THE EFFECT OF CRUSTAL MASSES ON GEOID ANOMALIES

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Abstract

An important question in geosciences is the physical interpretation of global geoid forms and the improvement of our knowledge on the inner structure of the Earth. The authors suggest a new method which separates geoid heights due to upper known density inhomogenities from geoid heights of inner unknown mass distributions. The interpretation of remaining geoid forms becomes presumably simpler after removing the effect of known masses from the full geoid. This paper deals with the mathematical solution of the effects of known surface mass distributions capable of computer computation and presents some results of initial numerical computations.

Recent terrestrial and satellite measurements make it possible to determine global geoid reliable up to a few meters. Hence, characteristic quantities of geoids constitute perhaps the most accurate data available regarding the geophysical information of the total Earth.

At present it is not possible to explain the physical background of large geoid anomalies; this fundamental task is in connection with the internal constitution of the Earth. Accordingly, the physical background of geoid anomalies in which we are interested -- the 3-D density function $\vartheta(x, y, z)$ of Earth's inhomogeneous density distribution -- have to be determined from the Earth's known potential field $W(r, \Theta, \lambda)$ or geoid shape. This is the famous geophysical inverse problem which has, unfortunately no unambiguous mathematical solution [5]. Owing to this fact, the physical interpretation of global geoid anomalies has not yet been given.

In the following a new and simple method is presented which offers the possibility of determining the Earth's density distribution more precisely [13].

The basic method of solution is to separate the effects of known and unknown masses responsible for geoid undulations. First, geoid anomalies due to known masses on and near the Earth's surface are determined (i.e. geoid anomalies which correspond to the distribution of topographic masses along the surface, isostatic compensating masses and, among others, plate tectonic density models are calculated). In the second step, geoid undulations of well-known mass distributions are subtracted from the real geoid undulations of the Earth; and finally in the third step, we try to explain the remaining simple geoid shapes. As on expects, these remaining geoid anomalies show the global effect of deeper unknown density distributions inside the Earth. On constructing plausible earth density models from all the geophysical (seismic, geomagnetic, geothermic) data available, the interpretation of the remaining geoid undulations can be achieved, but the geoid anomalies of these models have to be evaluated. From such Earth models only one may be accepted which produces the picture of the remaining geoid undulations. This final step of physical interpretation of global geoid anomalies is the most difficult one.

This paper aims at the evaluation of the first two steps.

1. Evaluation of influences of topographic and isostatic masses

First let us have a brief look into the strategy of the computational method. The gravitational potential of a body comprised in the domain σ of density $\vartheta(x, y, z)$ in an external point P is given by the integral expression

$$V_p = k \int_\sigma \frac{dm}{l} = k \iiint_\sigma \frac{\vartheta(x, y, z)\, dx\, dy\, dz}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}} \tag{1}$$

where the notations are seen in Fig. 1 and k is Newton's constant of gravitation.

When we consider the effect of topographic and isostatic masses, σ is the domain bounded by the physical surface of the Earth and density distribution demonstrated in Fig. 2. This model is capable of computation in such a way that the Earth is subdivided into two parts with regular but unknown inhomogeneous density distribution and on upper part with known inhomogeneous density distribution. This splitting up is performed so that the total mass and shape of the model must be the same as for the real Earth. Elements of mass required for integration were constructed according to Fig. 2 and Fig. 3. Individual mass elements lie between the compensation surface and the Earth's surface; in lateral direction they are bounded by meridian planes and vertical planes perpendicular to that of the meridians. The mass Δm_i of each element can be composed of several parts of different densities depending on the topography itself as the isostatic model, illustrated in Fig. 3, indicates [14]. The gravitational potential per unit mass of the i-th. mass element Δm_i at point P is

$$\Delta V_i = k \frac{\Delta m_i}{l_i} \tag{2}$$

where l_i denotes the distance between the centre of mass of a mass element Δm_i and point P according to Fig. 2. The total gravitational potential at P

