THE WATER-DEMAND AND GAP-VOLUME OF AGGREGATE FOR FERROCEMENT

NGUYEN HUU THANH

Department of Building Materials
Technical University, H-1521, Budapest

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Abstract

In designing concrete based on paste saturation we should know the water-demand of cement, aggregate and the Gap-volume of aggregate mixture. The water-demand assures the consistency of fresh concrete and the cement-setting. The paste saturated concrete can be designed if the Gap-volume of aggregate is known. The experimental results of the author in this investigation shows such analytical relationships from which the water-demand and the Gap-volume of aggregate can easily be determined for the case of the gradually and continuously graded aggregates with maximal particle size ($D = 1{:}2{:}4$ mm).

1. Introduction

Ferrocement is a type of reinforced concrete with fine aggregate ($D \leq \leq 4$mm), thickness is less than 50 mm combining fine mesh or other reinforcement (e.g. cane, bamboo). The design procedure of ferrocement concrete is based on practical experiments and the design of traditional concrete. These procedures are still underdeveloped. Experiments and practical work indicates that mixing of ferrocement differs from a traditional concrete mixture. There are differences in the amount of water absorbed by the aggregate, in the gap ratio, in the high cement content of concrete etc.

The specific surface of the aggregate has a major influence on the consistency and on the strength of concrete. For traditional concretes the specific surface of aggregate is chosen from certain limit values and thus the necessary water and consistency of concrete can be set by a certain sand-gravel ratio. If the aggregate is sand only the change in the amount of water needed is so great that it cannot be neglected.

The effect of the specific surface of the aggregate on the consistency has been known for a long time. One of the major rules in concrete technology after Abrams and Popovics is: aggregates with the same fineness modulus and specific surface are identical from the concrete technology point of view.

The influence of the aggregate on concrete does not only depend on the Abrams modulus ("m") but also on the specific surface "S" — a statistic character of the deviation of the grading. A changed specific surface modifies consistency and proves this. Concretes with identical water — cement ratio and fineness modulus have different workabilities [1, 2, 3, 4, 5, 6, 7, 8].

New results have been published on the grading characteristics of aggregates, for example, the two parameters (m and S) description of grading [9].

Concrete design based on paste saturation opened a new phase in concrete technology [10, 11]. One of the most important factors of design is the Gap-volume of the aggregate. For designing concretes from sand-gravel aggregate an approximate Gap-volume function which depends on the Abrams fineness modulus was developed [11, 12, 13, 14]. Later Ujhely introduced a "u" irregularity factor which depends on the fineness modulus and the characteristics of the grading curve of the aggregate which more accurately determines the Gap-volume.

The aggregate of concrete is a bulk of particles. Part of the cement paste fills the gaps between the particles. If the volume of the cement paste is equal to the Gap-volume the concrete is paste saturated, if the cement paste is less or more the concrete is under- or over-saturated respectively. Thus accurate information on the Gap-volume is essential for designing saturated concrete.

The accurate calculation of Gap-volume between spherical shaped particles is still an unsolved problem for engineers and mathematicians. At this time it is still imposible to calculate the accurate Gap-volume of a bulk of identical spherical aggregates. Practically the aggregate particles are not spherical and their diameter is not homogeneous, thus the accurate Gap-volume is determined from tests only.

2. The aim of the research

The specific surface of the aggregate determines the amount of water needed in the concrete. If the grading of the aggregate is finer its specific surface is greater, thus to achieve the same strength more water and cement are needed in the concrete.

The Gap-volume of the aggregate primarily depends on the grading characteristics — the ratio of identical sized particles compared to the whole bulk and their distribution in the bulk. The main characteristics which were found to influence Gap-volume according to investigations were: the maximal diameter particle (D), the fineness modulus (m) and the specific surface of the aggregate (S).

3. The method of experiments

The Palotás equation was used to calculate the specific surface of the aggregate, which assumes spherical particles [14]:

$$s_i = \frac{6}{\varrho_a \cdot d_{ai}} \tag{1}$$

Table 1

For example, how to compute specific surface of sand

Particle size mm		Average diameter mm	Specific surface m²/kg	Particle size mm	Average diameter mm	Specific surface m²/kg
0.0313(0)	−0.063	0.0442	51.4259	0.5−1	0.7071	3.2141
0.063	−0.125	0.0884	25.7130	1−2	1.4142	1.6071
0.125	−0.25	0.1768	12.8565	2−4	2.8284	0.8035
0.25	−0.5	0.3536	6.4282	4−8	5.6569	0.4018

$$S = \frac{\Sigma s_i \cdot c_i}{100} = \frac{0.06}{\varrho_a} \sum \frac{c_i}{d_{ai}} = 0.0227 \sum \frac{c_i}{d_{ai}} \qquad (2)$$

where: $\varrho_a = 2.64$ kg/dm³
For example, how to compute: $d_{ai} = \sqrt{d_i \cdot d_{i+1}}$
For further information see Table 1.

The Gap-volume of the aggregate can be calculated in the following ways:

1. to measure the dry bulk density of aggregate (in a dish with a unit volume),
2. to fill the gaps with water or fluid,
3. to measure the wet bulk density of the aggregate [15].

The first method is easy to apply but dry sand is hardly compactable without segregation. The problem with the second method is that if the aggregate is very fine the water or fluid cannot go between the particles and thus air bubbles can remain in the aggregate which causes false results. The third method was chosen for this research program as it avoids the disadvantages of the first two procedures and has some further advantages, i.e.

