ESTIMATING THE RELAXATION OF PRESTRESSING TENDONS

A. Erdélyi

Department of Building Materials
Technical University, H-1521, Budapest

Received March 30, 1989
Presented by Prof. Dr. Gy. Balázs

Abstract

The pure relaxation of stress relieved and stabilized (R1 and R2 class) wires was measured at ambient temperature and at different usual stress levels $f = \sigma_0/\mathrm{UTS}$ for 5 to abt. 20 thousand hours. In some cases the effect of extreme f levels (0.55 and 0.95) was also studied. The longer durations were considered to be sufficient to find the constants of three-parameters extrapolating functions. Other test supported the idea that the "final" (ie. practically for 10^6h, abt 114 years) relaxation percentages $r\%$ (compared to the initial prestress σ_0) fit fairly well to straights of the shape $r_{114} = Af + B$, where A and B depend from the type of tendon (R1 or R2, class of normal or low relaxation, resp). For standardization purposes an exponential function of two parameters was chosen to render the well-known S-shaped curve in log t scale. This function was to cross two points and tends to r_{114} in infinite time. The two points are: the specified max. relaxation at 1000 h (standards) *and* the arbitrarily but realistic chosen relaxation of 10^5 h (abt 57 years) $r_{57} \cong 0.95 \cdot r_{114}$.

The values r_{57} and r_{114} were compared with the prescriptions of German (FRG) so called suitability certifications. Hence one can conclude that the $r = Af + B$ straight may be a simplification for some types of tendon in class $R2$. The two-parameter function seems to be adequate if the time-thickening behaviour of the steel (ie. strainhardening due to segregations) is taken into account by a time variable t^c rather than t where the exponent $c < 1$ makes the function to follow the retarded relaxation phenomenon, typical for some low stress levels and steel products. The functions and the statements concerning pure relaxation and the "final" remaining effective stresses are of course valid only within the limits of maximum stresses allowed by the CEB/FIP Model Code.

1. Introduction

The initial stress (σ_0) of high tensile strength wires and strands used for pre- and post-tensioning of concrete structures decreases as time goes by. This decrease or stress loss is expressed as σ_{rel} (MPa) or as the ratio of stress loss and the initial stress ($r\% = 100 \cdot \sigma_{\mathrm{rel}}/\sigma_0$).

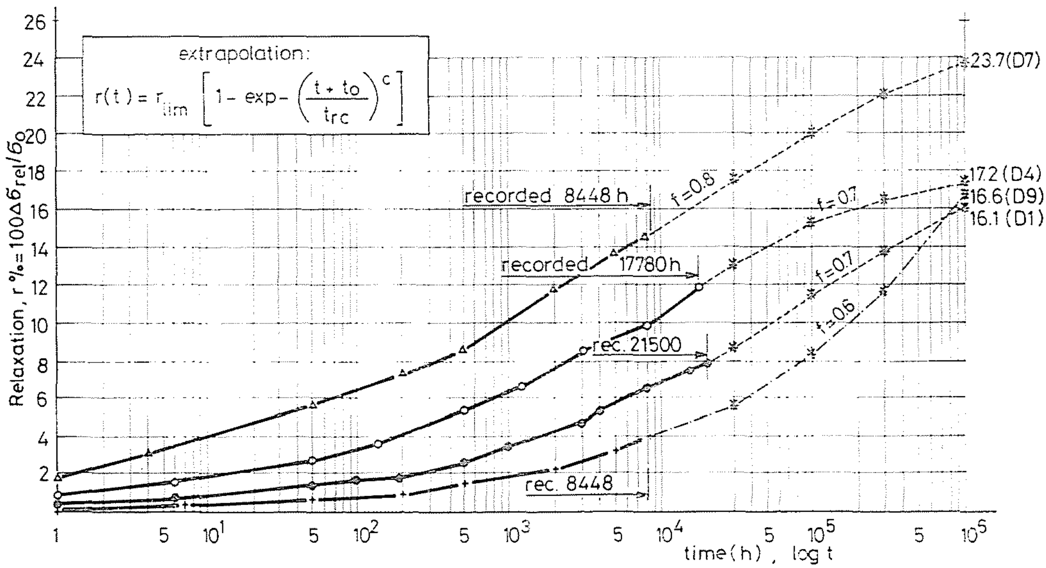
Designers are usually interested in the stress loss after a certain length of time — usually some hundred or thousand hours (41 days for transport and classing) and also at the "final stage" which is 30, 50 or even 100 years after the initial stressing. It is usually acceptable to estimate stress loss after $5 \cdot 10^5$ hours (approximately 57 years) and after 10^6 hours (approximately 114 years). The manufacturers of such high tensile strength wires usually supply the data or curves of stress loss after a few thousand hours but there is usually no data available for the "final value".

Sophisticated material research is expected to supply reliable stress loss data for standards. In the last 15 years the prescribed design values of relaxation loss $r\%$ increased according to several standards. After opening up old pre-stressed structures there were higher losses found than was estimated previously. The total loss — including loss from creep and shrinkage of concrete — of stress was sometimes even 50% of the initial stress [1].

