

RELATIONSHIP BETWEEN GRADING ENTROPY AND DRY BULK DENSITY OF GRANULAR SOILS

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Abstract

The paper provides a practical method for the calculation of the grading entropy of soils, further it gives the experimentally determined volumetric ratios of the solid phase (s) of continuously-, and gap-graded soil mixtures versus the quantities of grading entropy.

1. Entropy of a statistical system

The entropy is a quantity of the theory of probability and is determined by the following equation:

$$S = - \sum_x p(x) \log_2 p(x). \quad (1)$$

In case of statistical measurement, the values of the random variable are "distributed" into statistical cells and are practically measured, thus (1) can be written into the following form:

$$S = - \frac{1}{\ln 2} \sum_i \alpha_i \ln \alpha_i \quad (2)$$

where α_i is the frequency of the i -th statistical cell. If the elements of a multitude can not be differentiated from each other, the multitude is not statistical, its entropy equals zero. When mixing *two* systems, the entropy increment (ΔS) resulted by the process of mixing is a function of the mixing ratio of the components, as it can be seen in Fig. 1. This ΔS can also be calculated with the help of Equ. (2) if α_i -s are the frequencies of the mixed systems.

The maximum of ΔS is 1 and is reached when the ratios of the two components are equal ($\alpha_A = \alpha_B = 0.5$).

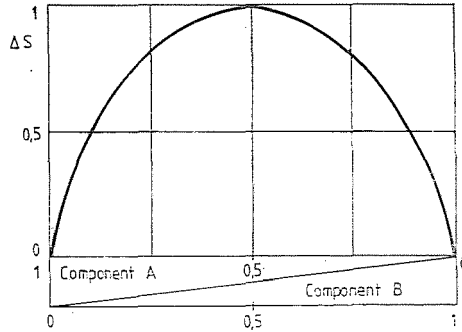


Fig. 1. Entropy increment vs mixing ratio of two multitudes

2. Grading entropy of soils

2.1 Eigenentropy of fractions

Grain size distribution of soils can be determined by means of sieves, or, if the diameter of particles is smaller than 0.063 mm the sedimentary tests are applied making use of Stokes' law. The soil fractions, that were used in the tests, had been separated with a sieve-set having the following mesh sizes:

$$d_{\text{mm}} = 0.063, 0.125, 0.25, 0.5, 1, 2, 4, 8, \dots,$$

namely each subsequent fraction had a double width.

Since the elementary statistical cells should have the same size, the fractions themselves also have entropy. The suggested and applied width of the elementary cell was:

$$z = 2^{-17} \text{ mm.}$$

With this very small cell size, any soil can be dealt with later, even colloidal clays. The distribution within one fraction was assumed to be uniform, therefore the frequencies of the elementary cells are

$$\alpha = \frac{1}{C}$$

where C = number of elementary cells in the fraction.

The eigenentropy of the i -th fraction of the soil:

$$S_{0i} = - \frac{1}{\ln 2} C_i \sum_i \frac{1}{C_i} \ln \frac{1}{C_i},$$

that is

$$S_{0i} = \frac{\ln C_i}{\ln 2}. \quad (3)$$

For example, the eigenentropy of the fraction between sieves 1 and 2 mm:

$$C = \frac{1 \text{ mm}}{2^{-17} \text{ mm}} = 2^{17}$$

$$S_0^{1-2} = \frac{\ln 2^{17}}{\ln 2} = 17.$$

On the same way, the eigenentropy of fraction 2—4 mm will be 18, and so forth. For fractions used in the tests introduced in this paper, the number of elementary cells (C_i) and the value of eigenentropy (S_{0i}) can be seen in Table I.

Table I

d_{mm}	(0.0625)—0.125	0.125—0.25	0.25—0.5	0.5—1	1—2
C_i	2^{13}	2^{14}	2^{15}	2^{16}	2^{17}
S_{0i}	13	14	15	16	17

2.2 Grading entropy of soils separated to fractions

Let the frequency of fraction i be x_i (to differentiate from the z frequency of the elementary cells). The sum of x_i frequencies will be 1 of course. The frequency of each elementary cell in fraction No 1, 2, . . . , n is:

$$\alpha_1 = \frac{x_1}{C_1}, \alpha_2 = \frac{x_2}{C_2}, \dots, \alpha_n = \frac{x_n}{C_n}$$

respectively.

