DESCRIPTION OF $\sigma-\varepsilon$ DIAGRAMS OF BUILDING MATERIALS USING THIRD DEGREE SPLINES

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Abstract

The spline technique has a very important role in CAD systems.

Complicated curves are drawn using "aid points"; these points are corner points of a polygon.

Curves which are fitted between points using special boundary condition parameters are the splines.

The drawing of $\sigma-\varepsilon$ diagrams of building materials using third degree polynomials is described in this article. Corner points of polygons are usually important points of the failure mechanism (stress, strain or modulus of elasticity).

The base points of a spline, which are closest to the $\sigma-\varepsilon$ diagram determined from experiment, are regarded as material properties.

Finding the relation between the polygon (base) points and the production technology of material helps to develop a material with given stress and strain properties.

1. Introduction

The description of the behaviour of building materials under load is usually done by using σ-ε diagrams. Each material has its own significant shaped σ-ε diagram. Figure 1 shows the tendency of some σ-diagrams.
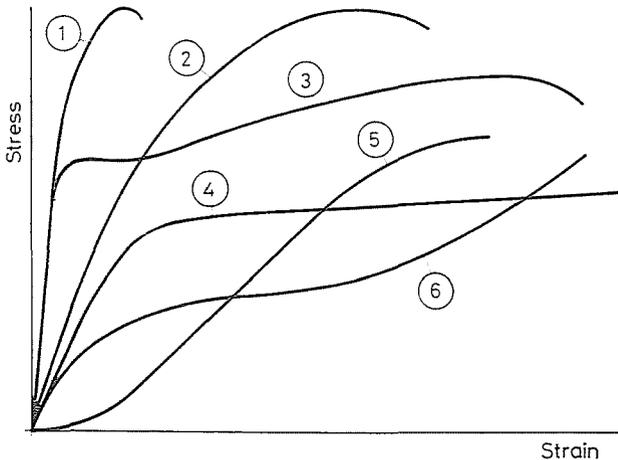
Fig. 1. $\sigma-\varepsilon$ Diagram. 1. High strength steel; 2. Concrete; 3. Mild steel; 4. Wood; 5. Stone; 6. Rubber

The appropriate description of σ-ε diagrams with theoretically supported mathematical functions is problematic even for only one type of material. Usually σ-ε diagrams are separated into phases with similar trends and each phase is approached by a function. The determination of parameters of such functions is complicated and tedious work because these functions have to be continuous and differentiable where the phases connect.

To find the physical relationship between function parameters and the tested material properties is even harder. For example, any third degree polynomial needs four parameters:

$$y = ax^3 + bx^2 + cx + d \tag{1}$$

Usually these parameters are concluded from test results using statistical methods.

It would be advisable to develop a general function (or procedure) to describe the σ-ε diagrams of each material which has a clearly defined relationship with the general materials properties.

The author of this paper suggests such a procedure. The recent development of computing, particularly the increasing size of computer capacity, has led to the presented solution.

2. The spline technique

The spline technique is connected to the development of interactive computer graphics and CAD systems. The two dimensional version is examined further.

Between the $x_{min} < x < x_{max}$ or $y_{min} < y < y_{max}$ limits a certain number of points are drawn. The points are connected with a straight line in any order and approached by a polynomial. (See Figure 2) The polynomial could be opened, closed or even looped. (See Figure 3).

The polygon is "smoothed" by approaching the joints with certain mathematical polynomials. The proper shapes of the curves are determined by the corner points of the polygon. These curves are called SPLINES.

The name spline has its origin in ship building. Planks attached to the ship frame on both ends created the shell of the boat. The stiff planks only had a limited curvature capacity.