This paper discusses how relaxation data for 5—10 thousand hours were analysed and extrapolated to gain 50—100 years relaxation values data for design purposes. Firstly, the effects of the initial stress level ($f = \sigma_0/R_{pt}$ or $f' = \sigma_0/R_{p\,0,1}$) on the relaxation procedure was examined. (R_{pt} — ultimate strength, $R_{p\,0,1}$ — proof stress 0.1%.)

2. Test results

Any extrapolated function is only reliable when the time interval of observing the relaxation process is long enough to obtain a true reflection of what is happening. The longest tests in Hungary were carried out in the Institution for Quality Controll in the Construction Industry-EMI (Dr. S. Veress — Budapest, S. Harangi — Debrecen) and in the Iron Industry Research Insti-



Captions

Fig. 1. The measured and extrapolated relaxation of 5 mm diameter, round D4D (Hungarian) stress relieved (*R1* class) wires. (Tests carried out by EMI, Budapest)

tution—VASKUT (S. Takács) under a contract with the Building Materials Department, T. U.B.

According to the CEB/FIP manual and the EURONORM 139 R1 class normal relaxation, stress releived, Hungarian made 5mm diameter round wires were used which were made by the D4D Wire Factory, Miskolc. Figure 1 shows the measured relaxation curves and the extrapolated ones. The initial stress levels ($f = \sigma_0/R_{pt}$) were 0.6—0.7—0.8. On the level of 0.7 stress ratio a higher strength wire (higher Mn and Si content) and a lower strength wire were used.

The effects of different initial stress levels can be seen even better on Figure 2 the testing of R1 class (stress reveiled) wires, the central wires of a 7 wire strand made in the D4D Wire Factory. The relaxation diagram of wires after 3 thousand hours observation have a different shape and thus the extrapolability is different too.

5000 hours tests on R2 class, low relaxation straight strands (core wires) are on Figure 3. These test results are presented in the pamphlet of D4D Wire Factory on R2 class strands. The shapes of these curves are different to those which were measured on R1 class wires. The curves of stabilized wires show low relaxation and strong curvature.

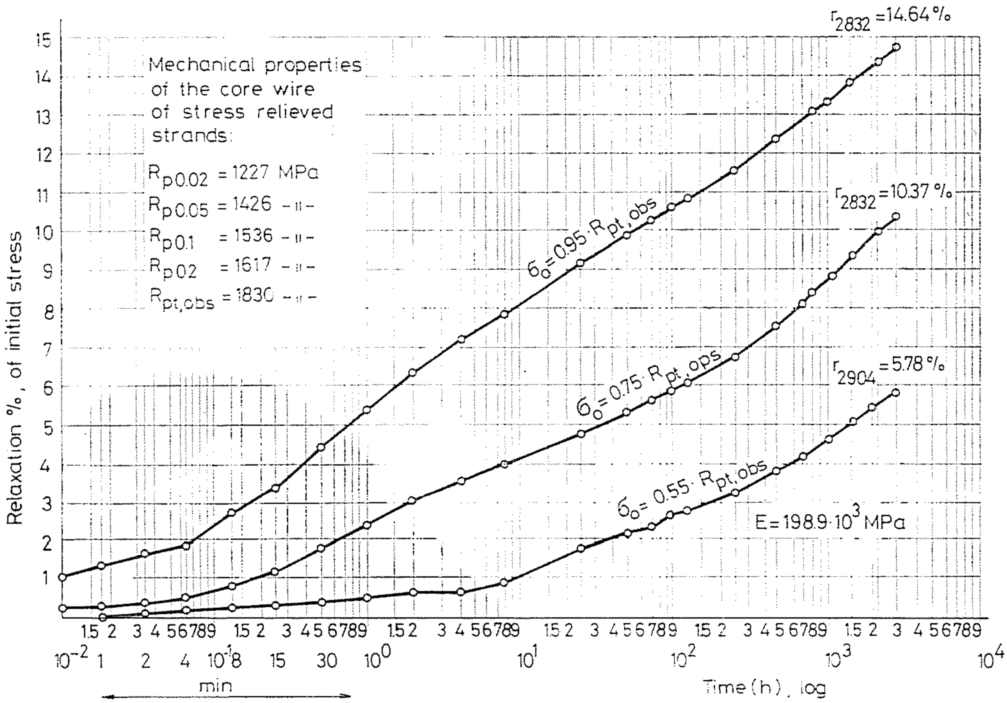


Fig. 2. The measured relaxation of a 4.4 mm diameter central wire from stress relieved (*R1* class) strands according to different initial σ_0 stresses. (Tests carried out by EMI, Debrecen)