— it is easy to carry out,

— sand is compactable with the hands or with machinery.

The tested aggregate (sand) was dried until its mass became constant and then it was graded using 0.063—0.125—0.250—0.50—1.0—2.0—4.0 mm sieves. Aggregates with any given maximal particle size, with gradual or continuous grading, with varying fineness modulus and specific surface were mixed. Three kilograms of material from every mixture was tested. The aggregate and the water (determined according to Table 2) [15] were placed into a drum mixer.

The wet sand was worked into three cylinders (diameter and height almost equal, $V = 0.5$ l). Dishes were filled in three layers. Each of the layers were hand compacted fifteen times with a sharply edged steel spoon then compacted with a portable table vibrator for 40 seconds. Finally, the surplus

Table 2

The water-need of sand mixture

Particle size (mm)	Mass of fraction (kg)	Water added		Specific surface of fraction (m²/kg)
		(m%)	mass (kg)	
0.000—0.063	0.0300	24	0.0072	1.5366
0.063—0.125	0.2652	20	0.0530	6.7920
0.125—0.250	0.3399	16	0.0544	4.3699
0.250—0.500	0.3849	12	0.0462	2.4742
0.500—1.000	0.5199	8	0.0416	1.6710
1.000—2.000	0.6400	7	0.0448	1.0285
2.000—4.000	0.8200	6	0,0492	0.6589
Sum total	3.000	—	0.2964	15.5312
Refer to 1 kg	100%	—	9.88%	6.1771

material was removed from the top of the dish using a metal ruler. The apparent Gap-volume of a dry sand can be calculated using the following equations:

$$V_p = 1000 - \frac{\varrho_{av}}{\varrho_a}\left[1 - \frac{w_a}{100 + w_a}\right]; \ \mathrm{dm}^3/\mathrm{m}^3 \qquad (3)$$

where $\varrho_a = 2.64$ g/cm³ (sand from Danube).

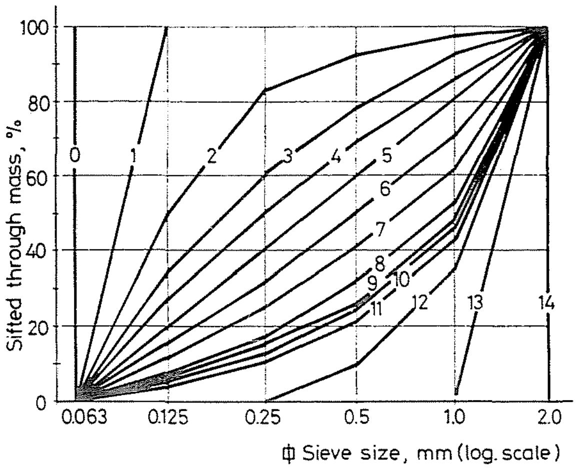
The water need of the sand mixture was determined using Table 2 and a 12—17 cm slump of the mixture was exceptable without water leaving the mixture.

4. Test program

During the experiments two kinds of grading — continuous and gradual — and three maximal diameters ($D = 1$, 2 and 4 mm) were used. The samples were marked as follows:

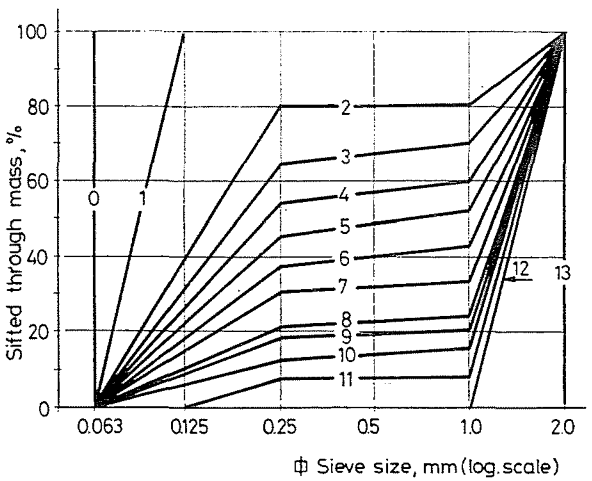
continuously graded : F1, F2, F4
gradually graded: L1, L2, L4.

The assumed grading was drawn using half log scale (see Figures 1 and 2). The measured Gap-volume of sands are summarized in Example 3 and Table 4.



Mixture		m	s	t
F2	0	0.00	51.22	0.000
	1	1.00	25.61	0.198
	2	1.79	17.77	0.425
	3	2.35	13.91	0.630
	4	2.72	12.00	0.785
	5	3.00	9.94	0.952
	6	3.30	9.37	1.078
	7	3.60	7.51	1.314
	8	3.90	6.05	1.586
	9	4.05	5.06	1.800
	10	4.10	4.75	1.881
	11	4.20	4.29	2.088
	12	4.55	2.49	2.883
	13	5.00	1.61	3.940
	14	6.00	1.14	5.620

Fig. 1. Continuous gradings and their properties. $d_{min} = 0.000$, $D = 2$ mm



Mixture		m	s	t
L2	0	0.00	51.22	0.000
	1	1.00	25.61	0.198
	2	2.20	15.71	0.555
	3	2.65	13.23	0.729
	4	3.00	11.46	0.886
	5	3.30	9.91	1.048
	6	3.60	8.47	1.237
	7	3.90	7.14	1.460
	8	4.20	5.55	1.783
	9	4.31	5.02	1.921
	10	4.50	4.00	2.250
	11	4.76	2.51	3.004
	12	5.00	1.61	3.941
	13	6.00	1.14	5.620

Fig. 2. Gradual gradings and their properties. $d_{min} = 0.063$; $D = 2$ mm

5. Processing the experimental results

It is known that in the case of a given D, grading is unambiguously determined with the m-S values. Describing some of the properties of the aggregate (e.g. Gap-volume, consistency or strength etc.) in the function of m and S

Table 3

Continuous gradings and their properties.