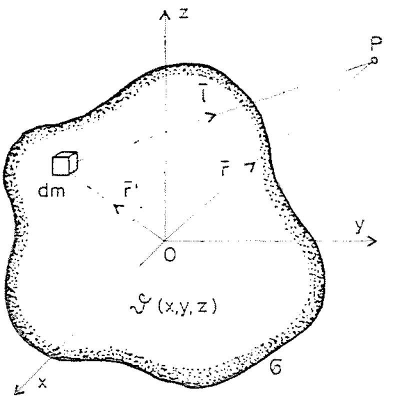

Fig. 1. Notations to evaluate gravitational potential of an arbitrary solid body

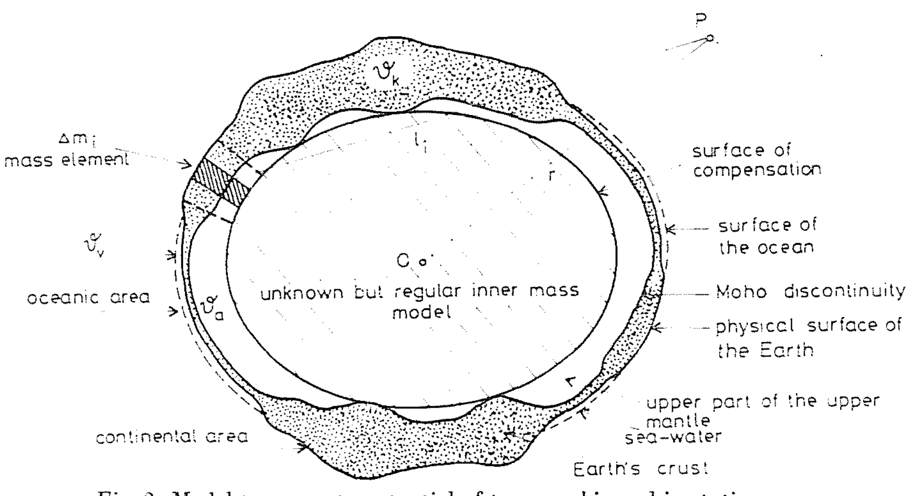

Fig. 2. Model to compute potential of topographic and isostatic masses

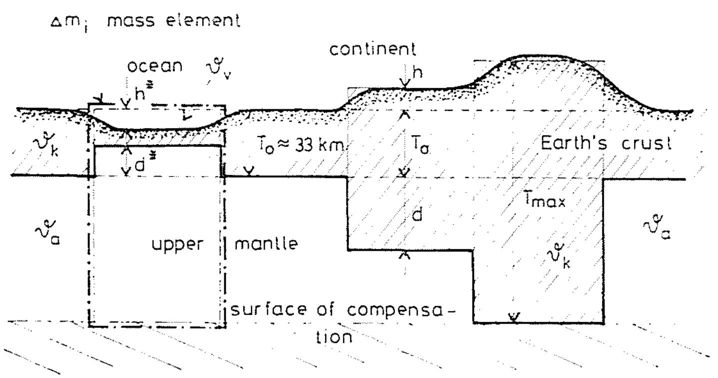

Fig. 3. Isostatic model of Airy and Heiskanen

can be expressed by numerical approximation of integral (1) and equation (2) as

$$V_P = V_B + \Sigma \Delta V_i = V_B + k \Sigma \frac{\Delta m_i}{l_i} \tag{3}$$

with V_B being the potential of the unknown inner part with an assumed regular density distribution.

Since the disturbing potential

$$T_P = V_P - U_P^* \tag{4}$$

is needed for computing geoid anomalies instead of the potential V_P, the gravitational part U_P^* of normal potential U_P have to be subtracted from gravitational potential V_P defined through (3). (The definition of normal potential will be dealt with later on.) We mention that in the preceeding only the gravitational potential was treated because the centrifugal potential V_F vanishes by subtraction: $T_P = W_P - U_P$, since $W_P = V_P + V_F$, $U_P = U_P^* + U_F$ and $V_F = U_F$.

Finally, the separation N_P between level surface of our model's gravity and normal potential can be expressed using the simplified Bruns' formula. With the notations of Fig. 4

$$N_P = \frac{T_P}{\gamma_Q}, \tag{5}$$

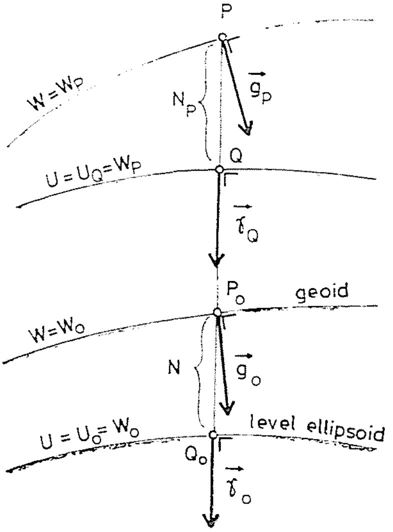

Fig. 4. Separation of geopotential and spheropotential surfaces

holds, where γ_Q is the intensity of normal gravity. When point P lies on the geoid, the separation $N = T_0/\gamma_0$ of the geoid above the ellipsoid (geoid undulation) can be determined.

In practical computations it is advantageous to develop T_P in (4) into a spherical harmonic series. The idea of this method is determine first the spherical harmonic coefficients of surface mass anomalies and then to use these coefficients to express the disturbing potential function and the required geoid undulations as well.

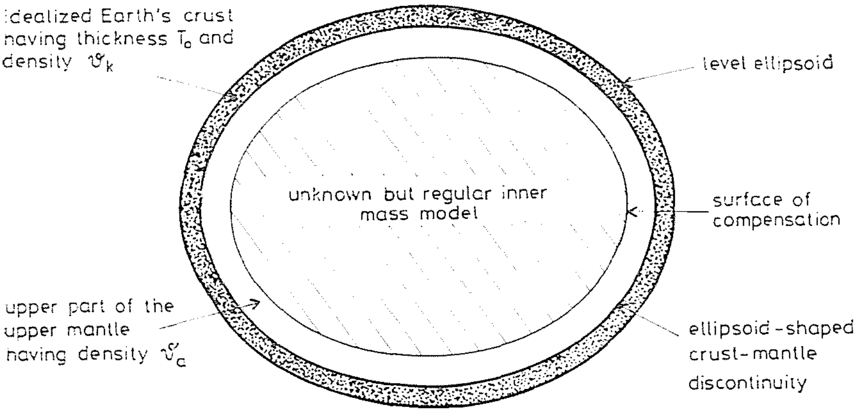

Fig. 5. Model to produce normal gravity field

In this case the inner mass distribution as well as the upper part are assumed to generate normal gravity fields as shown in Fig. 5, (i.e. idealized crust of uniform depth T_0 with homogeneous density ϑ_k and mantle lying between bottom of crust and isostatic compensation depth of density ϑ_a). The normal field is supposed to coincide exactly with the international normal gravity field of a level ellipsoid.

The evaluation of the gravitational potential of our model is split into two parts. The main part consist of a rotationally and equatorially symmetrical normal field generated by an unknown inner regular density distribution with a mantle of uniform thickness and homogeneous density ϑ_a above it; and finally, homogeneous crustal matter of density ϑ_k and thickness T_0. According to our hypothesis, the external bounding surface of this body coincides with the ellipsoidal level surface (level ellipsoid) of the international normal gravity field, and the normal potential U_0 of this ellipsoid equals that of the geoid. Hence the potential of this main part can be calculated by the well-known formulas of the international normal gravity field.