The entropy of the soil:

$$S = -\frac{1}{\ln 2} \left(C_1 \frac{x_1}{C_1} \ln \frac{x_1}{C_1} + C_2 \frac{x_2}{C_2} \ln \frac{x_2}{C_2} + \dots + C_n \frac{x_n}{C_n} \ln \frac{x_n}{C_n} \right) \quad (4)$$

After expanding this equation it can be written into the following form:

$$S = -\frac{1}{\ln 2} \sum_i x_i \ln x_i + \frac{1}{\ln 2} \sum_i x_i \ln C_i. \quad (5)$$

The first member of Equ. (5) has the same form with Equ. (2) and is the ΔS entropy increment, resulted by the "mixing of fractions". Selecting the i -th element out from the second member of Equ. (5):

$$x_i \frac{\ln C_i}{\ln 2} = x_i S_{0i} \text{ (see: Equ. (3))}$$

that is the product of the eigenentropy of the i -th fraction and its frequency. The total sum in the second member of Equ. (5) is the so called *base entropy* of the soil (S_0):

$$S_0 = \sum_i x_i S_{0i} \quad (6)$$

S_0 is the entropy of an imaginary system in which the fractions are not mixed but layered above each other. The ratio of the fractions in the layered system is their frequency. Finally, the grading entropy of the soil is:

$$S = S_0 + \Delta S. \quad (7)$$

In some countries the grading of soils is determined with sieves different from the ones indicated above. The value of the total S entropy will be the same if it was calculated with Equ. (4), but S_0 and ΔS will be different. Same S_0 and ΔS values can be obtained if the *grading curve* is divided into the fractions suggested in this paper. It also applies for the grading curve of soils finer than 0.0625 mm.

3. The maximum grading entropy

The extreme of grading entropy can be determined by differentiating Equ. 5. It is easy to see that this entropy will be maximum in the case of equality of frequency of the elementary statistical cells, namely if

$$\frac{x_1}{C_1} = \frac{x_2}{C_2} = \dots = \frac{x_F}{C_F} = \alpha. \quad (8)$$

If two neighbouring fractions are mixed with each other, then, since $C_2 = 2C_1$:

$$x_2 = 2x_1$$

In this case the S_0 base entropy will be:

$$S_0 = x_1 S_{01} + x_2 S_{02} = x_1 (S_{01} + 2S_{02}) \quad (9)$$

But, since $x_1 + x_2 = 1$, or $3x_1 = 1$, $x_1 = \frac{1}{3}$, and $S_{02} = S_{01} + 1$, Equ. (9) can be factored into the following form:

$$S_0 = S_{01} + \frac{2}{3}. \quad (10)$$

Equation (10) means that in case of two neighbouring fractions, the proportions of the fractions in the maximum entropy mixture:

2/3 coarser and 1/3 finer fraction.

When the relationship between grading entropy and dry bulk density was tested, many different soil mixtures were made. The grading curves of some mixtures can be seen in Fig. 2. Fig. 3 is representing the grading entropy (S) of mixtures B and C of Fig. 2.

4. Density of different soil mixtures

Very simple tests were made to determine relationship between grading entropy quantities and density of soil skeleton: the so called e_{\max} tests. The soil was mixed from fractions of given proportions, further it was poured into a funnel and let flow into a 10 cm high and 10 cm diameter cylinder so that the point of the funnel was just in touch with the soil surface. The density of the soil in the cylinder was expressed with the magnitude of the volumetric ratio of the solid phase:

$$s = \frac{m_d}{V \cdot \rho_s}$$

where m_d = dry mass

V = volume

ρ_s = density of particles.

Similarly to S_0 base entropy, the volumetric ratio of the solid phase of an imaginary system, in which the fractions are layered upon each other, can also be calculated, that will be:

$$s_0 = \frac{1}{\frac{x_1}{s_{01}} + \frac{x_2}{s_{02}} + \dots + \frac{x_F}{s_{0F}}} \quad (11)$$

where $s_{01}, s_{02}, \dots, s_{0F}$ are the volumetric ratios of solids in the loosest packing of the pure fractions.