Mixture (Mass of fraction / Water-demand of fraction)		0.00—0.063	0.063—0.125	0.125—0.250	0.250—0.500	0.500—1.000	1.000—2.000
	1	—	3.00 0.600	—	—	—	—
	2	—	1.50 0.300	0.96 0.1536	0.30 0.0360	0.15 0.0120	0.09 0.0063
	3	—	1.05 0.210	0.75 0.1200	0.54 0.0648	0.42 0.0336	0.24 0.0168
	4	—	0.84 0.1680	0.66 0.1056	0.57 0.0684	0.48 0.0384	0.45 0.0315
	5	—	0.60 0.1200	0.60 0.0960	0.60 0.0720	0.60 0.0480	0.60 0.0420
	6	0.06 0,0144	0.42 0.0840	0.48 0.0768	0.54 0.0648	0.80 0.0480	0.90 0.0630
	7	0.03 0.0072	0.33 0.0660	0.39 0.0624	0.48 0.0576	0.60 0.0480	1.17 0.0819
F2	8	0.03 0.0072	0.21 0.0420	0.27 0.0432	0.45 0.0540	0.60 0.0480	1.44 0.1008
	9	—	0.21 0.0420	0.24 0.0384	0.33 0.0396	0.63 0.0504	1.59 0.1113
	10	—	0.18 0.0360	0.21 0.0336	0.36 0.0432	0.63 0.0504	1.62 0.1134
	11	—	0.15 0.0300	0.18 0.0288	0.33 0.0396	0.63 0.0504	1.71 0.1197
	12	—	—	—	0.30 0.0360	0.75 0.0600	1.95 0.1365
	13	—	—	—	—	—	3.00 0.2100
	14	—	—	—	—	—	(3.00)
	15						
	16						

would result in a quite complicated and practically useless two variable equation. The introduced fineness factor,

$$t = \frac{m}{\sqrt{S}} \tag{4}$$

overcomes some of these problems and it has two major advantages:

a) The $\Psi = \Psi(m, S)$, two variable equation is transformed into $\Psi = \Psi(t)$ a one variable equation and this is easier to apply.

($D = 2$ mm; $d_{min} = 0.000$ mm)

Sum total		Finness modulus	Specific surface	Fineness factor	Gap-Volume of sand	
kg	%	m	S(m²/kg)	$t = m/\sqrt{S}$	individual	
3.00	100				443, 444, 446,	445
0.60	20.00	1.00	25.61	0.198	449	
3.00	100					
0.508	16.93	1.79	17.77	0.425	412, 412, 418	414
3.00	100					
0.445	14.84	2.35	13.91	0.630	376, 380, 382	379
3.00	100					
0.412	13.73	2.72	12.00	0.785	333, 340, 347	348
3.00	100					
0.378	12.60	3.00	9.94	0.952	295, 296, 301	297
3.00	100				271, 273, 276,	
0.351	11.70	3.30	9.37	1.078	283, 283, 285	278
3.00	100				255, 256, 266	
0.323	10.77	3.60	7.30	1.339	269, 284, 284	269
3.00	100	3.90	5.84	1.614	263, 264, 266	
0.295	9.84				274, 279, 283	272
3.00	100					
0.282	9.39	4.05	5.06	1.801	268, 274, 276	273
3.00	100				303, 303, 306,	
0.277	9.22	4.10	4.75	1.881	310, 316, 320	310
3.00	100				311, 311, 312,	
0.269	6.95	4.20	4.35	2.014	328, 329, 335	321
3.00	100				378, 380, 381,	
0.233	7.75	4.55	2.49	2.883	384	381
3.00	1 0				408, 412, 414,	
0.210	7.50	5.00	1.61	3.940	419, 421	415
3.00	100	6.00	1.14	5.620	—	450
3.00	100					
3.00	100					

b) The questioned is less ambiguous than when the $\Psi = \Psi(m)$ or $\Psi = \Psi(s)$

functions were used.

Experiments proved that the Ujhelyi method (see Table 2) was appropriate for calculating the percentage of water, however, it was very tedious work and it was very hard to estimate the water which was needed according to the specific surface of the material. In the first part of my research an analytic function was set to calculate the Water-need in the function of the specific

Table 4

Gradual gradings and their properties.