A much smaller irregular part, demonstrated in Fig. 6, caused by the upper part of the crust (physical surface of Earth) and the irregularities of

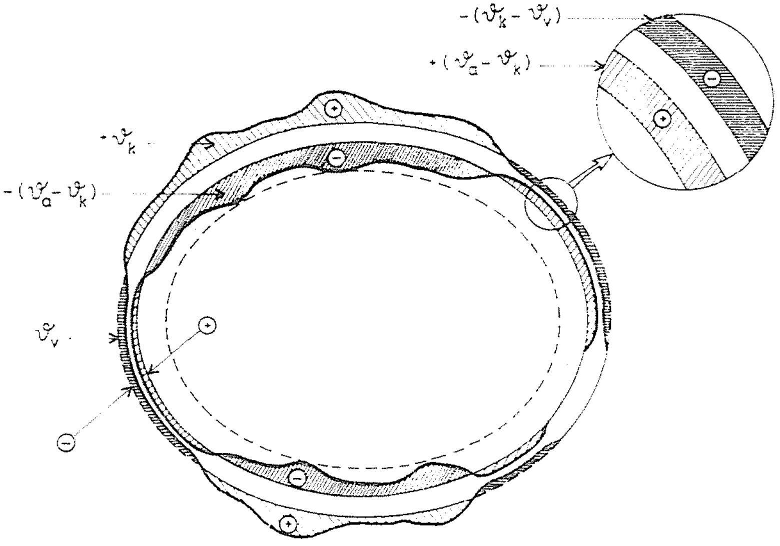

Fig. 6. Model of disturbing potential computation

the crust-mantle boundary is added to the main part mentioned above. The potential of this small irregular part is evaluated only under spherical approximation illustrated in Fig. 7.

On the basis of the previously introduced principle, the geoid computed by Bruns' formula now refers to the level ellipsoid of normal gravity field, i.e. the international reference ellipsoid. The potential U_0 of this ellipsoid equals the potential of the geoid but the inner mass distribution of our model (Fig. 2) still remains unknown. If this model — the potential of which we want to develop into a spherical harmonic series — is introduced as above, there will be no confusion at least in principle when the geoid heights of this model, computed by Bruns' formula, are subtracted from the global geoid since they are referred to the same normal gravity field and reference ellipsoid. After subtracting from the complete geoid, the resulting geoid heights will show geoid forms of a body which comprises internal masses of unknown distribution inside the earth and its external part will reflect the effects of masses not compensated according to the Airy—Heiskanen hypothesis.

2. Effect of neglecting flattening

It might cause considerable unjustified difficulties to use on ellipsoidal shape for the regularly distributed inner mass, therefore it is convenient to approximate the shape of this domain by a sphere and to measure ellipsoidal

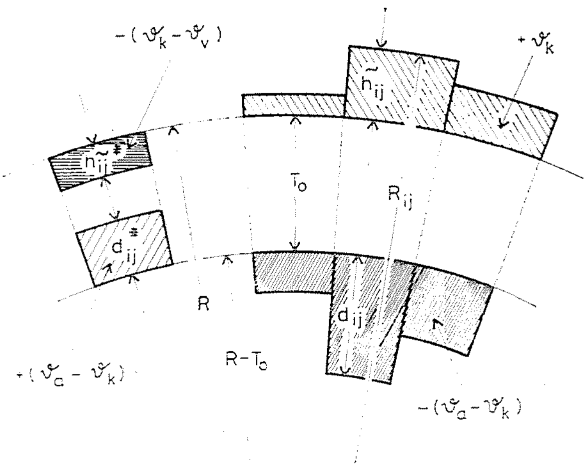

Fig. 7. Notations for disturbing potential computation

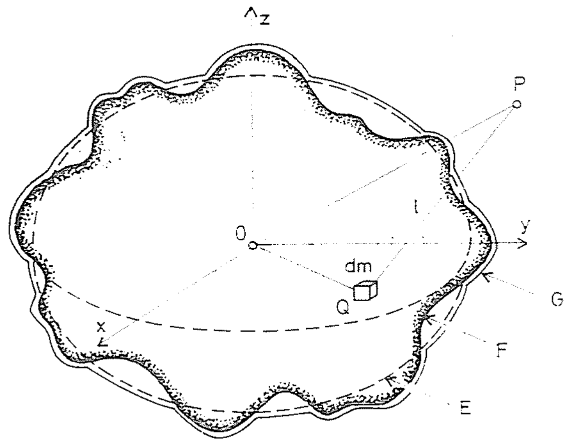

Fig. 8. Notations to investigate flattening neglection

topographic heights above this sphere. When the flattening of the ellipsoid is neglected, i.e. it is approximated by a sphere, an obvious error is committed during disturbing potential computation; in our case, however, this approximation can be justified [15].

To prove this, in Fig. 8. let F denote the domain bounded by the physical surface of the Earth, E denote a rotational ellipsoid which closely approximates the shape of Earth, and G be the domain bounded by E and F. Now the disturbing potential T_P can be expressed as

$$T_P = V_P - U_P^* = k \iiint\limits_F \frac{\vartheta_V(x, y, z)}{l}\, dx\, dy\, dz - k \iiint\limits_E \frac{\vartheta_U(x, y z)}{l}\, dx\, dy\, dz \qquad (6)$$

where ϑ_V is the density of model in Fig. 2. and ϑ_u is the density of a body producing the gravitational part of a normal gravity field. In the following let ϑ_g denote the density distribution for which the two integrals on the right side of (6) can be summed up into one integral over domain G:

$$T_P = k \iiint\limits_G \frac{\vartheta_g(x, y, z)}{l} \, dx \, dy \, dz \tag{7}$$

