EFFECT OF RIGIDITY OF STRUCTURE ON THE SETTLEMENT OF FOUNDATION

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Received May 9, 1990

Abstract

The paper deals with a computation method based on a combined soil model. The model, making use of Repnikov's assumption, is suitable for application and calculation of shallow foundation in case of medium high panel building thus saving construction costs.

1. Computer methods in the geotechnics

The development in computer technics aided the introduction of such new methods into the geotechnics that make possible gradually more and more realistic modeling of soil-structure interaction. Part of these methods, e.g. finite elements method, require big memory capacity and fast computer and even — due to inaccuracy of parameters — the application of these methods is not reasonable. The microcomputers recently so abundant require methods of less semianalytical computation work.

2. Introduction of combined soil model

The method suitable for the calculation of shallow foundation for medium high buildings was developed on the basis of Repnikov Soviet researcher's publication (Fig. 1).

The principle of the method is that in case of $\mu = 0.25$ Poisson's number (sands and silty soils) a half-space having moduls of elasticity that varies parabolically with depth:

$$E = E_0(1 + cz)^2$$

is equivalent — from the point of view of settlement — with a parallelly connected half space having E_0 and a Winkler bedding of k bedding factor combination.

The equivalency exists according to Barvasov and Plevanko if

$$C = 0.84 \frac{k}{E_0}$$

and the Poisson number equals 0.25.

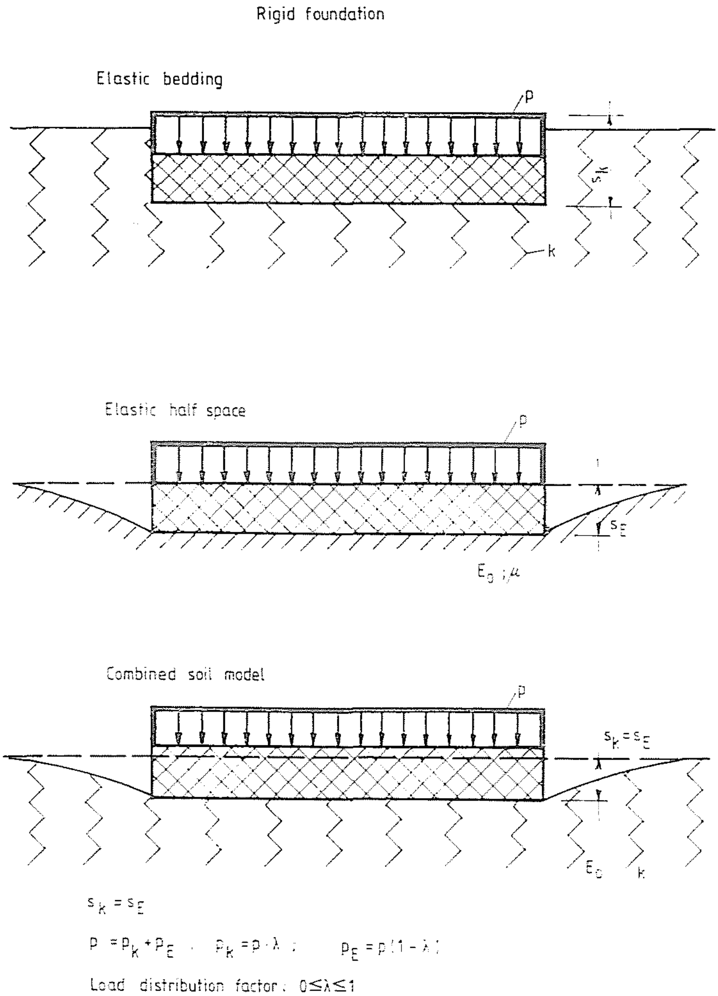


Fig. 1. Different models of soil

Similar relation exists for constant-volume materials ($\mu = 0.5$) if the Young's modulus increases linearly with depth from zero (Gibson Type soil):

$$E = c_1 z.$$

The equivalency with a Winkler bedding of k bedding factor exists if

$$c_1 = \frac{3}{2} k.$$

If the initial value of modulus of elasticity is E_0, then substitution with combined model is a good approximation.