Mixture	Mass of fraction / Water-demand of fraction					Sum total	
	0.063—10.125	0.125—0.250	0.250—0.500	0.500—1.000	1.000—2.000	kg	%
1	3.00 0.6000	—	—	—	—	300 0.60	100 20.00
2	1.20 0.2400	1.20 0.192	—	—	0.60 0.0420	300 0.488	100 16.27
3	0.975 0.1950	0.975 0.1560	0.075 0.0090	0.075 0.0060	0.90 0.0630	300 0.429	100 14.30
4	0.825 0.1650	0.825 0.1320	0.075 0.0090	0.075 0.0060	1.20 0.0840	300 0.396	100 13.20
5	0.69 0.1380	0.69 0.1104	0.09 0.0108	0.09 0.0872	1.44 0.1008	300 0.367	100 12.24
6	0.57 0.1140	0.57 0.0912	0.075 0.0090	0.075 0.0060	1.71 0.1197	300 0.340	100 11.33
7	0.465 0.0930	0.0465 0.0744	0.03 0.0036	0.03 0.0024	2.01 0.1407	300 0.314	100 10.47
8	0.33 0.0660	0.33 0.0528	0.03 0.0036	0.03 0.0024	2.28 0.1596	300 0.284	100 9.48
L2 9	0.285 0.0570	0.285 0.0456	0.030 0.0036	0.030 0.0024	2.37 0.1659	300 0.275	100 9.15
10	0.195 0.0390	0.195 0.0312	0.045 0.0054	0.045 0.0036	2.52 0.1764	300 0.256	100 8.52
11	—	0.24 0.0384	—	—	2.76 0.1932	300 0.232	100 7.72
12	—	—	—	—	3.00 0.210	300 0.210	100 7.00
13	—	—	—	—	(3.00)	300	100
14						300	100
15						300	100
16						300	100

surface. In the second phase of my research an equation was given which gives the Gap-volume of any aggregate mixture as a function of the fineness factor.

5.1 *The relationship between the water-demand and specific surface of sand*

It is known that the Water-demand of the aggregate depends on its total surface. As the surface of the aggregate increases it needs more water. Thus I justify determining the Water-demand of the aggregate as a function of the specific surface.

($D = 2$ mm; $d_{min} = 0.063$ mm)

Fineness modulus	Specific surface	Fineness factor	Gap-volume of sand			
m	S/m²/kg/	$t = m/\sqrt{S}$	individual			
1.00	25.61	0.198	443, 449	444,	446,	445
2.20	15.71	0.555	369,	375,	376	373
2.65	13.23	0.729	330,	331,	339	333
3.50	11.46	0.886	287, 294,	288, 298,	292, 301	293
3.30	9.91	1.048	268, 276,	270, 277,	276, 281	275
3.60	8.47	1.235	255, 262,	257, 265,	257 265	260
3.90	7.14	1.4595	246, 244,	252, 254,	254, 258	251
4.20	5.55	1.783	253,	259,	259	257
4.305	5.02	1.921	266,	272,	277	272
4.50	4.00	2.250	303,	322,	327	317
4.76	2.51	3.004	392,	402,	409	401
5.00	1.61	3.941	488, 419,	412, 421	414,	415
6.00	1.14	5.620				(450)

The specific surface of each grade and their water-demand according to Table 2 were determined during tests. (For example: Tables 3 and 4). The same values are shown on Figure 3a which shows the relationship between the water-demand (w_a) and specific surface (S) of aggregates. The w_a-S relationship was assumed to be

$$w_a = a \cdot S^b + c; \quad (\text{m}\%), \tag{5}$$

where a, b and c are unknown constants. The solution of the empirical equation (5) is in two steps. Firstly, the shape of the formula was chosen and only

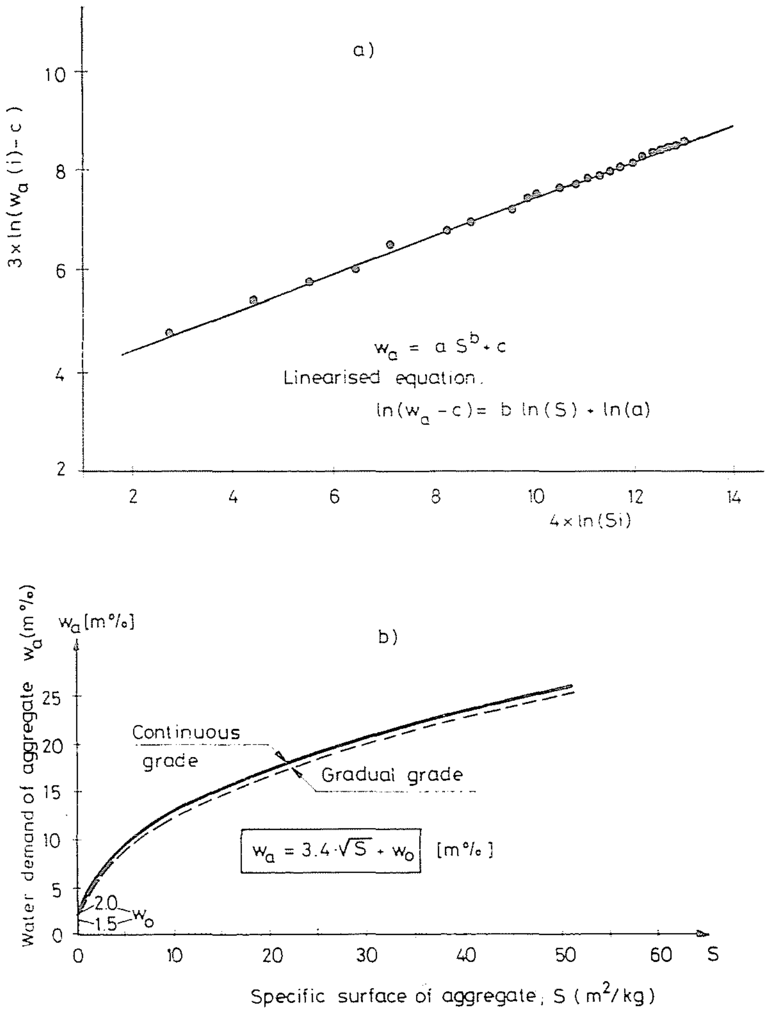


Fig. 3. The Water-demand of the aggregate in the function of the specific surface: a) linear scale, b) log scale

then were the numerical values of the parameters selected. Figure 3b shows the optimum estimations.