In the next step a coordinate transformation

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} D^{-1} & 0 & 0 \\ 0 & D^{-1} & 0 \\ 0 & 0 & D^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{8}$$

is introduced where the numerical value of D depends on both semi major and minor axes a and b of the rotational ellipsoid $E(a, b)$ or on the flattening $f = (a - b)/a$:

$$D = \sqrt[3]{\frac{a}{b}} \approx 1 + \frac{f}{3} + \ldots .$$

It can readily be seen that equation (8) transforms the (x, y, z) points of a rotational ellipsoid $E(a, b)$ into the points (x', y', z') of a sphere of equal volume to that of an ellipsoid. Differences of geographical latitudes of corresponding Q and Q' points remain below 6 minutes of arc using this transformation (and taking into account the numerical value of flattening, approximately 1/300). Next, the transformation (8) of integral expression (7) (note that density is not altered in corresponding points Q and Q') and then the Taylor expansion of $1/l$ in the integrand when $D = 1$ yields

$$\begin{aligned} T_P' = k \iiint\limits_{G'} \frac{\vartheta_g(x', y', z')}{l'} \, dx' \, dy' \, dz' + \\ (D-1)k \iiint\limits_{G'} \frac{\vartheta_g(x', y', z')}{l'} \left[3 \left(\frac{z_P' - z_Q'}{l'} \right)^2 - 1 \right] dx' \, dy' \, dz' . \end{aligned} \tag{9}$$

The term in braces is the cosine of the angle between $\overline{P'Q'}$ and plane $x'y'$; so the maximum absolute value of the bracketed expression is 2. The function $\vartheta_g(x', y', z')$ in the integrand may either be positive or negative, hence domain G has to be divided into two parts $^+G'$ and $^-G'$ according to its positive or negative sign, respectively. Now the disturbing potential $T_{P'}$ on the left side of (9) can also be expressed as the sum of positive $^+T_{P'}$ and negative $^-T_{P'}$ quantities. Accordingly, if the flattening is neglected and only the first terms of Taylor expansions of $^+T_{P'}$ and $^-T_{P'}$ are kept, then the following estimation holds for absolute values of both quantities: the error due to the second Taylor term is surely less than $2f/3 \approx 1/400$-th part in ${}^{-}T_{P}$, and ${}^{+}T_{P}$ [10].

3. The potential of a given mass distribution in terms of spherical harmonics

The gravitational potential V_P of an arbitrary density distribution in Fig. 9 over domain σ (the Earth) in an external point P is given by expression (1). With the notations of Fig. 9 this formula can be rewritten in spherical polar coordinates in the form

$$V_P = k \iiint_{\sigma} \frac{\vartheta(r', \Theta', \lambda')}{l} \, d\tau = \qquad (10)$$
$$= k \int_{r'} \int_{\Theta'} \int_{\lambda'} \frac{\vartheta(r', \Theta', \lambda') \, r'^2 \sin \Theta'}{l} \, dr' \, d\Theta' \, d\lambda'$$

Substituting the spherical harmonic expansion of $1/l$ into the above expression (if terms of 0th order and of rotational symmetry are written explicitly), the gravitational potential at an arbitrary outer point $P(r, \Theta, \lambda)$ is given by

$$V_P = \frac{kM}{r} \left\{ 1 - \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^n J_n P_n(\cos\Theta) + \right. \qquad (11)$$
$$\left. + \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left(\frac{a}{r}\right)^n \left[C_{nm} \cos m\lambda + S_{nm} \sin m\lambda \right] P_{nm}(\cos\Theta) \right\},$$

where the total mass of body is M and a suitably chosen distance ($a < r$) [1].

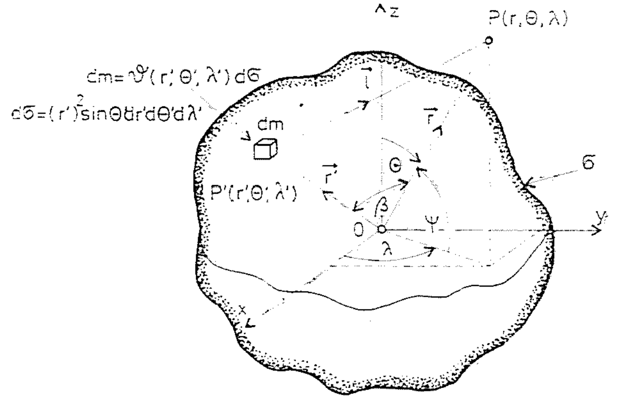

Fig. 9. Notations to evaluate gravitational potential of an arbitrary body in spherical coordinates

The corresponding coefficients J_n, C_{nm}, S_{nm} in (11) can be evaluated if the density distribution inside the given body is known.

If $m = 0$,

$$J_n = -C_{n0} = -\frac{1}{M\,a^n} \iiint_\sigma (r')^n\, P_n(\cos\Theta')\, \vartheta(r', \Theta', \lambda')\, d\sigma \tag{12}$$

holds true and for the case $m \neq 0$

$$\begin{Bmatrix} C_{nm} \\ S_{nm} \end{Bmatrix} = \frac{2}{M\,a^n} \frac{(n-m)!}{(n+m)!} \times \\ \times \iiint_\sigma (r')^n\, P_{nm}(\cos\Theta') \begin{Bmatrix} \cos m\lambda' \\ \sin m\lambda' \end{Bmatrix} \vartheta(r', \Theta', \lambda')\, d\sigma \tag{13}$$

is valid where $d\sigma$ denotes the volume element:

$$d\sigma = (r')^2 \sin\Theta'\, dr'\, d\Theta'\, d\lambda' \,.$$

If we substitute the following normalized form

$$\overline{P_{nm}}(\cos\Theta) = \sqrt{i(2n+1)\,\frac{(n-m)!}{(n+m)!}}\; P_{nm}(\cos\Theta)\ ;$$