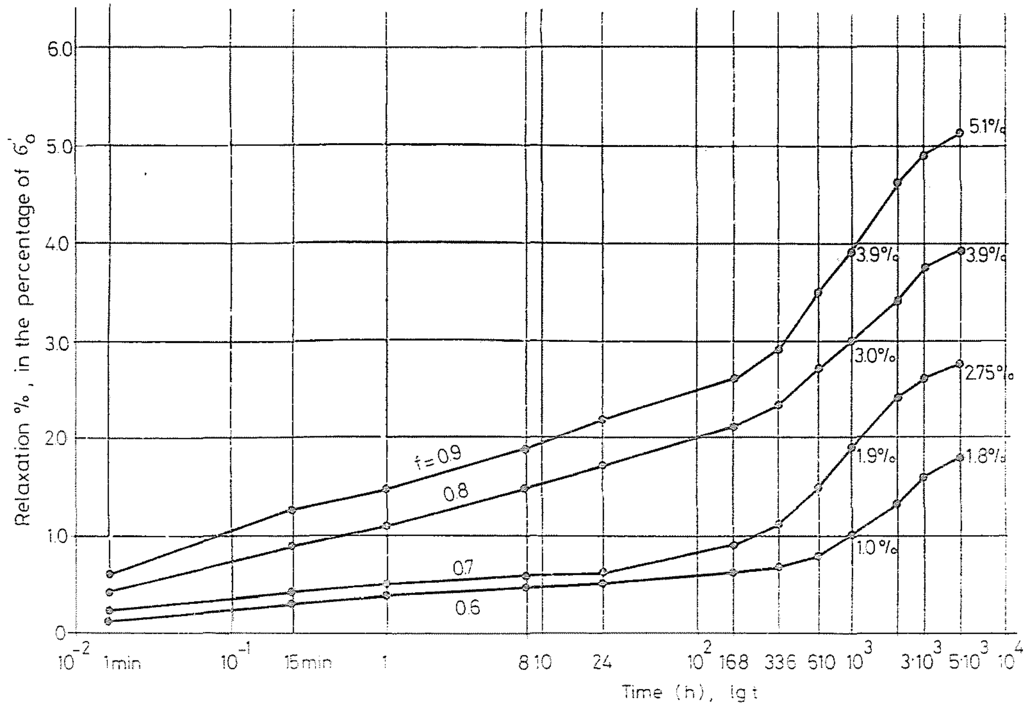


Fig. 3. The measured relaxation of a 4.4 mm diameter central wire from stabilized (*R2* class) strands. (Tests carried out by IRI-Vaskút, Budapest)

3. The extrapolation

3.1 *The initial model*

The creep and relaxation diagrammes with a log scale usually have an "*S*" or a shorter or longer part of an "*S*" shaped curve.

It is easier to describe this symptom with the phenomenon of creep caused by the constant stress tha nwith the self slowing relaxation. Creep curves on a constant stress level (σ_0 = constant) close to the ultimate stress are already producing almost the full "*S*" to be studied after 1000 hours of testing. Under a lower stress level only the first part of the "*S*" curve is drawn ($\sigma_0 = 1100$—1500 MPa), see Figure 4. [3]. When the initial stress is higher the "*S*" curve is almost completed. The creep curves of wires which are as cold drawn show, on a $\sigma_0 = 1650$ MPa initial stress level, that creep corresponding to the first part of the "*S*" curve occurs already during the loading period thus this creep is undetectable.

The test curves with different initial stress levels measured between 1 and 3000 hours are chosen and demonstrated on Figure 2. Firstly, the inflection

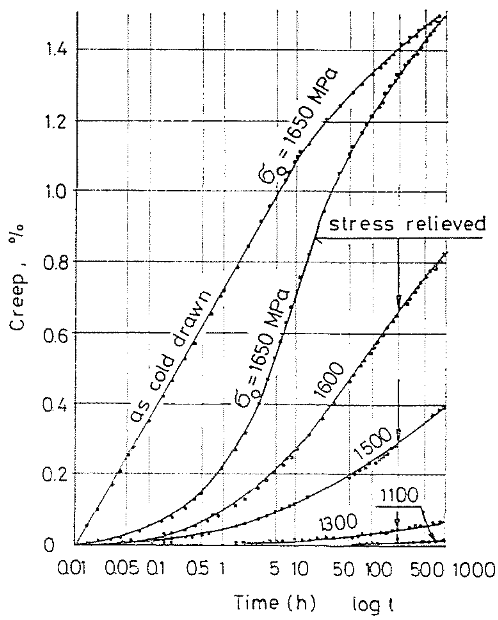


Fig. 4. The "S" shaped creep curves of wires manufactured with different technologies. [3]

part of the curve (when $f = 0.95$) can be measured and the upwards curving section only with $f = 0.55$. At first it was straight then curved upwards and the second (inflection) part was observed in the first 5000 hours with low relaxation (R2 stabilised) wires. It is assumed that for our time dependent process (in this case the relaxation) the observed quality (in this case σ_{rel}) approaches a limit value and this value is always lower than the original (in this case σ_0) thus in our case it means that the major part of the initial stress stays in the wire after infinite time. It is already stated here that an estimating function suitable for a 50—100 years period can possibly give an extremely high stress loss after an infinite time interval. (e.g. $\sigma_{\text{rel, lim}} \geqq \sigma_0$).

The Thompson-Pointing 3 parameters model or the Standard Linear Solid (SLS) time dependent model, which contains two parallel, linear (Hooke) springs and a Maxwell unit (a spring plus a piston) : H parallel to M (see Figure 5) give an adequately correct description of creep and relaxation of prestressing wires.

This model is able to produce both temporary strain and creep or relaxation. As creep disappears slowly after unloading and there is no permanent strain this model is not entirely realistic.