The constant parameters of the assumed equation (5) were determined using the "smallest error squares" method with the condition that if the diameter of the aggregates $d >= 63$ mm, then the specific surface $S <= 0.5$ and the water-demand is constant. After the calculations I got the following result:

$$w_a = 3.4 \cdot \sqrt{S} + w_0; \quad (\mathrm{m}\%) \tag{6}$$

where $w^0 = 2.0$ (m%) for a continuously graded aggregate and
$w^0 = 1.5$ (m%) for a gradually graded aggregate.

5.2 *The relationship between Gap-volume and fineness factor of sand*

5.2.1 *General description*

The determination of Gap-volume of spherically shaped, mixed diameter aggregate is still an unsolved problem, however I would like to point out a very interesting and important finding.

The Gap-volume of spherical particles with identical diameter, regularly placed independently of the particle size, are identical.

The following cases are examples which explain this statement. First case: each of the spherical particles with identical diameters were placed in a cube.

$$V_p = 1 - \frac{\frac{4}{3}\pi r^3}{8 \cdot r^3} = 1 - \frac{\pi}{6} = 1 - 0.52 = 0.48 \tag{7}$$

Second case: the centre points of the four spherical particles create a regular tetrahedron (with 6 edges).

$$V_p = 1 - \frac{\frac{4}{3}\pi r^3}{4\sqrt{2}r^3} = 1 - \frac{\pi\sqrt{2}}{6} = 1 - 0.74 = 0.26 \tag{8}$$

Third case: the centre points of the 6 adjacent spherical particles create a regular octahedron (with 12 edges).

$$V_p = 1 - \frac{\frac{4}{3}\pi r^3}{3\sqrt{2}r^3} = 1 - \frac{\pi\sqrt{2}}{6} = 1 - 0.74 = 0.26 \tag{9}$$

Fourth case: 10 spherical particles are attached to one, then

$$V_p = 1 - \frac{\frac{4}{3}\pi r^3}{6 \cdot r^3} = 1 - \frac{\pi \cdot 2}{9} = 1 - 0.70 = 0.30 \tag{10}$$

The following are concluded:

It is impossible to achieve any looser structure of these spherical particles than was introduced in the first case thus the upper limit of Gap-volume (V_{pHf}) can be assumed to be 0.48.

The second and third cases resulted in the most dense structure. This also means that the lower limit of gap volume is

$$V_{pHa} = 0.26 \tag{11}$$

Thus the limit of gap volume of identical diameter balls is

$$0.26 <= V_p <= 0.48 \tag{12}$$

5.2.2 *The selection of the equation describing the relationship between the Gap-volume and fineness factor of sand*

Research results found in the bibliography show that the change in the Gap-volume can be described as a function of the fineness modulus. It has been proven to be moving in the right direction in the research of concrete technology. The main aim of the research is to find the fineness modulus or a range of this where the Gap-volume is minimum. Fortunately this optimum fineness modulus is almost the same as the fineness modulus of those aggregates found in nature. Thus most of the researchers have carried out experiments using aggregates which had almost the optimal modulus of fineness although this was not planned. When analysing the gap volume of an aggregate bulk using the fineness modulus it is only valid on this narrow interval. The major aim of my research was to determine a function to describe the changing Gap-volume of sand. This function is supported by theory and proved by the necessary number of experiments. The analysis of this function is necessary on the whole domain.

This analysis takes two parts. The first part gives a theoretical analysis of grading and Gap-volume of those cases where experiments were impossible to carry out. The second part of the analysis was based on experimental data.

The connection between any respectively chosen $d_{\min}-D$ values and fineness modulus is synonymous only if the fineness modulus and specific surface is given, in other words, the *fineness factor* (t) is given. Previously it was stated that there are more gradings which belong to a given $d_{\min}-D$ pair with identical fineness modulus but different specific surface or vice versa. Thus, the introduced

$$t = m/\sqrt{S} \tag{13}$$

factor is enough to analyse the grading of an aggregate bulk.
The function describes the Gap-volume as follows:

$$V_p = V_p(t) \qquad (\mathrm{dm}^3/\mathrm{m}^3). \tag{14}$$

Let us analyse the boundary conditions of this function:

a) The domain of the function: $t >= 0$
b) The range of the function: $0 <= V_p\,(t) <= 400$
c) The $V_p = V_p\;(t) = 0$ if $t = 0$ or $\mathrm{m} = 0$. This means that every particle in the mixture would fall through the fictitious 0.0313 sieve, in other words, the mixture is homogeneous with constant Gap-volume
d) $t = t_{\max}$ if $m = m_{\max}$ and the particle sizes are uniform, thus $d = D$. The Gap-volume of the mixture is identical to the Gap-volume of an aggregate with uniform particles.

e) The Gap-volume of an irregularly placed aggregate with uniform particles;

$$260 = V_{pHa} <= V_p <= V_{pHf} = 480; \quad (dm^3/m^3) \tag{15}$$

From the tendency of the test results and also from considering the theoretical upper limit of the Gap-volume ($V_{pHf} = 480$) the Gap-volume of an aggregate with identical particles was expected to be

$$V_{p,max} = A = 450; \quad (dm^3/m^3). \tag{16}$$

Thus:

$$V_p(t = 0) = V_p(t = t_{max}) = A = 450 \quad (dm^3/m^3 \tag{17}$$

Experimental results show that the $V_p = V_p(t)$ function in the $0 \leq \leq t \leq t_{max}$ domain only has one minimum value when $t = t_0$. This also means in the case of a given $d_{min} - D$ that among all the existing and possible grading curves there is one with minimum Gap-volume.