$$i = \begin{Bmatrix} 1, \text{ if } m = 0 \\ 2, \text{ if } m \neq 0 \end{Bmatrix} \tag{14}$$

of Legendre polynoms $P_n(\cos\Theta)$ and associated Legendre functions $P_{mn}(\cos\Theta)$ into spherical harmonic series (1) then, of course, coefficients (12) and (13) also have to be normalized:

$$\begin{Bmatrix} \overline{C_{nm}} \\ \overline{S_{nm}} \end{Bmatrix} = \sqrt{\frac{(n+m)!}{i(2n+1)(n-m)!}} \begin{Bmatrix} C_{nm} \\ S_{nm} \end{Bmatrix}$$

$$i = \begin{cases} 1, \text{ if } m = 0 \\ 2, \text{ if } m \neq 0 \,. \end{cases} \tag{15}$$

Since in our case integrals (12) and (13) have to be determined for the model demonstrated in Fig. 2, the physical surface of the earth represents the limits of integration. Once the coefficients of the spherical harmonic series (11) are determined, disturbing potential T_P can also be evaluated by the same coefficients (as will be shown later).

4. The evaluation of spherical harmonic coefficients by numerical quadratures

In the following a numerical quadrature method is introduced which is approximate over the entire Earth surface but exact within each mass compartment. Numerical quadrature is accomplished by dividing the surface of a sphere approximating the Earth by $p - 1$ parallel circles and s meridians into ps area compartment. Topography is estimated by average heights over these compartments. With notations of Fig. 10 let us denote

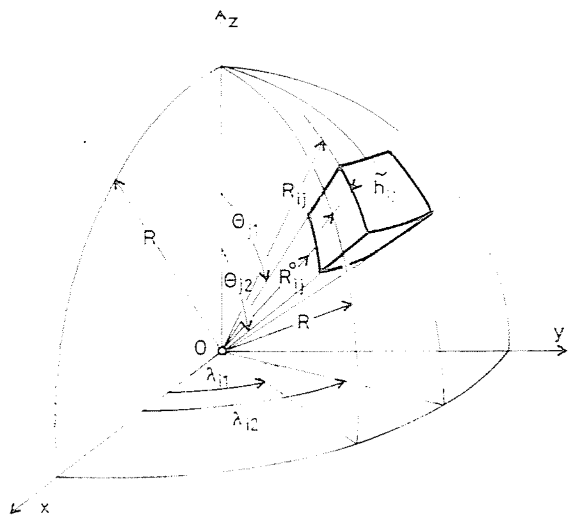

Fig 10. Notations for numerical integration

$$R^0_{ij} = R - T_0 - d_{ij}$$

and

$$R_{ij} = R + h_{\tilde{ij}} ,$$

where $h_{\tilde{ij}}$ is the average height over a given area compartment, d_{ij} is the isostatic root-thickness, T_0 is the average crustal depth and R is radius of the Earth's equivolumal sphere.

First let us evaluate the triple integral in the right side of expression (13) within integration limits shown in Fig. 10. One may readily evaluate the simpler equation (12). Let us neglect for the moment the constant factor on the left side of integral and introduce the following notation:

$$I = \iiint (r')^n P_{nm}(\cos \Theta') \begin{Bmatrix} \cos m\lambda' \\ \sin m\lambda' \end{Bmatrix} \vartheta(r', \Theta', \lambda')\, d\sigma$$

Using the above mentioned partitioning

$$I = \sum_{i=1}^{s} \sum_{j=1}^{p} \int_{R^{\circ}_{ij}}^{R_{ij}} \int_{\Theta_{j1}}^{\Theta_{j2}} \int_{\lambda_{i1}}^{\lambda_{i2}} (r')^{n+2} P_{nm}(\cos \Theta') \times$$
$$\times \begin{Bmatrix} \cos m\lambda' \\ \sin m\lambda' \end{Bmatrix} \vartheta(r', \Theta', \lambda') \sin \Theta' \, dr' \, d\Theta' \, d\lambda' \, . \tag{16}$$

holds. Since the density function $\vartheta(r', \Theta', \lambda')$ is independent of Θ' and λ' within a single compartment and integration limits are constants with respect to Θ' and λ', the triple integral (15) can be factored into three single integrals:

$$I = I_R \begin{Bmatrix} I_C \\ I_S \end{Bmatrix} I_P \, , \tag{17}$$

where we have denoted:

$$I_R = \int_{R-T_0-d_{ij}}^{R+\tilde{h}_{ij}} \vartheta(r') \, (r')^{n+2} \, dr' \tag{18}$$

$$\begin{Bmatrix} I_C \\ I_S \end{Bmatrix} = \int_{\lambda'_{i1}}^{\lambda'_{i2}} \begin{Bmatrix} \cos m\lambda' \\ \sin m\lambda' \end{Bmatrix} d\lambda' \tag{19}$$

$$I_P = \int_{\Theta'_{j1}}^{\Theta'_{j2}} P_{nm} (\cos \Theta') \sin \Theta' \, d\Theta' \tag{20}$$