If an effective force p_0 at time $t = 0$ in the system causes strain then because the piston does not move at $t = 0$ moment

$$p_0 = (\mu_1 + \mu_2) \cdot \varepsilon_0 = p_1 + p_2 \tag{1}$$

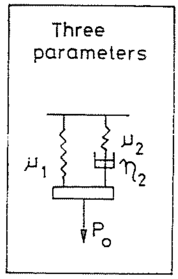


Fig. 5. The 3 parameters Thompson-Pointing model which is able to demonstrate creep and relaxation

If this is held constant then a constant force $p_1 = \mu_1\,\varepsilon_0$ remains in line 1 (there is no relaxation) and in the second (Maxwell) line after an infinite time the total force $p_2 = \mu_2\,\varepsilon_0$ is relaxed by the displacement of the piston which is caused by the continuously decreasing force in the spring. According to the known equation, at any time t,

$$p_{2t} = \mu_2\varepsilon_0\left[\exp\left(-\frac{t}{\eta_2/\mu_2}\right)\right] \tag{2}$$

and after infinite time $t = \infty$, $p_{2t} = 0$.
The expression $\eta_2/\mu_2 = t_r$ is the quotient of viscosity and the modulus of elasticity has a time dimension. It is called "relaxation time". After $t = t_r$ time of the start of relaxation, then

$$p_{2t} = \mu_2\varepsilon_0\cdot \mathrm{e}^{-1} \tag{3}$$

Thus during this time, 63% of the force, corresponding to the p_2 force in the Maxwell unit, has already relaxed and ($1/\mathrm{e} = 0.37$) only 37% of this force remains. LAZAN regards the relaxation time t_r as an "internal time unit" of the system [5]. If the length of tests (t_t) are marginally shorter than t_r then only a very minor part of this phenomenon was measured. This is usually the case of pre-stressing steels. If t_r is very short compared to the testing time then the questioned phenomenon has already played down well before the observation starts (eg. the cases of some plastics).
Thus stress (force) in the SLS model after time t is

$$p_t = p_{1t} + p_{2t} = \mu_1\varepsilon_0 + \mu_2\varepsilon_0[\exp(-t/t_r)] \tag{4}$$

or rearranging it and adding $+$ and $-$ $\mu_2\varepsilon_0$:

$$p_t = (\mu_1 + \mu_2)\varepsilon_0 - \mu_2\varepsilon_0[1-\exp\,(-t/t_r)] \tag{5}$$

The remaining stress after an infinite time is

$$p_{t_\infty} = \mu_1\varepsilon_0 \tag{6}$$

The stress loss according to (1) is

$$\Delta p_t = p_0 - p_t = \mu_2 \varepsilon_0 [1 - \exp(-t/t_t)] \tag{7}$$

and the loss compared to the initial p_0 is $r\% = 100 \cdot p_t/p_0$ which can be expressed as

$$r = 100 \cdot \frac{\mu_2 \varepsilon_0}{(\mu_1 + \mu_2)\varepsilon_0} [1 - \exp(-t/t_r)] \tag{8}$$

$$r_r(\%) = \frac{100\mu_2}{\mu_1 + \mu_2}\left[1 - e^{-\frac{t}{t_r}}\right] = r_{\lim}\left[1 - e^{-\frac{t}{t_r}}\right] \tag{9}$$

It is concluded that according to (1)—(9) valid for SLS models:

— There is no such limit of initial stress which would not be followed by relaxation, thus after any $\varepsilon_0 \neq 0$ initial strain causes relaxation.

—Every different ε_{i0} initial strain (also σ_{i0} initial stress) induces different $p_{t_\infty} = \mu_1 \varepsilon_{0i}$ final remaining stress, thus the supposition by STÜSSI is not correct. (This claims that from any practical σ_{i0} initial stress the final remaining σ_t stress is always the same [6].)

The function, exponentially approaching the relaxation limit $r_{\lim}\%$ describes (with a logarithmic time scale) an "S" shaped curve. It may be appropriate to extrapolate the measured data sequences. It is possible to find the two unknowns ($r_{\lim}$ and t_r) in equation (9) from the most perfectly fitting function.

3.2 *The fitted functions*

Functions consistent with equation (9) were fitted to long data sequences. It was found that they fitted *well* on the data sequences only when the time thickening of steel (caused by long time stretching) and the increased time-hardening (from increased segregation) were also considered. Researchers usually estimate the continuously increasing viscosity, but in this case a "time slowing" factor was introduced — a $c < 1.0$ exponent and t/t_{rc} instead of t/t_r and thus the function fitted marginally better. This modification is necessary. The t_{rc} means that this is not the classic t_r relaxation time (see above) because there is a $(t_r)^c$ used instead of it.

To improve the fitting of functions on the points of short term measurements the starting (time) point was shifted by a time t_0 and thus the non-measurable relaxation while taking the initial load could be considered and resulted in the next function (EXP4) with 4 parameters:

$$r_t = r_{\lim}[1 - \exp - \{(t + t_0)/t_{rc}\}^c] \tag{10}$$

To estimate the final limit value $t_0 = 0$ gives a perfect result and the result is an exponential function (EXP 3) with three parameters ($c < 1.0$).

$$r_t = r_{\lim}[1 - \exp - (t/t_{rc})^c] \tag{11}$$

The data of EXP4 functions fitted on longer test results are listed in Table 1.