Finally, the $V_p = V_p\ (t)$ function in the $t = t(0, t_{max})$ domain is continuous and has an upper and lower limit and because of this it has two inflexion points as in the diagram of the function (Fig. 4). Experimental results also show that the shape of the $V_p = V_p\ (t)$ function is similar to a slanting bell curve. The following formula gives the $V_p = V_p\ (t)$ function:

$$V_p = A - a \cdot t^b \cdot \exp(- c \cdot t^q) \quad (dm^3/m^3) \tag{18}$$

where $A = 450\ dm^3/m^3$,

$$a > 0,\ c > 0,\ b > 1,\ q >= 0$$

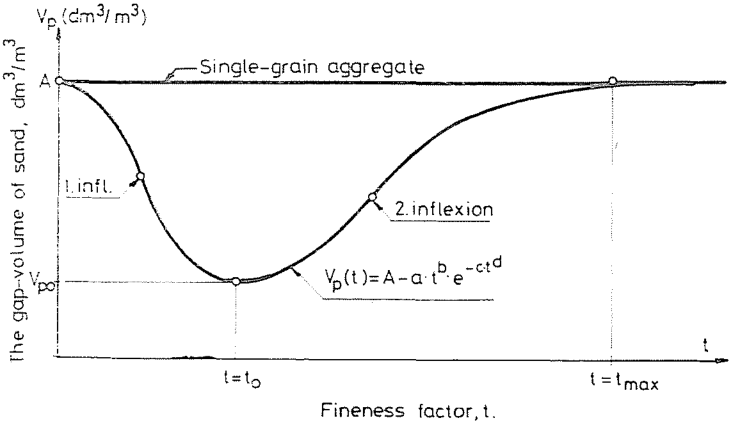


Fig. 4. The typical points of $V_p = V_p(t)$ function

5.2.3 *Determining the parameters of the $V_p = V_p\ (t)$ functions*

Before the detailed analysis of the parameters some distinctive points of the function are examined.

$$\lim_{t \to 0} V_p(t) = A - a \cdot \lim_{t \to 0} \left(t^b \cdot \exp(-c \cdot t^q)\right) = A \tag{19}$$

$$\lim_{t \to \infty} V_p(t) = A - a \cdot \lim_{t \to \infty} \left(t^b \cdot \exp(-c \cdot t^q)\right) = A \tag{20}$$

Thus the function satisfies one of the conditions, it has an upper limit. To determine the lowest limit of the function the first and second derivatives are:

$$V_p'(t) = -a \cdot t^{b-1} \cdot (b - cqt^q) \cdot \exp(-ct^q) \tag{21}$$

$$V_p''(t) = -a \cdot t^{b-2} \cdot \left((b - cqt^q)(b - cqt^q - 1) - cq^2t^q\right) \cdot \exp(-ct^q) \tag{22}$$

The minimum point of the function:

$$t = t_0 = \sqrt[q]{b/(cq)} \tag{23}$$

the abscissas of the two inflexion points from the $V_p''(t) = 0$:

$$t_{1,2} = \sqrt[q]{\frac{(2b + q - 1) + \sqrt{(2b + q - 1)^2 - 4b(b-1)}}{2cq}} \tag{24}$$

To determine the parameters of the assumed function (18) a method comprising two parts was developed.

1. Fitting the function on the measured points (First Approach). In this phase three previously chosen points are used to determine the shape of the function to fit on the measured points. The program runs on a Personal Computer. The abscissas of the three chosen points is $t_1 < t_0 < t_2$. To introduce

$$k = q \cdot t_0^q = b/c \text{ or } b = k \cdot c \tag{25}$$

into equation (18).

Reconstructing the function on the three points:

$$450 - V_p = a \cdot t^b \cdot \exp(-ct^q)$$

$$\ln(450 - V_{p0}) = \ln(a) + b \cdot \ln(t_0) - c \cdot t_0^q \tag{26a}$$

$$\ln(450 - V_{p1}) = \ln(a) + b \cdot \ln(t_1) - c \cdot t_1^q \tag{26b}$$

$$\ln(450 - V_{p2}) = \ln(a) + b \cdot \ln(t_2) - c \cdot t_2^q \tag{26c}$$

which means

$$c_{01} = \frac{\ln(450 - V_{p1}) - \ln(450 - V_{p0})}{k\left(\ln(t_1) - \ln(t_0) - (t_1^q - t_0^q)\right)} \tag{27a}$$

$$c_{02} = \frac{\ln(450 - V_{p2}) - \ln(450 - V_{p0})}{k\left(\ln(t_2) - \ln(t_0) - (t_2^q - t_0^q)\right)} \tag{27b}$$

$$c_{12} = \frac{\ln(450 - V_{p2}) - \ln(450 - V_{p1})}{k\left(\ln(t_2) - \ln(t_1) - (t_2^q - t_1^q)\right)} \tag{27c}$$

The values of $c_{i,j}$ only depend on q. The q which gives good solutions is when

$$c_{01}/c_{02} = c_{01}/c_{12} = c_{02}/c_{12} = 1 \tag{28}$$

The accuracy of $c_{i,j}$ depends on the accuracy of the q solutions. In this case the 0.05% accuracy of q solutions is satisfactory. After this the first approached values of parameters are calculated as follows:

$$c^{+} = (c_{01} + c_{02} + c_{12})/3 \tag{29}$$

$$b^{+} = c^{+} \cdot k \tag{30}$$

$$a^{+} = (450) - V_{p0}) \cdot t_0^{-b^{+}} \cdot \exp(c \cdot t_0^q) \tag{31}$$

2. With the knowledge of the previously calculated parameters, in the second phase we determine the questioned function considering it can be either continuously or gradually ($D = 1, 2,4$ mm) graded. (Second Approach).