We evaluate the first integral (18) in two basic cases, i.e. over continental and oceanic areas. As it can be seen in Fig. 7 the continental case becomes

$$I_R = \int_{R-T_0-d_{ij}}^{R+\tilde{h}_{ij}} \vartheta(r') \, (r')^{n+2} \, dr' =$$
$$= (\vartheta_k - \vartheta_a) \int_{R-T_0-d_{ij}}^{R-T_0} (r')^{n+2} \, dr' + \vartheta_k \int_{R}^{R+\tilde{h}_{ij}} (r')^{n+2} \, dr' \tag{21}$$

where mean height $h_{\widetilde{ij}}$ is positive and T_0 is the mean crustal thickness, and

$$d_{ij} = \frac{\vartheta_k}{\vartheta_a - \vartheta_k} h_{\widetilde{ij}}$$

according to the isostatic model by Airy. On performing integration in (21) and performing elementary manipulations, the expression

$$I_R = \frac{1}{n+3}\left\{\vartheta_k R^{n+3}\left[\left(1+\frac{h_{ij}}{R}\right)^{n+3}-1\right]+\right.$$
$$\left.+(\vartheta_a-\vartheta_k)(R-T_0)^{n+3}\left[\left(1-\frac{\vartheta_k h_{ij}}{(\vartheta_a-\vartheta_k)(R-T_0)}\right)^{n+3}-1\right]\right\} \qquad (22)$$

results where ϑ_k and ϑ_a denotes average crustal and mantle density ($\vartheta_k \doteq 2670$ kg/m^3, $\vartheta_a \doteq 3270$ kg/m^3). In the same way the integral (18) can also be evaluated over oceanic areas covered by sea water. If $\tilde{h}^*_{ij}$ denotes mean oceanic depth,

$$I^*_R = \frac{1}{n+3}\left\{(\vartheta_k-\vartheta_v)R^{n+3}\left[\left(1+\frac{\tilde{h}^*_{ij}}{R}\right)^{n+3}-1\right]+\right.$$
$$\left.+(\vartheta_a-\vartheta_k)(R-T_0)^{n+3}\left[\left(1-\frac{(\vartheta_k-\vartheta_v)\tilde{h}^*_{ij}}{(\vartheta_a-\vartheta_k)(R-T_0)}\right)^{n+3}-1\right]\right\} \qquad (23)$$

is found where $\vartheta_v \doteq 1030$ kg/m^3 is the density of sea water and

$$d^*_{ij} = \frac{\vartheta_k-\vartheta_v}{\vartheta_a-\vartheta_k}\tilde{h}^*_{ij}$$

denotes anti-root thickness from Airy's isostatic equilibrium hypothesis.

The integral expressions (19) may be evaluated to yield equations

$$I_C = \frac{1}{m}(\sin m\lambda_{i2} - \sin m\lambda_{i1}) = $$
$$= \frac{2}{m}\cos m\frac{\lambda_{i1}+\lambda_{i2}}{2}\sin m\frac{\Delta\lambda}{2},$$
$$I_S = \frac{1}{m}(\cos m\lambda_{i2} - \cos m\lambda_{i1}) = \qquad (24)$$
$$= \frac{2}{m}\sin m\frac{\lambda_{i1}+\lambda_{i2}}{2}\sin m\frac{\Delta\lambda}{2}$$

where

$$\Delta\lambda = \lambda_{i2} - \lambda_{i1} = \text{const}.$$

Let us finally evaluate integral (20)! After introducing the new variable $t = \cos\Theta'$ as above,

$$I_P = \int_{t_2=\cos\Theta'_{j2}}^{t_1=\cos\Theta'_{j1}} P_{nm}(t)\,dt\,. \qquad (25)$$

holds true. This integral may be evaluated by a recursive method suitable for computer calculations.

For this purpose let us start with the following expression which can easily be verified by differentiating (14):

$$P_{nm}(t) = \sqrt{1-t^2}\,\frac{d\bar{P}_{n,m-1}(t)}{dt} + (m-1)\frac{t}{\sqrt{1-t^2}} P_{n,m-1}(t) \tag{26}$$

Moreover, it can be deduced from the differential equation of Legendre functions that

$$P_{nm}(t) = \frac{1}{n(n+1) - m(m+1)} \times \left[2(m+1)\frac{t}{\sqrt{1-t^2}} P_{n,m+1}(t) - P_{n,m+2}(t)\right] \tag{27}$$

holds. Note that expressions (26) and (27) became undetermined at poles, i.e. when $t = \cos\Theta$ or $t = \sin\psi$. Integration of (26) between limits t_1 and t_2 and applying (27) produces the expression

$$\int_{t_1}^{t_2} P_{nm}(t)\,dt = \frac{1}{n(n+1) - m(m+1)} \times \left\{-\frac{2(m+1)}{m+2}\left[\sqrt{1-t_2^2}\,P_{n,m+1}(t_2) - \sqrt{1-t_1^2}\,P_{n,m+1}(t_1)\right] + \frac{m}{m+2}\int_{t_1}^{t_2} P_{n,m+2}(t)\,dt\right\} \tag{28}$$

It is evident that integrals of $P_{nn}(t)$ are also needed in the above formula for recursive computation. The desired expression can be gained by the integration of (cf. [4])

$$P_{nn}(t) = 1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)(1-t^2)^{n/2} \tag{29}$$

which yields

$$\int_{t_1}^{t_2} P_{nn}(t)\,dt = \frac{1}{n+1}\left[P_{nn}(t_2)t_2 - P_{nn}(t_1)t_1 + n(2n-1)(2n-3)\int_{t_1}^{t_2} P_{n-2,n-2}(t)\,dt\right]. \tag{30}$$

Note that for the recursive computation of (30) a very disadvantageous error accumulation occurs which can be avoided by using the method of [11].

To summarize, the integral I_p can now be calculated recursively — suitable for computer calculation — by formulas (26), (27), (28), (2o), and (30).

5. Results of initial numerical computations

Numerical test computations were performed by the authors on the basis of the previously described procedure. Computer programs were developed in the FORTRAN language on an IBM PC/AT computer. The first program system computes C_{nm}, S_{nm} spherical harmonic coefficients from input mean surface heights — using the above described process — for the Earth model sketched in Fig. 2. The second program of the system creates geoid heights over previously given grid points from input C_{nm}, S_{nm} coefficients. The third program interpolates contour maps of geoid heights.

