320
185: IF U$="A" THEN 330
190: IF U$="G" THEN 340
195: IF U$="A" THEN 350
200: IF U$="G" THEN 360
205: IF U$="A" THEN 370
210: IF U$="G" THEN 380
215: IF U$="A" THEN 390
220: IF U$="G" THEN 400
225: IF U$="A" THEN 410
230: IF U$="G" THEN 420
235: IF U$="A" THEN 430
240: IF U$="G" THEN 440
245: IF U$="A" THEN 450
250: IF U$="G" THEN 460
255: IF U$="A" THEN 470
260: IF U$="G" THEN 480
265: IF U$="A" THEN 490
270: IF U$="G" THEN 500
275: IF U$="A" THEN 510
280: IF U$="G" THEN 520
285: IF U$="A" THEN 530
290: IF U$="G" THEN 540
295: IF U$="A" THEN 550
300: IF U$="G" THEN 560
305: IF U$="A" THEN 570
310: IF U$="G" THEN 580
315: IF U$="A" THEN 590
320: IF U$="G" THEN 600
325: IF U$="A" THEN 610
330: IF U$="G" THEN 620
335: IF U$="A" THEN 630
340: IF U$="G" THEN 640
345: IF U$="A" THEN 650
350: IF U$="G" THEN 660
355: IF U$="A" THEN 670
360: IF U$="G" THEN 680
365: IF U$="A" THEN 690
370: IF U$="G" THEN 700
375: IF U$="A" THEN 710
380: IF U$="G" THEN 720
385: IF U$="A" THEN 730
390: IF U$="G" THEN 740
395: IF U$="A" THEN 750
400: IF U$="G" THEN 760
405: IF U$="A" THEN 770
410: IF U$="G" THEN 780
415: IF U$="A" THEN 790
420: IF U$="G" THEN 800
425: IF U$="A" THEN 810
430: IF U$="G" THEN 820
435: IF U$="A" THEN 830
440: IF U$="G" THEN 840
445: IF U$="A" THEN 850
450: IF U$="G" THEN 860
455: IF U$="A" THEN 870
460: IF U$="G" THEN 880
465: IF U$="A" THEN 890
470: IF U$="G" THEN 900
475: IF U$="A" THEN 910
480: IF U$="G" THEN 920
485: IF U$="A" THEN 930
490: IF U$="G" THEN 940
495: IF U$="A" THEN 950
500: IF U$="G" THEN 960
505: IF U$="A" THEN 970
510: IF U$="G" THEN 980
515: IF U$="A" THEN 990
520: DIM A$(3,2)
COORDINATION BY TRIGONOMETRIC METHODS

![Fig. 21](image-url)
Fig. 22
Fig. 23
E. PAPP

955: DIM A(2,2), A$: 1280: COSUB 1550: 1145: IF (A+B)=WOR
2, P(2): 1000: IF (A$="A")+ (G+E) THEN
: 1590: C = 1: U = 1:
: 1030: INPUT "t(A)" D: 1790.
: 1090: IF $="@" THEN 1055.
: 1230: GOSUB 1120: GOSUB 1070:
: 1050: T2: 1150: IF $="@" THEN 1070:
: 1095: U4=1: GOSUB 1: GOSUB 1700:
: 1085: U4=0: GOTO 114.

960: USING "HHH", "HHH", "HHH", "HHH", "HHH".

: 1100: U4=0:CLS: 1160: I=2: J=3:
: 1110: CLS: INPUT "t(A) P N".
: 1120: TAB 3: LPRINT 3: U=S:

1025: IF (T1>T2) THEN 1780.

1040: IF (K$="Y") THEN 1290.

1050: PRINT ": ".

2, P(3): 1000: IF (A$="@")+ (G+E) THEN
: 1350: D=290.
: 1090: IF $="@" THEN 1055.
: 1230: GOSUB 1070.
: 1050: T2: 1150: IF $="@" THEN 1070:
: 1095: U4=1: GOSUB 1: GOSUB 1700:
: 1085: U4=0: GOTO 114.

1050: PRINT ": ".

Fig. 24
COORDINATION BY TRIGONOMETRIC METHODS

```
1236: LPRINT "": 1280: TAB 3: LPRINT 1340: CLS: INPUT ""
1382: GOSUB 1740: S
40: S=RB
1455: IF (K$="Y") THEN PRINT "": 1480: GOSUB 1740
45: IF (K$="N") THEN PRINT "": 1480: GOSUB 1740
40: S=RB
1455: IF (K$="Y") THEN PRINT "": 1480: GOSUB 1740
45: IF (K$="N") THEN PRINT "": 1480: GOSUB 1740
1236: GOSUB 1500
1290: IF (A+RB=RB) THEN PRINT "": 1370: SIZE 3:
1365: IF (A+RB=RB) THEN PRINT "": 1370: SIZE 3:
1236: IF (A+RB=RB) THEN PRINT "": 1370: SIZE 3:
1365: IF (A+RB=RB) THEN PRINT "": 1370: SIZE 3:
```

Fig. 25
Fig. 26
COORDINATION BY TRIGONOMETRIC METHODS

Fig. 27
### INTERSECTION

<table>
<thead>
<tr>
<th>Intersection</th>
<th>Adjusting Point</th>
<th>828.400</th>
<th>1424.000</th>
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</thead>
<tbody>
<tr>
<td>X = 150.155,530</td>
<td>Y = 789.535.512</td>
<td>X = 150.155,530</td>
<td>Y = 789.535.512</td>
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</tbody>
</table>