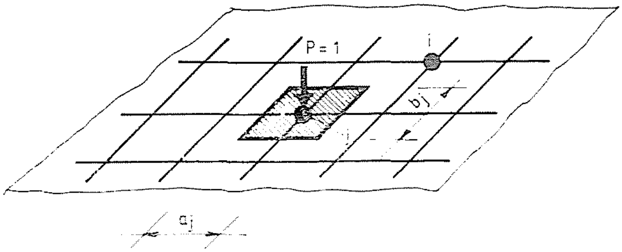


Fig. 2. Mesh to the FEM analysis

Soils having stiffness increasing with depth are very frequent, so the application possibilities of this method are very wide.

In case of finite elements method the stiffness matrix can be determined easily—after division of the ground surface — with the help of Boussinesq's and Steinbrenner's settlement equations and the relations concerning Winkler bedding.

If we take a net of rectangles on the surface of the tested half-space (Fig. 2), then the elements of the flexibility matrix of the elastic half-space are the following:

$$f_{ij} = \frac{1-\mu^2}{\pi E_0} \cdot \begin{cases} \frac{2}{b_j} \ln \frac{b_j + c_j}{a_j} + \frac{2}{a_j} \ln \frac{a_j + c_j}{b_j}, & \text{if } i = j \text{ (Steinbrenner)} \\ \frac{1}{r_{ij}}, & \text{if } i \neq j \text{ (Boussinesq); equations have to use} \end{cases}$$

$$c_j = \sqrt{a_j^2 + b_j^2}$$

$$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}.$$

The elements of the rigidity matrix of the Winkler bedding:

$$k_{ij} = \begin{cases} k a_j b_j & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}.$$

The rigidity matrix of the combined soil model is the sum of the rigidity matrix of the elastic half-space (the inverted flexibility matrix) and the rigidity matrix of the Winkler bedding.

This stiffness matrix can be connected with stiffness matrices of other elements that are worked out on the basis of variation principles (plates, disks, bodies).

In simple case, when the lengths width ratio of the building is bigger than 3, the applied equations are totally analytical. In this case the building can be modelled as a beam. The relationship between the load of the beam, the base stresses and the deformation of the elastic line of the beam can be determined easily.

By using of dimensionless coordinates:

$$\xi = \frac{x}{A},$$

where A = the half length of the raft.

The stress transferred by the raft in polynomial form:

$$q_t(\xi) = \sum_{i=0}^{n} b_i \xi^i.$$

The part of the stress acting on the elastic half-space:

$$q_E(\xi) = \sum_{i=0}^{n} a_i \xi^i,$$

and that on the bedding:

$$q_k(\xi) = q_t(\xi) - q_E(\xi).$$

The load should be also given in polynomial form:

$$P(\xi) = \sum_{i=0}^{n} P_i \xi^i.$$

The deformation of the raft can be expressed with the help of the differential equation of the elastic line:

$$s = s_0 - A\xi \tan \varphi_0 + \frac{A^4}{E \cdot J} \left(- \frac{M_0}{2A^2} \xi^2 - \frac{T_0}{2A^2} \xi^3 + \right.$$
$$\left. + \frac{P_0 - b_0}{1 \cdot 2 \cdot 3 \cdot 4} \xi^4 + \frac{P_1 - b_1}{2 \cdot 3 \cdot 4 \cdot 5} \xi^5 + \dots \right)$$

where:

s_0 = settlement of the centre of the raft,
φ_0 = angular displacement at the centre of the raft,
EJ = flexural rigidity of the raft
M_0 = moment at the raft centre:

$$M_0 = -A^2 \left(\frac{P_0 - b_0}{2} + \frac{P_2 - b_2}{4} + \dots \right)$$

T_0 = shear force at the raft centre:

$$T_0 = -A \left(\frac{P_1 - b_1}{2} + \frac{P_3 - b_3}{4} + \dots \right).$$

The settlement can be determined from the share of stress falling to the elastic half-space:

$$s(\xi) = \frac{2(1-\mu^2)}{\pi E_0} \sum_{j=0}^{m} \left[\left(\ln 4 \frac{A}{B} - d_{2j} \right) a_{2j} + \sum_{\substack{i=0 \\ i \neq j}}^{m} \frac{a_{2i}}{2(i-j)} \right] \xi^{2j}$$

in case of symmetric stresses,

$$s(\xi) = \frac{2(1-\mu^2)}{E_0} \sum_{j=0}^{m} \left[\left(\ln 4 \frac{A}{B} - d_{2j+1} \right) a_{2j+1} + \sum_{\substack{i=0 \\ i \neq j}}^{m} \frac{a_{2i+1}}{2(i-j)} \right] \xi^{2j+1}$$

in case of antisymmetric stresses.