There are several curve properties, mathematical descriptions and calculation procedures in the computer applications. The approaching curves should go through all of the given points also. These curves are drawn as Lagrange-type interpolation polynomials. There is also an arrangement when the spline touches each side of the polynomial. Another solution could be when splines only follow the corner points of the polynomial.
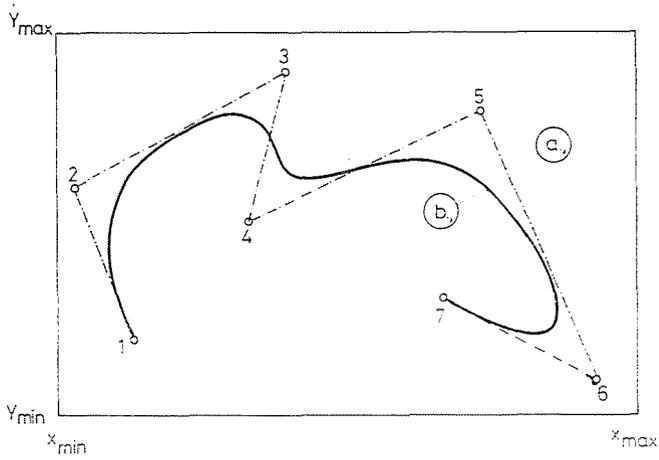
Fig. 2. Polygon and Spline
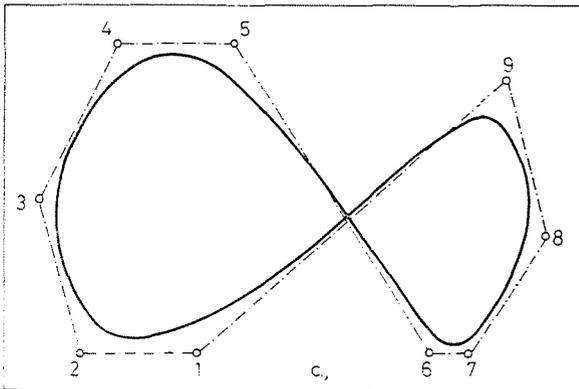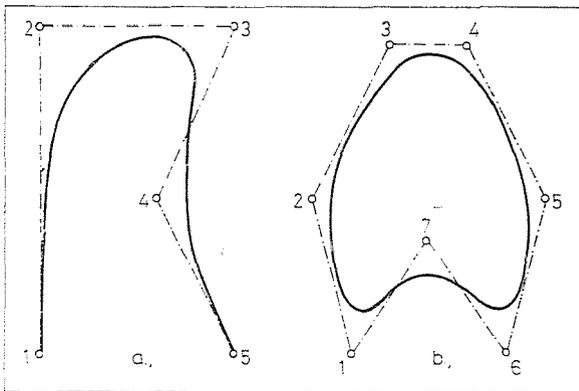
Fig. 3. Open, Closed and Loop Spline

Today there are several procedures known to determine the shape of the approaching polynomial. Beizer, the designer of Renault, used so called "weight functions" to determine the curves. He used his method for the cars chassis design.

In this paper the simplest approaching function is the third degree polynomial. Basic procedure and computer programme (BASIC) was developed by Weber [i] and applied by the author using CASIO fx 7000G graphic pocket calculator.

3. The principle of third degree splines

Only the approaching procedure is shown without the detailed verification of the theory (Figure 4).

A third degree polynomial approaches 4 points. Four points which are following each other are chosen. A polynomial can be fitted between the second and third points.

The $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$ coordinates and a u "aid function" helps to determine the x, y coordinates of each point of the polynomial. The u "aid function" satisfies the $0 > n > 1$ conditions between the second and third points.

The position of each of the points on the polynomial is influenced by the position of the fourth point because the weight of each point changes while the u "aid function" increases from 0 to 4 and while the curve between the second and third points is computed.

Each approaching procedure differs in the number of, and values of, the polynomial parameters.

The weight functions of the third degree polynomial:

$$D_{1(n)} = 1/6*(1-n)^3 \tag{2}$$
$$D_{2(n)} = 1/2*n^3 - n^2 + 4/6 \tag{3}$$
$$D_{3(n)} = -1/2*n^3 + 1/2*n + 1/2*n + 1/6 \tag{4}$$
$$D_{4(n)} = 1/6*n_3 \tag{5}$$

Any x coordinate of the polynomial is given by

$$x = D_{1(n)} x_1 + D_{2(n)} x_2 + D_{3(n)} x_3 + D_{4(n)} x_4. \tag{6}$$

The calculation of the y coordinate is similar to x. Repeating the calculations for the next polynomial point after cyclically increasing n.