Table 1

Parameters of long data sequences and fitting functions

Parameters	(Wire)			
	D-9	D-1	D-4	D-7
test time (h)	8448	21504	17748	8448
$f = \sigma_0/R_{pt}$	0.6	0.7	0.7	0.8
$r_{1000h}\%$ (tested)	1.5	3.3	6.0	9.5
r_{1y} (1 year)% (tested)	4.0	6.6	~10.5	14.5
$r_{57y}\%$ (estimated)	13.5	14.9	16.8	22.8
$r_{114}\%$ (estimated)	16.6	16.1	17.2	23.7
r_{57y}/r_{1000h}	9	4.5	2.8	2.4
r_{114y}/r_{1000h}	32.5 (!)	4.9	2.9	2.5
$r_{lim}\%$	48.9 (!)	18.3	17.4	25.3
t_0 (h) time gap	4.2	0.5	~0	−0.5
c slowing factor	0.3534	0.3381	0.3215	0.245
$t_{rc} = (t_r)^c$ "relaxation time"	$1.24 \cdot 10^7$	$1.08 \cdot 10^5$	$1.13 \cdot 10^4$	$1.62 \cdot 10^4$

The CEB/FIP Model Code suggests estimating the long term relaxation as the 1000 hours rate multiplied by a factor of 3. Tests using good quality wires indicate a ratio of (r_{57y}/r_{1000h}) 2.4 — 2.8 and supports this suggestion.

It was concluded from Table 1 and Figure 1 that in the case of different $f = \sigma_0/R_{pt}$, the functions fitted on test data of wires had not only different relaxation limit, r_{lim}, but they had also very different relaxation time, t_{rc}, and the difference in relaxation times is marginal.

This also means in the case of a different f, even identical testing times are not equivalent. When the initial stress is low (or similar to this, when using very good, low relaxation wires) the 10.000 hours test gives an obviously wrong final relaxation limit (here: $r_{lim} = 48.9\%$) because the testing time was marginally shorter than the t_{rc} relaxation time of the tested steel.

In spite of this obviously wrong limit after 10.000 hours, the estimated relaxation after 57 years is not unfounded; the estimated stress loss when $f = 0.6$ and 0.8 were respectively 13.5% and 22.8%. Comparing tests (See Fig. 1) D-9 and D1 it is clear that the obtained results after 114 years are impossible because a lower initial stress must have a lower relaxation also.

These estimations of relaxations are only real if the testing time is almost equivalent to the internal timing t_{rc} (or t_r) of the material.

4. Practical extrapolation

4.1 *Formulas for estimating the final relaxation value*

Similar to the examples which were discussed in Chapter 3, several foreign and even some Hungarian long term test data sequences were analysed and extrapolated. The acceptably estimated relaxations after 114 years ($r_{114y}\%$ as final design values) in the function of $f = \sigma_0/R_{pt}$ were collected. Our results were compared to those prescribed in foreign codes and suggested design values for relaxation of a given type of wire. (These design values are like the "suitability certifications — Zulassung" in FRG).

One can conclude from curves presented in Figure 3 that relaxation loss is a linear function of $f = \sigma_0/R_{pt}$ and $\sigma_0/R_{p0,02}$ and thus it is possible to estimate the final relaxation value using an $r_{114y}\% = A.f - B$ type function, which can substitute the previous fittings of individual fuctions (see Figure 6). The factor B in the equation means that under a practical limit there is no relaxation — contradicting the theoretical SLS model.

The relaxation loss of stabilized R2 class, or generally, of wires and strands with low relaxation, is estimated according to the linearity of Figure 6 and some other sources in the literature such as

$$(\mathrm{R2})\ r_{114y}\% = 50 \cdot f - 25, \tag{12}$$

where $f_0 = 25/50 = 0.5$ is the zero-relaxation limit.

According to Figure 7 the suggestion in (12) is acceptable but the unsufficient duration of 5000 hours of our test also has to be considered. It was questioned whether to accept the lower limit $f_0 = 0.5$ according to Figure 6 because at this stress level there was still relaxation but without this it would not have been possible to obtain a "calibrated" straight line around the $f =$ $= 0.75$ range. For a higher degree "final value function" additional measured data would be necessary.

Using a similar procedure but comparing the theory with much more experimental data, to find the relaxation loss of wires and strands with normal relaxation the following equation is suggested:

$$(\mathrm{R1})\ r_{114y}\% = 70.f - 28, \tag{13}$$

where $f_0 = 28/70 = 0.4$ is the supposed zero-relaxation limit.