Furthermore, the results were worked up in the following forms:

— the entropy quantities were in the form of

$$A = \frac{S_0 - S_{0\min}}{S_{0\max} - S_{0\min}},$$

where S_0 = the base entropy of the mixture

$S_{0\min}$ = the eigenentropy of the finest fraction

$S_{0\max}$ = the eigenentropy of the coarsest fraction in the mixture:

A may vary between 0 and 1 for any grain size range.

$A = 0$ means that only the finest fraction takes place in the "mixture",

$A = 1$ when we have only the coarsest fraction.

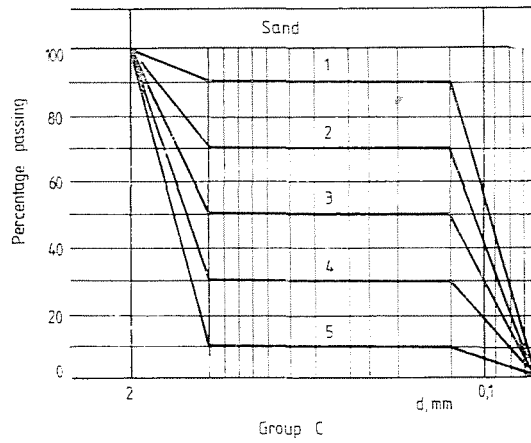
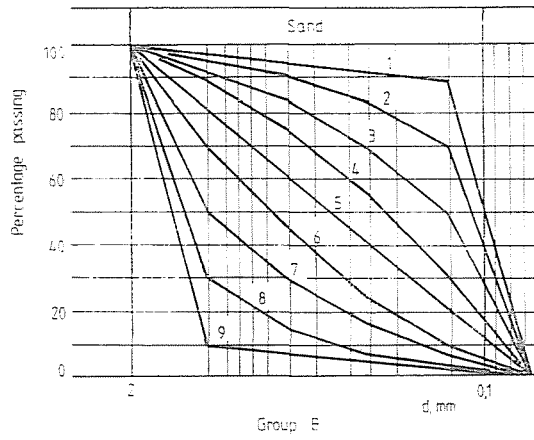
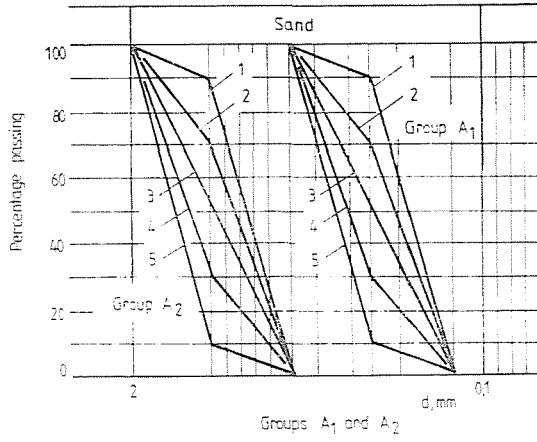