During the calculations the author of this paper tried to obtain and express the connection between the F1, F2, F4 or L1, L2, L4 series and the $V_p = V_p(t)$ function.

It was only possible if the three points on the three curves have similar properties.
On the F1, F2 and F4 curves:

(t_{i0}, V_{pi0}) points on the $g_{0f} = 450 \cdot \exp(-0.3875 \cdot t)$ curve (32a)

(t_{i1}, V_{pi1}) points on the $g_{1f} = 450 \cdot \exp(-0.3000 \cdot \mathrm{t})$ curve (32b)

(t_{i2}, V_{pi2}) points on the $g_{2f} = 450 \cdot \exp(-0.0500 \cdot t)$ curve (32c)

On the L1, L2 and L4 curves:

(t_{i0}, V_{pi0}) points on the $g_{0l} = 450 \cdot \exp(-0.4170 \cdot t)$ curve (33a)

(t_{i1}, V_{pi1}) points on the $g_{1l} = 450 \cdot \exp((-0.3000 \cdot \mathrm{t})$ curve (33b)

(t_{i2}, V_{pi2}) points on the $g_{2l} = 450 \cdot \exp(-0.0500 \cdot \mathrm{t})$ curve (33c)

where $i = 1,2$ and 4, see Figure 5.

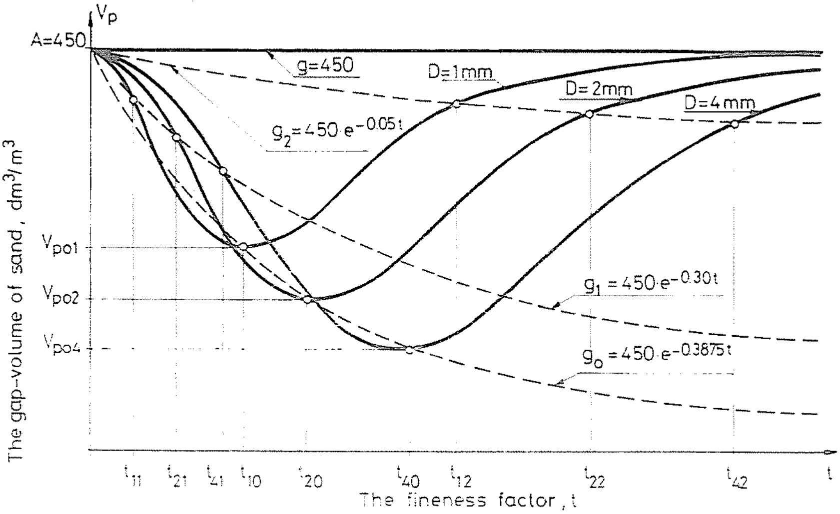


Fig. 5. The selection of t_{i1}, t_{i0}, and t_{i2} points

From the results of the approximate fitting process it is clear that for the gradual and the continuous gradings the parameter q is practically constant and only slightly changes with the changing D. q is approximately 1. The values of the parameter b^+ for gradual and continuous gradings were between 2.557 — 2.915 or 2.267 — 2.456. For further use a value of 3 for b is suggested. Thus instead of equation (18) we can assume the following:

$$V_p = 450 - a^+ \cdot t^3 \cdot \exp(-3 \cdot t/t_0) \tag{34}$$

In this simplified formula only a^+ and t_0 are influencing the result — $D =$ $= 1, 2, 4$ are also assumed.

Introducing the $a^+ = e^h$, then (34) is going to be:

$$V_p = 450 - t^3 \cdot \exp\big((-3 \cdot t/t_0) + h\big) \tag{35}$$

If the (t_{i0}, V_{pi0}) are known the following formula gives the values of h_{fi} and h_{li}:

$$h_{fi} = \ln(450 - V^f_{pi0}) - 3 \cdot \ln(t^f_{i0}) + 3 \tag{36}$$

$$h_{li} = \ln(450 - V^l_{pi0}) - 3 \cdot \ln(t^l_{i0}) + 3 \tag{37}$$

In the case of a given D the coordinates of the minimum point are known, thus h_{fi} are regarded as knowns. This also means that h_{fi} and h_{li} only depend on D. Table 5 summarizes the values of h_{fi} and h_{li}.

Table 5

The parameters of the Gap-volume function

Grading	D (mm)	Co-ordinates of Min.		h_{fi} and h_{li}
		t_{io}	V_{pio}	
Continuous	1	1,050	300	7.8643
	2	1.330	265	7.3648
	4	1.775	230	6.6722
Gradual	1	1.050	290	7.9288
	2	1.400	250	7.2889
	4	1.850	210	6.6351

Introducing the

$$K_f = (-3 \cdot t/t_{f0}) + h_f \quad (38a)$$

$$K_l = (-3t/t_{l0}) + h_l \quad (38b)$$

the (34) equation becomes

$$V^f_{pi} = 450 - t_3 \cdot \exp(K_{fi}) \quad (39)$$

$$V^l_{pi} = 450 - t^3 \cdot \exp(K_{li}) \quad (40)$$

where K_{fi} and K_{li} only depend on D_i. After building D into the K_i then (39) and (40) give a series of curves. Figure 6 shows the curves in a $(t_0-D)/(h-D)$ coordinate system, values from Table 5.