Here

$$d_k = \sum_{r=1}^{k} \frac{1}{r}$$

and B = half width of k the raft.

Followingly, by means of making equal the corresponding coefficients in polynomes, the base stresses, the settlement and the stresses in the building can be calculated.

According to our experiences, the method gives favourable results, the stress peaks, the settlements and the building stresses are lower. The calculation may be done with microcomputer very quickly.

3. Introduction of the tested static models

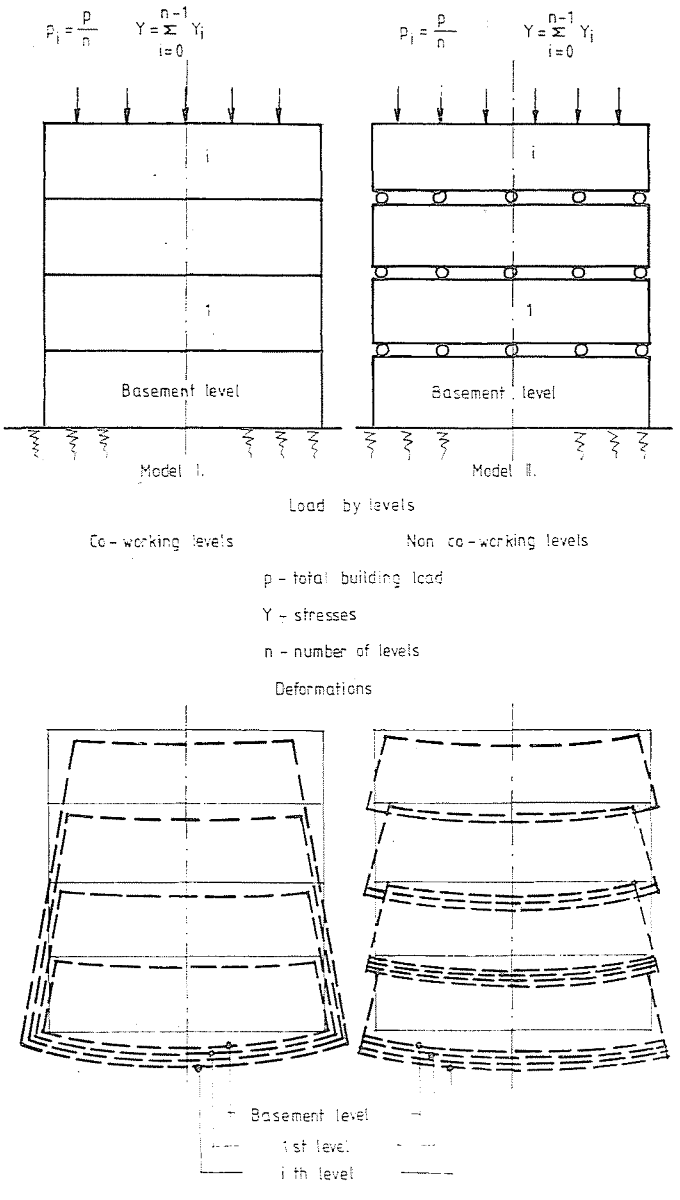
Four different models were in the calculations (Fig. 3).

Model I. follows the course of the construction. The load of each constructed level acts directly on the previous level, thus the loads are continously superimposed. To determine stresses in the so-called reception level, the construction order should be followed, the loads will be summarized by levels. Perfect interworking between the levels is assumed.

Model II. differs from Model I. only in one point: no interworking between levels is assumed.

Consequently, the loads are taken by levels and the incements in deformation caused by the different levels are superimposed. This process can be seen in Fig. 2 where deformations caused by the different levels are represented in case of Model I. and II.