Fig. 2. Grading curves of some tested mixtures

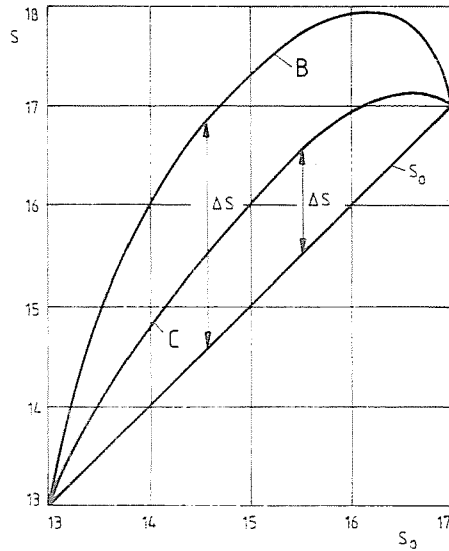


Fig. 3. Grading entropy quantities of mixture groups B and C vs S_0 base entropy

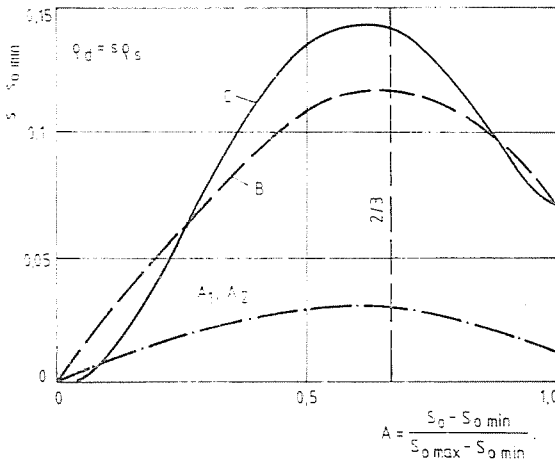


Fig. 4. Variation of density vs A

— the s volumetric ratios were taken with

$$s - s_{0min}$$

where s = the measured volumetric ratio of the solid phase in the test,
 s_{0min} = the volumetric ratio of solids of the finest fraction.

The A and $s - s_{0min}$ values are plotted in Fig. 4, for the mixtures of Fig. 2. In case of two neighbouring fractions — A_1 and A_2 — the density

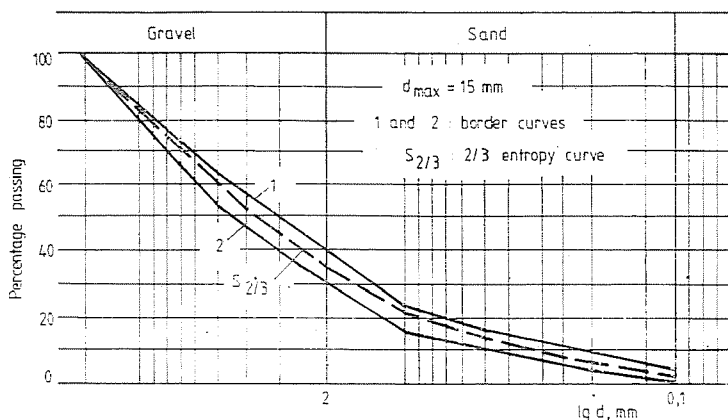


Fig. 5. Border curves of "good" concrete aggregates ($d_{\max} = 15$ mm) and the $S_{2/3}$ curve

increases only slightly with A , has a maximum, further decreases. When 5 fractions are involved — Group B —, the density increment is more significant. In Group C only the finest and the coarsest of 5 fractions were mixed with each other in different proportions.

Many other mixtures were tested and it was found that the maximum density situates around $A = 2/3$, that is when

$$S_0 = S_{0\min} + 2/3(S_{0\max} - S_{0\min}) \quad (12)$$

For two neighbouring fractions it means that maximum density mixture can be obtained with $2/3$ coarser and $1/3$ finer fraction. This mixture has the maximum entropy in the same time (see: Equ. (10)).

In case of more-than-two-component-, or gap-graded mixtures the maximum density and maximum grading entropy do not coincide, but maximum density can be found about $A = 2/3$.

KABAI [2] tested many soil mixtures from the point of view of compactibility. On the basis of his own results and taking into consideration FURNAS's [1] and KÉZDI's [4] results, he proved theoretically, that in case of $d_2/d_1 \sim 0$ — it means that a skeleton of solid particles is filled up with a liquid of the same density —, the densest packing can be obtained with $2/3$ "coarse" $1/3$ "fine" ratio.

$A = 2/3$ condition gives only the S_0 base entropy. The frequency of the fractions in this densest soil can be obtained with finding extremum of ΔS to the predetermined S_0 . The frequencies of the "two-third entropy" mixtures depend on the number of fractions.

An example is given in Fig. 5. where border curves of "good" concrete aggregate (max. grain size = 15 mm) are plotted together with the $2/3$ entropy curve of the same grain size range.

5. Summary

With the help of frequencies of soil fractions, the grading entropy of the soil can be calculated. The fractions have their eigenentropy (S_{0i}), since they consist of elementary statistical cells. In consequence of the "mixed condition" of the soils, there exists a ΔS entropy increment. The grading entropy of soils is a sum of the base entropy and the entropy increment.

The densest skeleton in the e_{\max} tests can be obtained with the so called $S_{2/3}$ entropy mixture, that is — only in case of two neighbouring fractions — the S_{\max} maximum entropy mixture in the same time.

$S_{2/3}$ entropy mixtures can be designed for any grain size range and locates well about the middle of the interval between the border curves of good concrete aggregates of the same diameter range.

References

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