After accepting equations (12) and (13) the remaining stress after 10^6h $= 114$ years is easily given as

$$\sigma_{114y}/R_{pt} = \frac{\sigma_0}{R_{pt}} \cdot \frac{\sigma_{114y}}{\sigma_0} = f \cdot \frac{100 - r_{114y}\%}{100} \tag{14}$$

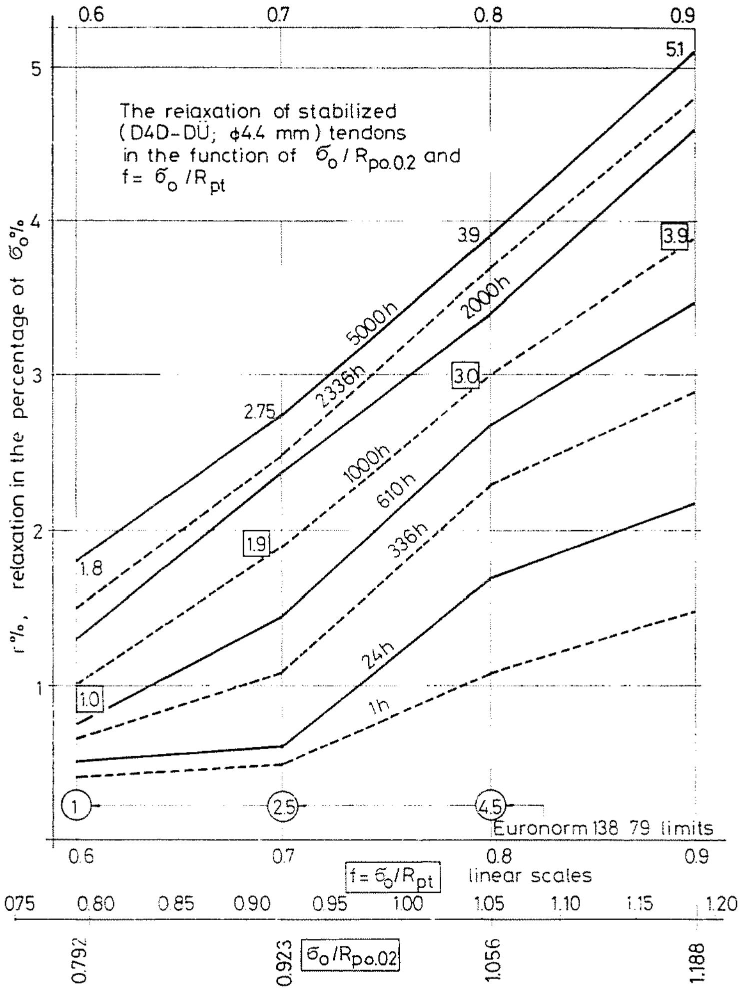


Fig. 6. The relaxation of stabilized (*R2* class) core wires vs. the initial tensioning ratio

where here equations (12) or (13) should replace $r_{114y}\%$. The peaks of the obtained second degree parabolas (see Figure 8) are not consistent with those valid codes which do not assume initial stress higher than $\sigma = R_{pt}/1.33 = 0.75\ R_{pt}$ because there are very few test results where f was higher than 0.8 thus over $f = 0.75(0.80)$ the linear (12) and (13) equations cannot be proven.

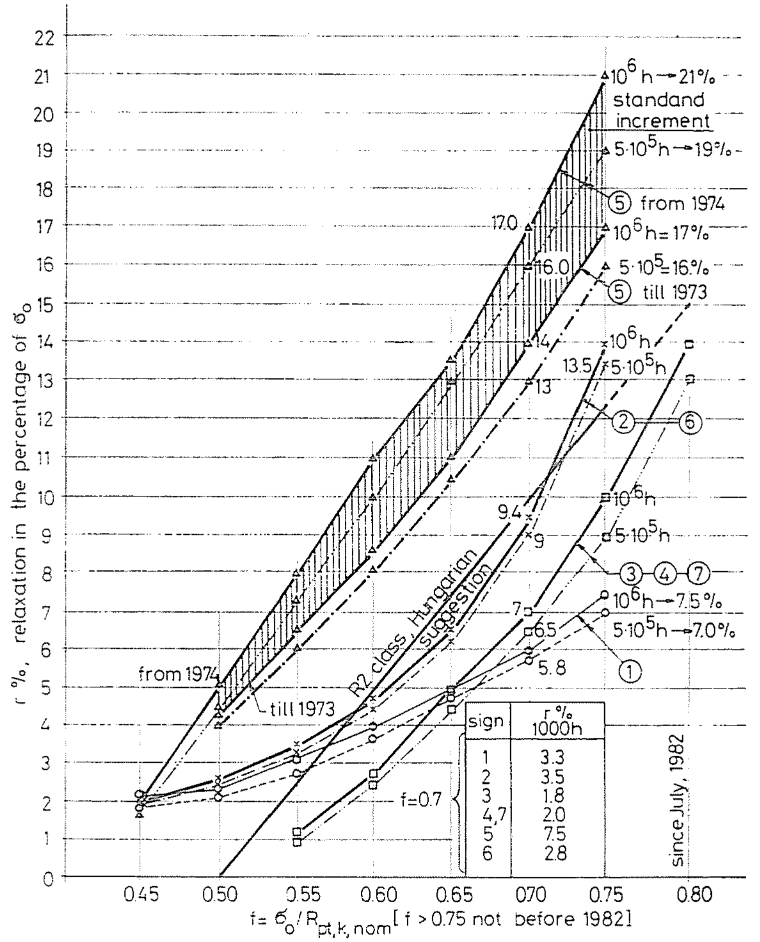


Fig. 7. Data of suitability certifications ("Zulassung") from FRG, with "design values" for calculated relaxation of different prestressing wires and the suggested Hungarian function for *R2* tendons

[1] Hot rolled, cold streched and stress reveiled St 835/1030 (KRUPP) diameter 26... 36 mm bar, FR Germany.