If the coordinates of the next (fifth) point is also givne the curve between the third and fourth points are drawn using the previously introduced weight functions. (Point 5 is not the end of the polynomial). In this case the weight of the fifth point is $D_{4(n)}$ and all the others respectively.

If the polygon is closed, going around the points every curved part can be drawn using the u "aid function". If the polygon is open the end points need
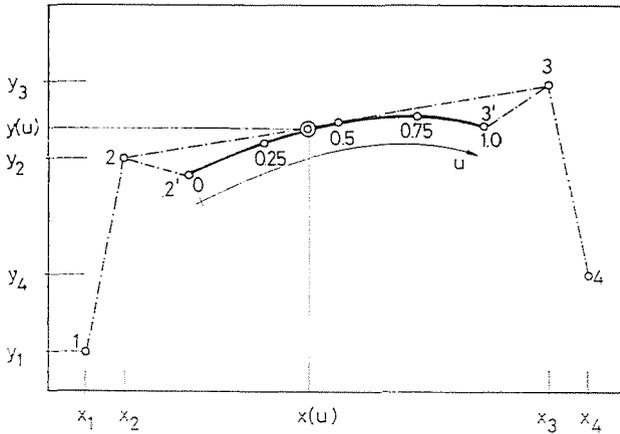
Fig. 4. Fitting Theory of Splines
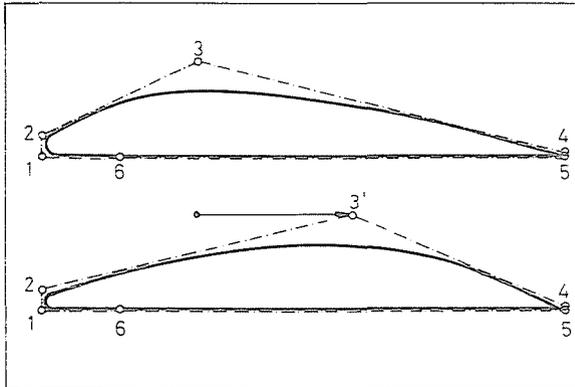
Fig. 5. Aircraft Wing Cross-section

different weights because the approaching polynomial goes through the end and also touches the polygon. (1 = 1'; 5 = 5')

The approximately chosen weight functions ensure that the following third degree polynomials have a common tangent at the given point, thus the approaching curve is continuous and differentiable.

The position of the given points determines the shape of the approaching polynomial. If one point is moved away it effects only the shape of the two neighbouring curves and a resultant curve remains continuous and differentiable.

These properties of splines are used during an interactive graphic design. Figure 5 shows an approaching polynomial which has an aircraft wing construc-

tion shape. If point 3 is moved away the characteristic of the wing changes and a so called laminar wing cross-section results. (These wing cross-sections have not been tested in wind channels yet.)

There are many fields where this spline technique could be useful. One application for drawing σ-diagrams is described next.

4. The points of the σ—ε diagrams

Every σ-ε diagram needs at least four given points. (See Figure 6). The first point is 1, where $x_1 = \varepsilon_1 = 0$ and $y_1 = \sigma_1 = 0$. The second, third and fourth points represent different physical quantities for each type of material. The fourth point is most frequently determined by the ultimate strength $(y_4 = \sigma_{max} = R_n)$ and the total strain $(x_4 = v_{total})$. The third point can be a fictitious quality when the initial tangent crosses the level of ultimate strength and gives the elastic deformation also. The real material would go through that point only if it was perfectly elastic and consistent with Hook's law. Point 2 could mean a proportional limit or a change in the state of material. (E.g. increasing solidity).