Model III. assumes perfect interworking again. This model corresponds to the conventional calculation method since it assumes that the total building load acts only on some load bearing levels of the building, causing stresses only in them.



A

Fig. 3. Applied building structure models

In Model IV. also only some levels are loaded but there are not shearing forces between the levels. Here, the loads on the load bearing levels are taken in one step.

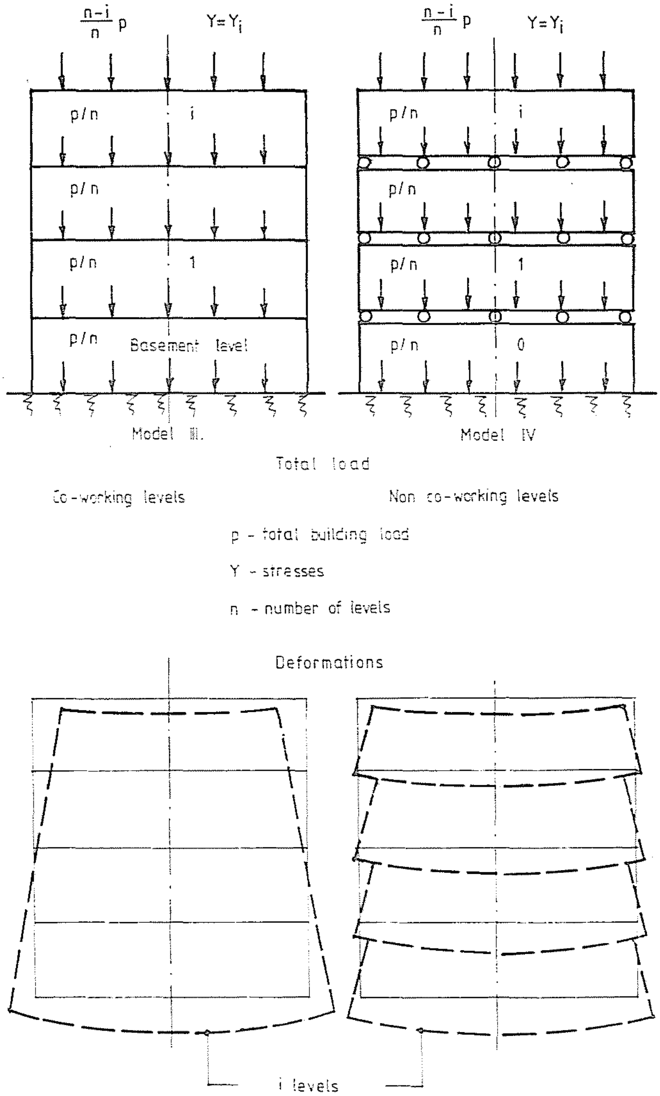


Fig. 3/B

4. Numerical analysis and evaluation of the Models

Experimental building was built on prefabricated raft foundation in Szeged, Hungary. The foundation was calculated with the method developed by the Geotechnical Department, Technical University of Budapest. The subsoil was river sediment (silt, sand and clay layers), here similar buildings were founded by piles.

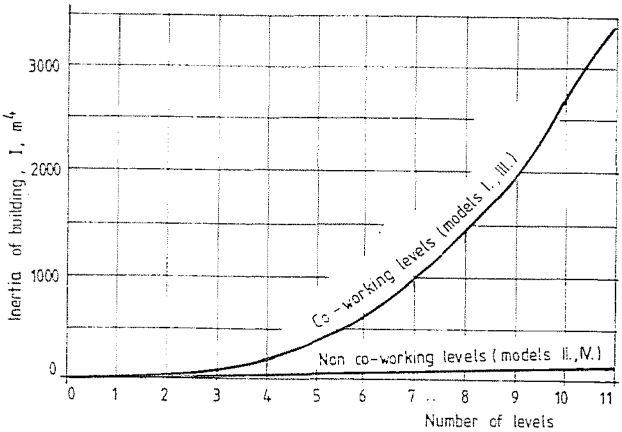


Fig. 4. Building inertia v. s. number of levels

The soil characteristics were taken from laboratory test results:

$$E_0 = 13 \text{ MPa}$$

$$k = 1.3 \text{ MN/m}^3$$

$$\mu = 0.32.$$

The building itself was an 11 level high structure. At the ground level there were garages, while above flats were built. In the calculations a prefab raft was taken as zero level.