[2] LR(R2) class St 160/180, 7 wire strand (52 ... 140 mm^2), FAGERSTA, SWEDEN.

[3] Plain, round quenched and tempered alloy steel bar (KRUPP), dia 6—14 mm, St 1420—1570.

[4] VLR very low relaxation 7 wire strand St 160.180 ARBED—F + G (Cologne), dia 9.3—15.3 mm (52 to 140 mm^2), FRG.

[5] Normal relaxation (class R1), 7 wire strand St 160/180 ARBED—F + G (Cologne), FRG

[6] R2 class, 7 wire strand, dia. 9.3 to 15.3 mm, WDI, (Hamm), FRG.

[7] St 1570/1770, 7 wire strand *R2* class, KLOCKNER DRAHT GmbH, (Hamm), FRG.

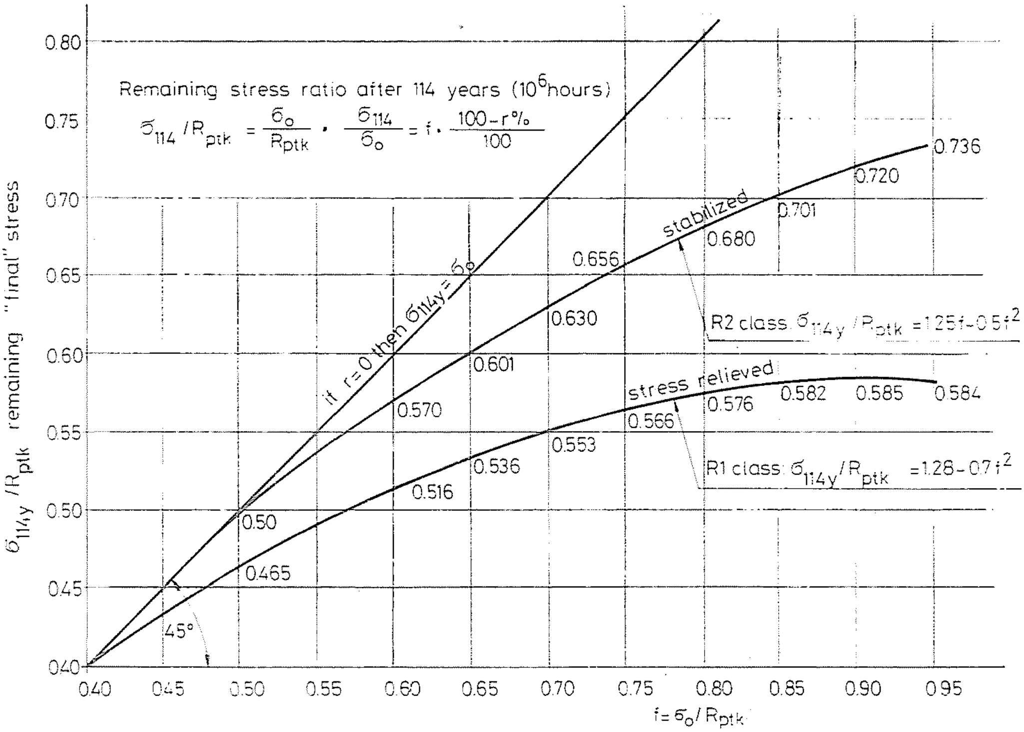


Fig. 8. The remaining final stresses calculated from equations (12) (13) and (14) respectively

4.2 *Further modifications for design purposes*

Relaxation is always measured on a steel which was previously tensioned with a certain percentage of stress of actual real tensile strength (for example $f = 0.75$ or 75%). Strands in the real structures are naturally tensioned according to their nominal tensile strength ($P_{pt,k,\mathrm{nom}}$). Thus the effective, average tension level is lower than the level which was designed. The ratio of $k = f_{\mathrm{eff}}/f_{\mathrm{des}}$ is the same as the ratio of 5% characteristic strength $R_{pt,k,\mathrm{nom}}$ and average strength, that is $k = R_{pt,k,\mathrm{nom}}/R$, where $R = R_{pt,k,\mathrm{nom}} + 1\cdot645$ sand "s" is the standard deviation.

According to (12) and (13) the final relaxation should be calculated from $f_{\mathrm{eff}} = k.f_{\mathrm{des}}$ instead of f_{des}, where $k < 1$.

If — to be on the safe side—we suppose, that:

— the effective strength is equivalent only with the characteristic strength ($f_{\mathrm{eff}} = f_{\mathrm{des}}$).

— the relaxation after the first 1000 hours corresponds to the upper limit of relaxation given in the code (allowed maximum).