By changing the positions of these four given points curves with any shape can be drawn. These points influence the shape of the polygon and polynomial. Comparing the calculated and tested material qualities gives the distinctive parameters of the curves or gives the coordinates of basic points.
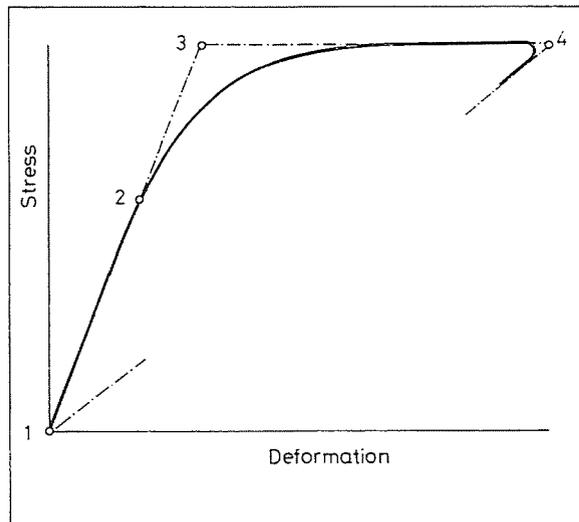
Fig. 6. σ—ε Diagram

From these few coordinates, most of which are tested qualities, it is appropriate to select those parameters which influence the σ-v diagrams of the materials or determine the production technology of any material with the required deflection properties.

The fact that the σ-ε diagrams of any material is drawn from 4—8 given points, using the same program for different materials, makes the procedure ideal for computer application. Of course, in special cases, the number of points can be increased to desribe more complicated behaviours.

5. σ—ε Diagrams on a CASIO fx 7000G graphics pocket calculator

A computer program for third degree polynomials, originally written for IBM PC's ,was applied to run on a CASIO fx 7000G. This calculator has a screen using 95 × 65 points and this allows reasonable accuracy for drawing polynomials. The memory capacity of the calculator is modestonly 422 bytes. The maximum 26 data are stored. The program needs 398 bytes and it leaves no free memory.

Only with the shortest version, that is, the routine of the closed polygon, was there enough memory to handle the maximum of 9 corner points of the polygon.

It was also possible to draw open polygons of up to 4 points when the coordinates of the starting and finishing points were doubled in the memory and the so called "jump routine" enabled the drawing of open polynomials
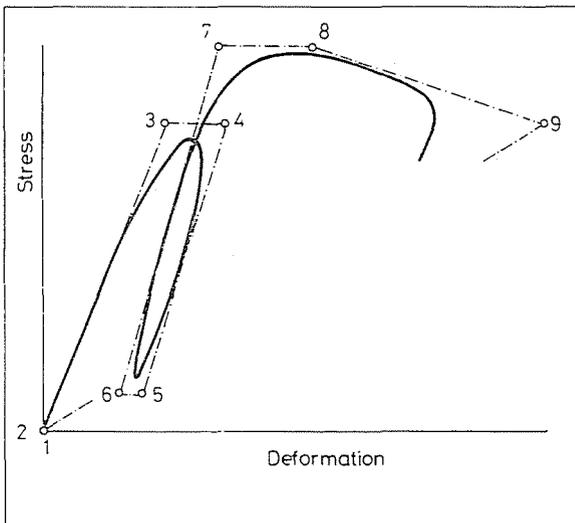
Fig. 7. σ—ε Diagram of a Fully Loaded / Unloaded Cycle

also. Another solution for drawing open polynomials was to double every point in the memory and the drawn closed polynomial gave the open curve.

A full loading cycle (of concrete) is shown on Figure 7 using all of the possible 9 polygon points. One of the built-in utilities of the calculator makes it possible to use unlimited coordinates. A numeric and graphic input for the coordinates of the polygon points was designed. In the graphic input the cursor was moved to the positions of the polygon points and fixed in the memory.

A separate routine draws the polygon and the polynomial. In another routine the chosen points are selected and the repositioning is done by the graphic routine. The unnecessary functions are deleted but the data on the polygon points are stored. Variations of curves are drawn on top of each other. The calculator stores data even after it is switched off (e.g. polygon points, scale data and the last graphic on the screen).

6. Examples of applications

Examples indicate some of the applications of the introduced spline technique.