Mean topographic heights were introduced over $1° \times 1°$ area blocks into the calculation (this implies 64 800 data for the entire Earth). Spherical harmonic series of disturbing potential were determined up to degrees $n = m = 36, 50, 90, 180$; however, since geoid shape due to topographic and isostatic masses does not vary significantly with increasing degree (and, on the contrary CPU times increased rapidly) the following test were accomplished only up to $n = m = 90$.

Geoid undulations due to topographic and isostatic masses can be seen in Fig. 11. It can be established that geoid heights computed by spherical harmonic series of disturbing potential are reasonable; maximum geoid undulations of $\pm 10 : \pm 30$ m were obtained depending on the characteristics of topography. We mention also here that since spatial positions of crust — atmosphere (-ocean) and crust — mantle boundaries are not known precisely, a minor translation of level surfaces of computed potential field may occur. This translation, however, can be neglected for our purposes since the computed geoid is needed for only interpretational purposes.

Our final goal is to interpret major geoid forms physically by separating the effects due to well-known density anomalies; hence the next step is to separate our computed topographic — isostatic effect from the full global geoid shape. Fig. 12 illustrates the RAPP 1981 geoid which we chose to interpret. Remaining geoid forms are demonstrated in Fig. 13 which were obtained by subtracting geoid heights of Fig. 11 from the RAPP 1981 geoid. Fig. 13 shows that, unfortunately, our problem has not been simplified significantly since geoid forms which do not contain the effect of topographic and isostatic masses have not become simpler or easier to interpret. Anyway, the separation process have to be continued, i.e. additional known mass inhomo-