The total building load was 65.8 MN. This load was divided equally between the 1.—11. levels. The weight of the raft, which does not cause stresses, was neglected, thus the total load of the modelling beam was 1.57 MN/m and the levels had 0.143 MN/m load each except level 0.

The dimensions of the equivalent beam:

half length: $A = 20.9$ m

half width: $B = 6.65$ m.

The inertia of building as a function of number of levels (Fig. 4) is as follows:

In case of co-working levels

(Mods I. and III.):

$$I_i = r_i \sum_{k=0}^{i} (I'_k + y_k^2 A'_{xk}).$$

In case of non co-working levels

(Mods II. and IV.):

$$I_i = \sum_{k=0}^{i} I'_k$$

where

I_i = inertia of the building that is ready up to the i-th level: reference axis is the same with the axis of gravity.

I_k = inertia of the cross-section of the k-th level taking opening into account.

r_i = factor depending on the building height/width ratio to take disk effect into account,

$$0.25 \leqq r \leqq 1$$

y_k = distance between the centre of gravity of the k-th level and that of the existing building,

A'_{xk} = cross-section area of the level.

In the case of basement level — onto which the panels are mounted and is matched to the panel modul sizes in layout — not the general building moment is the important factor (it may not be determined in case of Model I. and III.) but other loads calculated from this moment and act on important structural elements (Figs 5, 6). In this very case that was the lengthwise normal force acting on the raft and the upper floor at the reception level.

Normal forces in any structural element, after level i is completed, can be determined as follows:
in case of Model I.

$$N = v \sum_{k=j}^{i} t_k \frac{M_k}{I_k} y_k$$

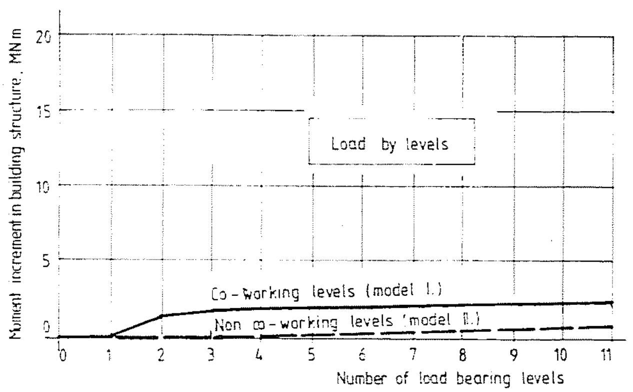


Fig. 5. Moment increment v. s. number of load bearing levels

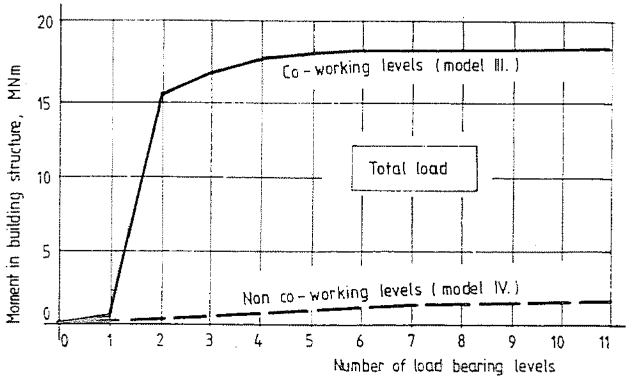


Fig. 6. Total moment in building structure v. s. number of load bearing levels

in case of Model II.

$$N = vy_j \sum_{k=j}^{i} \frac{M_t}{kI_j}.$$

The method for the determination of the normal force in any structural element if the number of the co-working levels is i, is the following:
in case of Model III.

$$N = vt_i \frac{M_i}{I_i} y_i.$$

in case of Model IV.

$$N = v \frac{M_i}{iI_j} y_j$$

where

N = normal force
v = thickness of structural element
t_k = reducing factor in case of building ready up to level k

$$-0.02 \leq t_k \leq 1.17$$

M_k = moment increment on the first k levels due to the load of level k
I_k = inertia of the first k levels
y_k = distance between the examined element and the centre of gravity of the k level building
y_j = distance between the examined element and the centre of gravity of the examined level
I_j = inertia of the examined level (the level that contains the examined element)

t_i = reducing factor when i levels are taken into account
M_i = moment of the total load when i levels are taken into account
I_i = building inertia in the same case
y_i = distance between the examined element and the centre of gravity of the i level building.