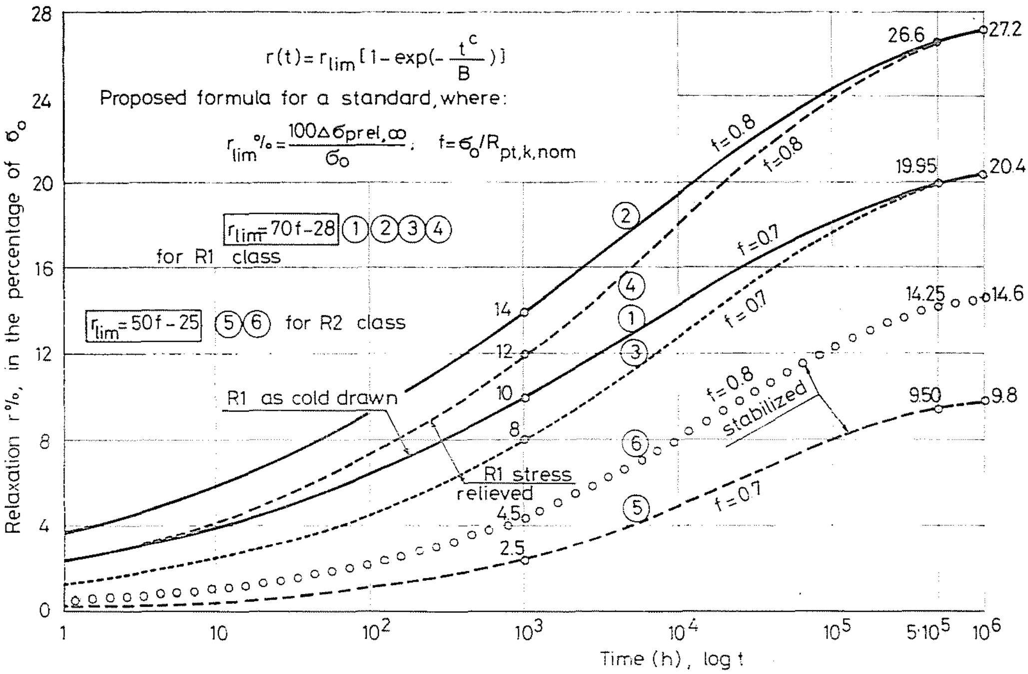


Fig. 9. 2-parameter functions tending to give final values ($t = \infty$) for estimating relaxation design values for different stress levels and tendons. ($R_{pt,k,\text{nom}} = R_{\text{char},5\%}$)

— instead of $r_{\lim}$ — the final value — the calculated relaxation after 114 years is assumed from (12) and (13).

— after 57 years ($5 \cdot 10^5$ hours) 95% of the total stress loss had already taken place then the curves given in Figure 9 are applicable. The parameters (c and t_{rc}) are determined for each curve so as to correspond to $r_{1000h}\%$, given in the codes and to $t = 5 \cdot 10^5$ and $r = 0.95 \cdot r_{\lim} = 0.95 \cdot r_{114y}$ points [7].

These curves indicate the highest possible relaxation. If similar curves are drawn on each possible f level then any value of the f_{eff} (not equal to f_{des}) can be considered.

It is also possible to start the curve from the factory given average relaxation data after 1000 hours, in this case using the f_{eff} instead of f_{des}, the $r_{\lim}$ also changes (decreases) and the result is going to be an average relaxation function.

Acknowledgements

The essential contributions of Dr. A. Tóth (Ass. Prof., Physics Department, TUB) and A. Kápolnai (Senior Research Officer, Computing Centre, TUB) concerning mathematical background and computer techniques during the research period is acknowledged. Furthermore the author is grateful to those researchers from the ÉMI and from the Iron Research Institute who were mentioned previously for helping with the project, as well as to Mr Z. Szombathy, Res. Officer at our Department.

References

1. PAPP L., BODÓ L., BALOGH T.: Előfeszített szerkezetek alakváltozásának meghatározása, ÉTI 1908/1982 témaszámú jelentés, Szentendre.
2. ENGBERG, E.—WALLIN, L.: Long time creep relaxation tests on high-tensille steel presstressing wires, Nordisk Betong, 1966. *3.* sz. p. 231—236.
3. PAPSDORF, W.—SCHWIER, F.: Kriechen und Spannungsverlust bei Strahldraht, insbesondere bei leicht erhöhten temperaturen, Stahl und Eisen, 1958. jul. No. 14. p. 937—947.
4. ERDÉLYI, A.: Losses on stress due to steam curing. FIP Eighth Congress, London, Proceedings Part 2 (1978. May 2.) p. 35—45 ISBN 0 7210 1102 0 Cement and Concrete Association, Wexham Springs.
5. LAZAN, B. J.: Stress—Strain-Time relationship for idealized materials, ASTM, STP No. 325, 1962. p. 3.
6. STÜSSI, F.: Zur Relaxation von Stahldraehten, Memoires (Abhandlungen) Publications AIPC/IVBH/IABSE 19. kötet, 1959. p. 273—286, Zürich, 1959.
7. ERDÉLYI, A.: Feszítőbetétek relaxációjának számítási értékei. Kandidátusi disszertáció Budapest, 1984. (Design values of tendon relaxation, C. Sc. Dissertation, in Hugarian)

Dr. Attila ERDÉLYI H-1521, Budapest