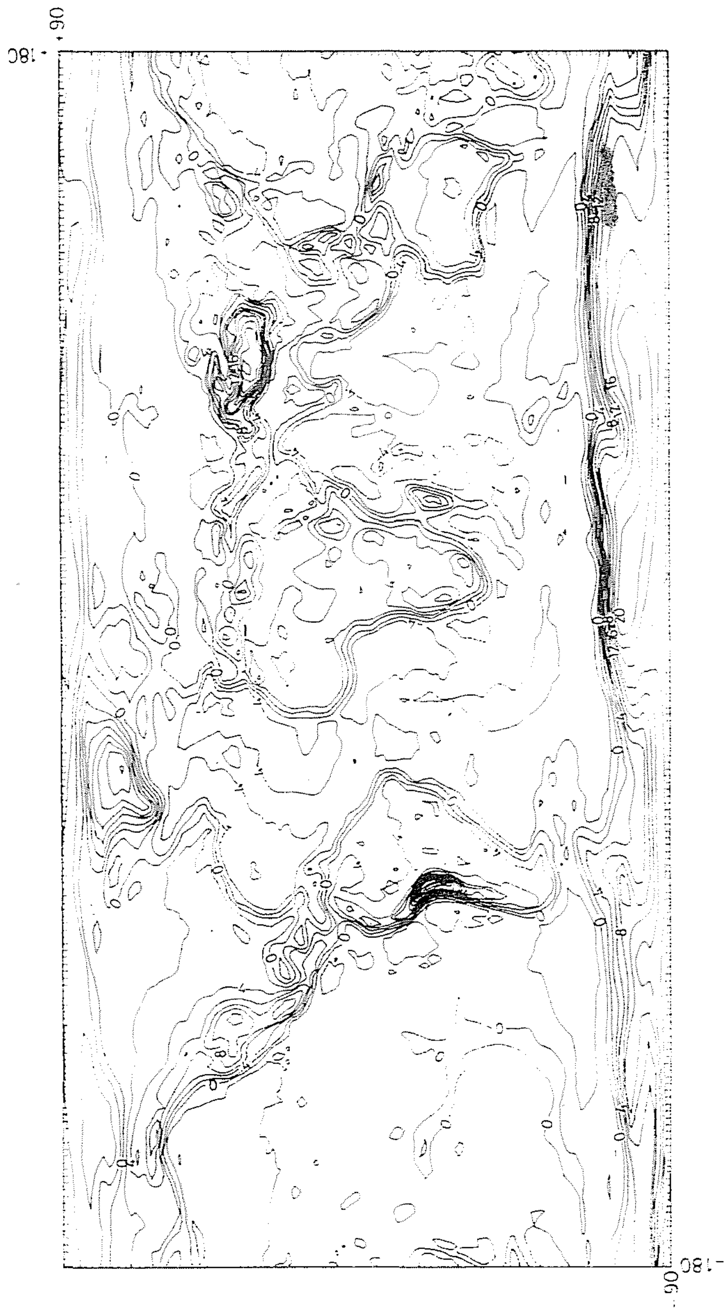

Fig. 11. **Geoid heights of topographic and isostatic masses**

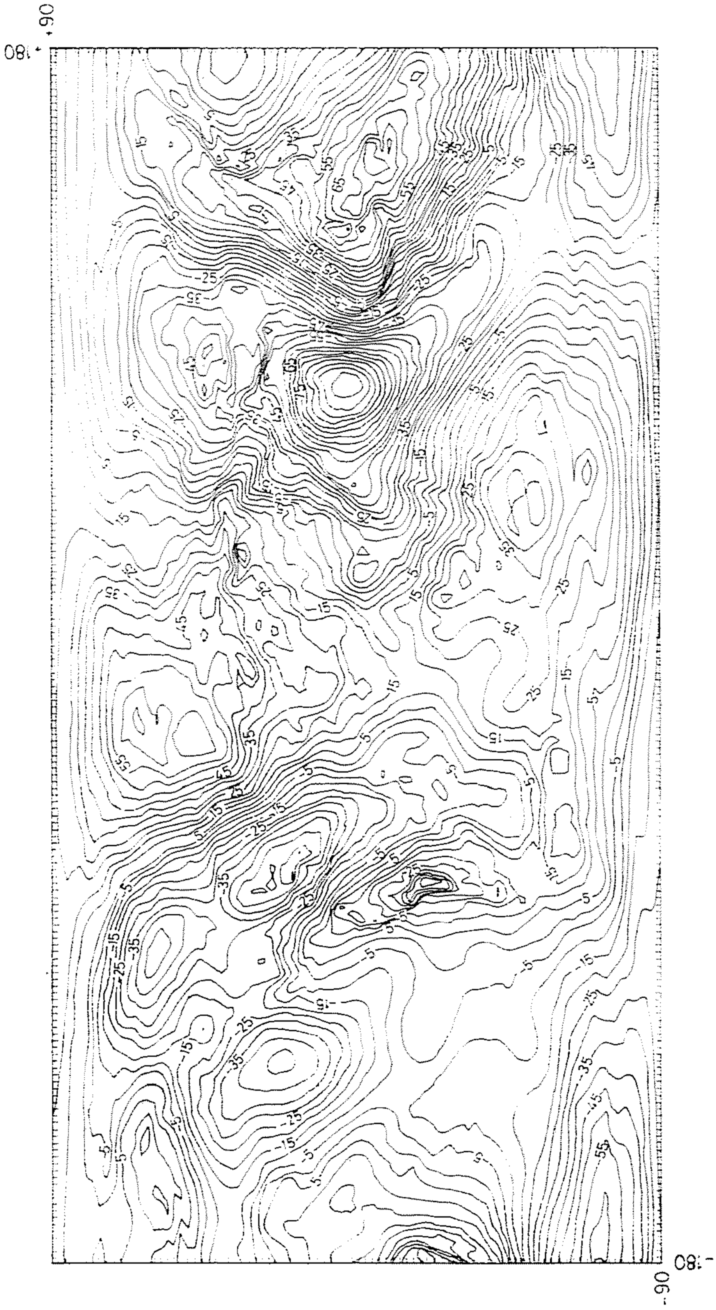

Fig. 12. **Contour map of RAPP 1981 geoid**

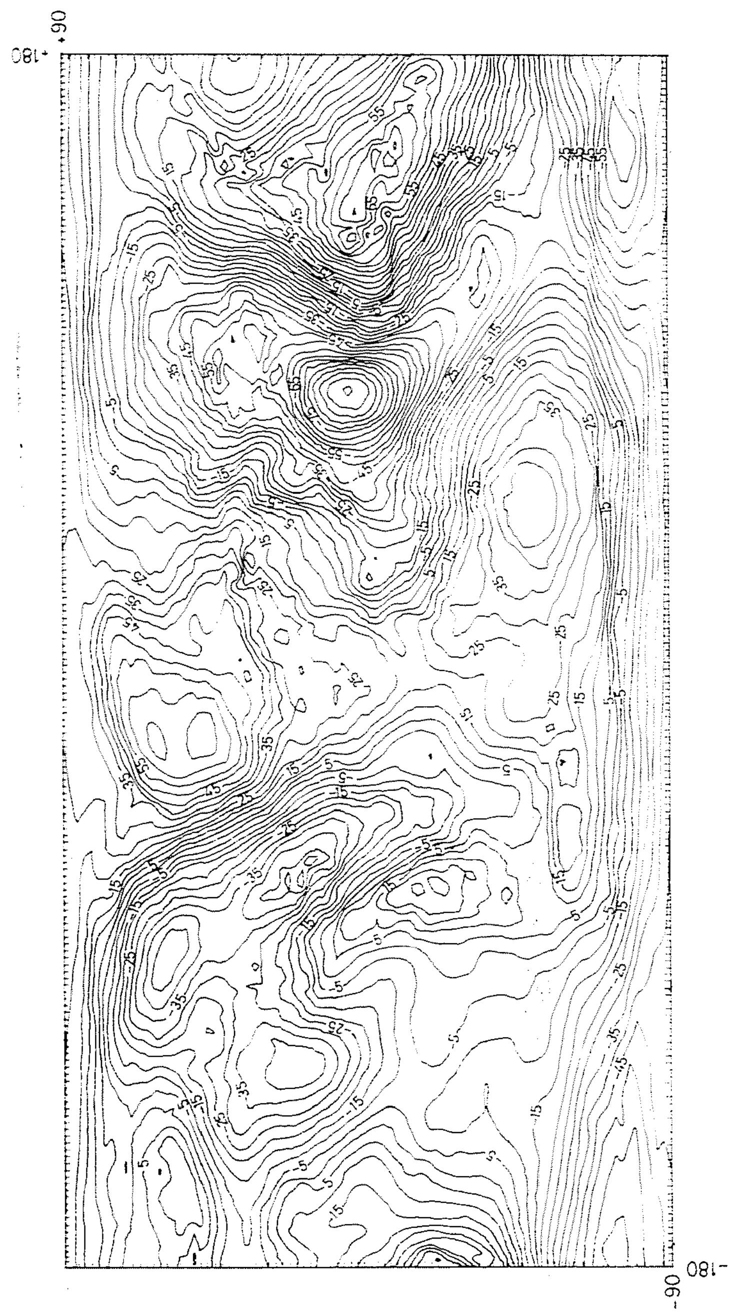

Fig. 13. **Geoid height differences of RAPP 1981 geoid and topographic-isostatic geoid**

genities (e.g. density irregularities of plate tectonic models, etc.) have to be considered and computable geoid heights due to masses have to be subtracted from geoid heights demonstrated in Fig. 13. To achieve this, further investigations, additional computer software, collection and consideration of other geophysical data are needed.

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