At Model I. the graph of normal forces converges slower to an end value than the moments (Figs 7, 8). The normal force acting on the upper floor changes sign after the 4th level is completed since not only the building stiffness increases but the centre of gravity gets higher.

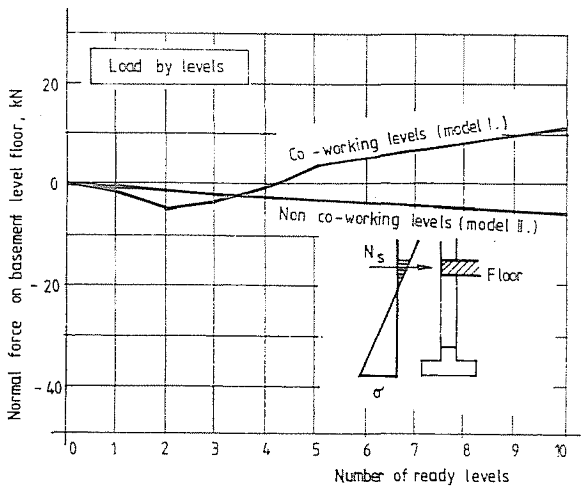


Fig. 7. Normal force on basement level floor v. s. number of ready levels

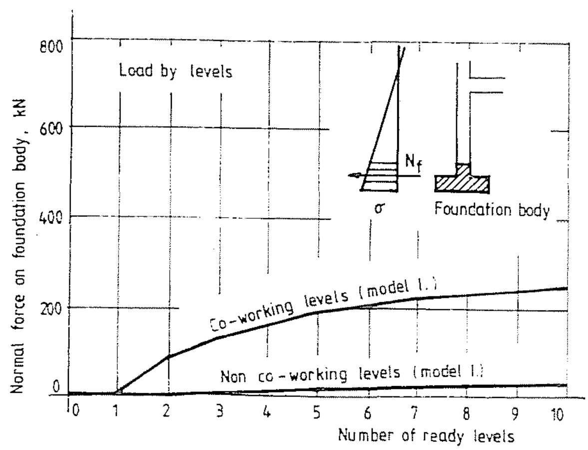


Fig. 8. Normal force on foundation body v. s. number of ready levels

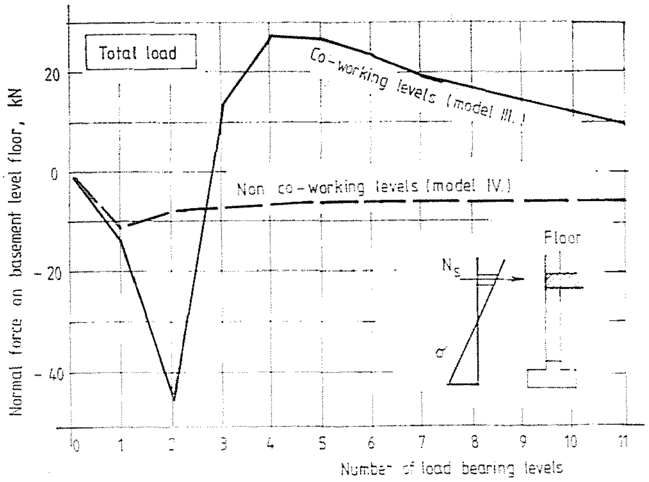


Fig. 9. Normal force on basement level floor v. s. number of load bearing levels

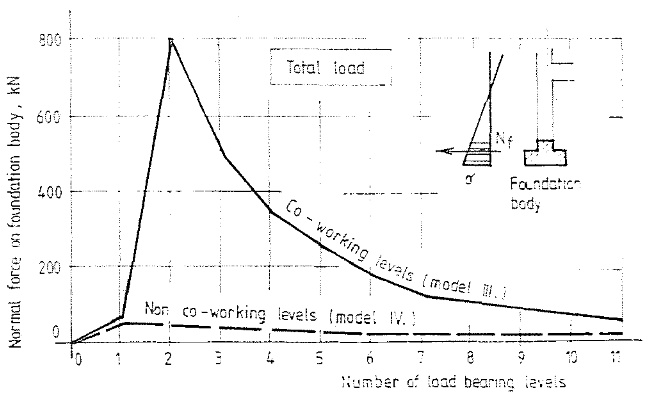


Fig. 10. Normal force on foundation body v. s. number of load bearing levels

In case of Model III. beside the well known moment graph, the normal forces develop curiously (Figs 9, 10). It can be clearly seen that the number of levels the stiffness of which is taken into account in the calculation, may not be decided on the basis of moment graph.

The stress graphs of Model II. and IV. show the same result (Figs 7—10).

5. Comparison of calculated and in-situ test results (Fig. 11)

There is no use to decide between different calculation methods on the basis of theoretical data only, but the results prove clearly that tracking of construction course in the calculation may not be substituted by taking a number of interworking levels (Models III. and IV.).

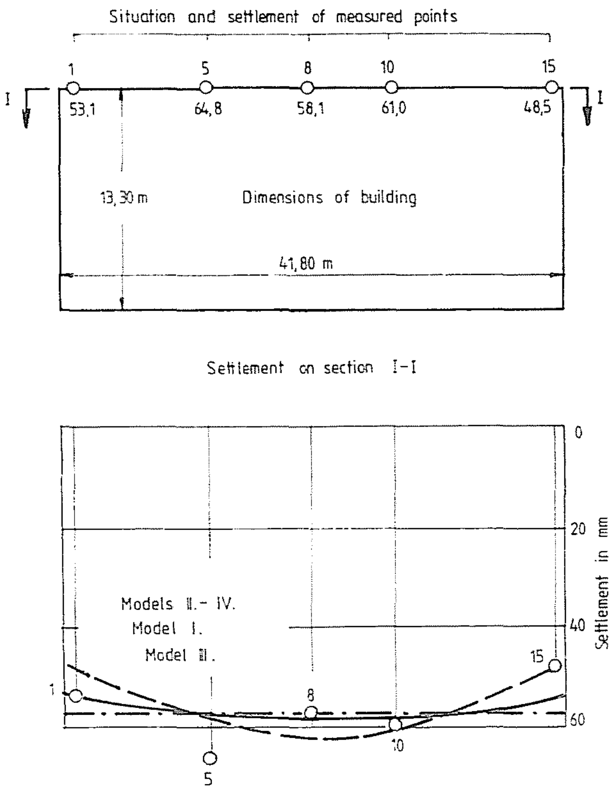


Fig. 11. Comparison of calculated and measured settlement

By comparing calculated and measured settlements it can be concluded that the interworking between levels in panel buildings is limited, the shear forces transmitted by the levels are very small.

In the sense of absolute value of forces, the stresses originating from the interworking are small, smaller than the concrete tensile strength. Since the settlement differences did not endanger either the orderly use of building or the public utilities, or the soil load bearing, the high cost of deep foundation could be saved.

6. Conclusions

The combined soil model is suitable for the tracing of building behaviour. The calculated results are in good agreement with the measured ones. Among the Models tested in this paper, Model II. approximates best the real building behaviour. Model II. traces the course of construction and interworking between levels is not assumed.

According to the calculations this Model gives much smaller moments and normal forces than the others. At the same time, building behaviour calculated with this Model is the most similar to the real one.

7. Summary

The paper presents a method for calculations of foundation for medium high buildings based on combined soil model. The principles of combined soil model are described and the applicability of the method for soils having stiffness increasing with depth improved. The calculations are very simple.

Based on the data of an 11 level panel building, four different models were tested. The course of construction and interworking between levels was taken into consideration differently. The gained result were compared with in-situ measured data.

Among the statical models, the best approximation was gained by the one that followed strictly the course of construction and did not assume interworking between levels. This model gave the smallest stresses, also.

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