Figure 5 shows the drawing of some streamlined bodies.

Figure 8 shows some σ-ε diagrams where the limit of elastic behaviour changes from 0, 32, 64 and 96% of the ultimate strength. The limit where the change in elastic behaviour occurs can be easily estimated during a test using Figure 8.
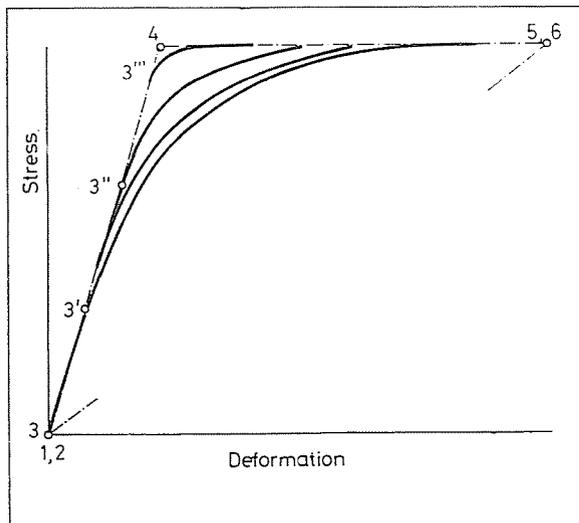
Fig. 8. A Series of $\sigma-\varepsilon$ Diagrams
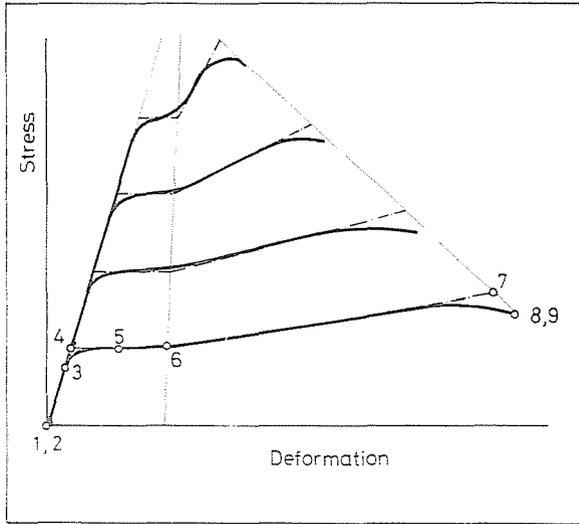
Fig. 9. σ—ε Diagrams of Mild Steels
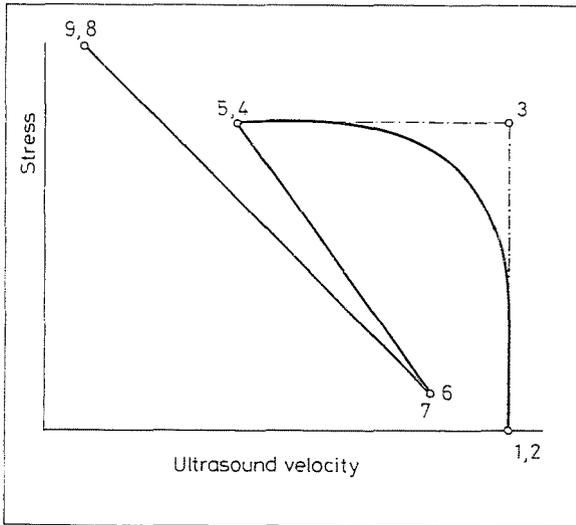
Fig. 10. Testing Concrete Failure with Ultra-Sound

The tensile strength test of some mild steels with different yield and ultimate stress limits are shown on Figure 9. The shape of these diagrams depends on the manufacturing technology. Using the spline technique to find the relationship between the shape of the curves and the manufacturing procedure could be a useful and interesting task.

The failure of concrete detected by ultrasound testing is shown on Figure 10 using the spline technique. When the load is high enough the concrete cracks, thus the velocity of ultrasound drops. The result of a full load cycle is shown on the figure drawn by a CASIO fx 7000G calculator.

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