<table>
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<th>1424.000</th>
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</thead>
<tbody>
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<td>X = 149.434,750</td>
<td>Y = 789.363,370</td>
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### 142.900

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<td>Y = 788.122,750</td>
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### 148.000

#### 1424.000

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<tr>
<td>X = 152.328,510</td>
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<td>X = 152.328,510</td>
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</tbody>
</table>

<table>
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</tr>
</thead>
<tbody>
<tr>
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<td>Y = 798.603,520</td>
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</table>

### 1404.000

#### 1404.000

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<th>134.000</th>
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</thead>
<tbody>
<tr>
<td>X = 150.731,465</td>
<td>Y = 788.937,969</td>
<td>X = 150.731,465</td>
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</tbody>
</table>

<table>
<thead>
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<th>1424.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>X = 152.328,510</td>
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### TRAVERSE

#### INTERSECTION

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<tbody>
<tr>
<td>X = 150.731,465</td>
<td>Y = 788.937,969</td>
<td>X = 150.731,465</td>
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<th>1424.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>X = 152.328,510</td>
<td>Y = 798.603,520</td>
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</tbody>
</table>

<table>
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<tr>
<th>Intersection</th>
<th>Adjusting Point</th>
<th>138.000</th>
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</thead>
<tbody>
<tr>
<td>X = 152.101,617</td>
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<td>X = 152.101,617</td>
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---

**Fig. 28**
<table>
<thead>
<tr>
<th>Co-Ordination by Trigonometric Methods</th>
<th>Hansen</th>
<th>Marek</th>
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<tbody>
<tr>
<td><strong>Double Resection</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1224 stone</td>
<td>1424 stone</td>
<td>1403 stone</td>
</tr>
<tr>
<td>$X = 152,329.510$</td>
<td>$X = 152,731.460$</td>
<td>$X = 150,278.320$</td>
</tr>
<tr>
<td>$Y = 728,603.520$</td>
<td>$Y = 769,862.860$</td>
<td>$Y = 750,992.333$</td>
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<tr>
<td>1225 stone</td>
<td>1464 stone</td>
<td>1224 stone</td>
</tr>
<tr>
<td>$X = 157,860.260$</td>
<td>$X = 151,657.710$</td>
<td>$X = 152,320.510$</td>
</tr>
<tr>
<td>$Y = 781,528.140$</td>
<td>$Y = 791,547.970$</td>
<td>$Y = 780,602.422$</td>
</tr>
<tr>
<td>2062 stone</td>
<td>1433 stone</td>
<td>2262 stone</td>
</tr>
<tr>
<td>$X = 153,410.382$</td>
<td>$X = 256-50-03$</td>
<td>$X = 155,410.912$</td>
</tr>
<tr>
<td>$Y = 735,513.320$</td>
<td>$Y = 232-36-00$</td>
<td>$Y = 752,723.900$</td>
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<td>1425 stone</td>
<td>1424 stone</td>
<td>2339 stone</td>
</tr>
<tr>
<td>$L = 222-00-00$</td>
<td>$L = 043-39-22$</td>
<td>$L = 151,268.110$</td>
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<td>$L = 125-12-25$</td>
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<td>$L = 123-19-44$</td>
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<tr>
<td>105 stone</td>
<td>105 stone</td>
<td>105 stone</td>
</tr>
<tr>
<td>$X = 151,657.710$</td>
<td>$X = 152,329.512$</td>
<td>$X = 000-00-22$</td>
</tr>
<tr>
<td>$Y = 731,547.971$</td>
<td>$Y = 750,602.518$</td>
<td>$L = 128-19-44$</td>
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<tr>
<td>1405 stone</td>
<td>1405 stone</td>
<td>1405 stone</td>
</tr>
<tr>
<td>$X = 152,319.320$</td>
<td>$X = 151,657.710$</td>
<td>$L = 128-19-44$</td>
</tr>
<tr>
<td>$Y = 781,184.930$</td>
<td>$Y = 791,547.973$</td>
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<td><strong>RESECTION</strong></td>
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<td>4243 stone</td>
<td>4243 stone</td>
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</tr>
<tr>
<td>$X = 169,244.122$</td>
<td>$X = 152,613.321$</td>
<td>$X = 752,734.550$</td>
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<td>$Y = 809,114.510$</td>
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<tr>
<td>$X = 169,984.175$</td>
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<tr>
<td>$Y = 825,222.422$</td>
<td>$Y = 752,734.550$</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 29
Then, the number/name of the fixed points are entered which will be followed by the relevant coordinates. Initial data are also printed at the same time.

When entering the degree, it should be clearly stated if readings to directions or calculated angles involved in the process. Distance and coordinates are considered in mm, while angles are taken in degrees, minutes and seconds or in grades and its decimals when they appear on the display. Small dots will separate them from each other.

In the flowchart, input data will be followed by quotation mark whilst data blocks will be followed by an equation mark and a question mark alike. Finally, results are listed and printed, then the computation can continue.

The possibility of the solution of the problem will also be investigated and if there is no solution, running of the program halts

NO SOLUTION!

is printed, and

*** END ***

remark can be seen on the display.

The author expresses his thanks to dr. F. A. Shepherd (University of Nottingham) for his advices.

References


Erik Papp H-1521 Budapest