Figure 6 shows that neither the $t_0 = t_0(D)$ nor the $h = h(D)$ are linear functions, thus the fitting function is chosen as a parabolic or hyperbolic curve.

$$t_0 = a \cdot D^2 + b \cdot D + c \quad (41)$$

$$h = m/(D + n) + p \quad (42)$$

After substituting (41) and (42) into (38a) and (38b),

$$K = \frac{k_1 \cdot t \cdot (D + n) + p \cdot D^3 + k_2 \cdot D^2 + k_3 \cdot D + k_4}{D^3 + k_5 \cdot D^2 + k_6 \cdot D + k_7} \quad (43)$$

where:

$$k_1 = (-3/a);\ k_2 = m + n \cdot p + (b \cdot p/a)$$

$$k_3 = ((m + n \cdot p) \cdot b + c \cdot p)/a$$

$$k_4 = ((m + n \cdot p) \cdot c)/a;\ k_5 = (n + b/a)$$

$$k_6 = (c + b \cdot n)/a; \quad k_7 = c \cdot n/a$$

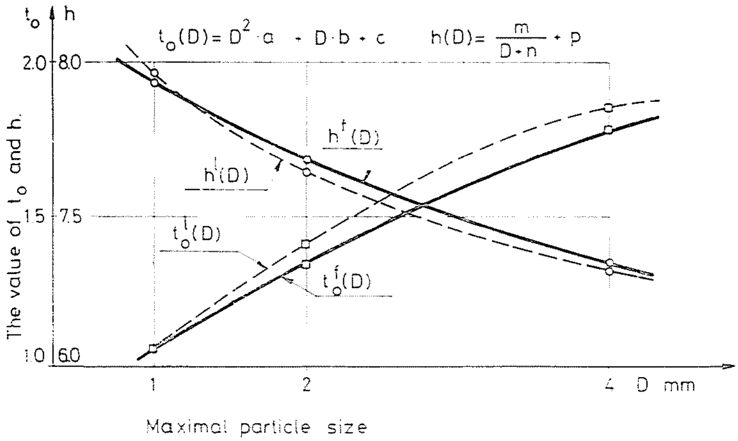


Fig. 6. The relationship between D and t_0 or h

After all these calculations the following formulas are obtained to calculate the Gap-volume of gradually or continuously graded sand.

$$V_p^f = 450 - t^3 \cdot \exp(K_f(D, t)) \tag{44}$$

$$V_p^l = 450 - t^3 \cdot \exp(K_l(D, t)) \tag{45}$$

where

$$K_f = \frac{156 \cdot t \cdot (D + 5.00) + 4 \cdot D^3 - 21 \cdot D^2 - 1021 \cdot D - 1886}{D^3 - 12 \cdot D^2 - 140 \cdot D - 221} \tag{46}$$

or

$$K_l = \frac{72 \cdot t \cdot (D + 2.15) + 5.3 \cdot D^3 - 40 \cdot D^2 - 300 \cdot D - 290}{D^3 - 9 \cdot D^2 - 39 \cdot D - 31.4} \tag{47}$$

The optimal Gap-volume

$$V_{p0}^f = 450 - t_{f0}^3 \cdot \exp\left(K_f(D, t_{f0})\right) \tag{48}$$

$$V_{p0}^l = 450 - t_0^3 \cdot \exp\left(K_l(D, t_0)\right) \tag{49}$$

where

$$t_{f0} = -0.01917 \cdot D^2 + 0.3375 \cdot D + 0.7317 \tag{50}$$

$$t_{l0} = -0.04170 \cdot D^2 + 0.4750 \cdot D + 0.6170 \tag{51}$$

Diagrams to make calculations easier are also given. It is also stated here that equations (43) and (44) only give accurate results in the $D = 1 - 4$ mm range. If $D < 1$ or $D > 4$ the function is not accurate enough because K_f and K_l were calculated in the $D = 1; 2; 4$ interval. If we want to use (44) or

(45) in a larger range then the $t_0 = t_0\ (D)$ and $h = h(D)$ in the exponents of K_f or K_l have to be approached with a higher degree polynomial. We would obtain the accurate solution if in the case of n types of D the $t_0 = t_0\ (D)$ or $h = h(D)$ functions are approached with an $(n - 1)$ degree polynomial.

Practically, the concrete of ferrocement structures comprises the $D = 1-4$ mm particles. From this point of view equations (44) and (45) are regarded as accurate.

6. Summary

The following are only valid if the density of the aggregate is between

$\varrho_a = 2500 - 2800$ kg/m³.

1. An analytic function related to the specific surface of the aggregate was developed to determine the water need of a sand-water mixture which had standard slump 12—17 cm without the water leaving the mortar. The following equation gives the necessary and satisfactory amount of water needed to coat the sand particles (see Figure 3):

$$w_a = 3.4 \cdot \sqrt{S} + w_0; \quad (\mathrm{m}\%)$$

where w_0 is a factor depending on the grading of the aggregate, when

$w_0 = 2.0$ (m%) if the grading is continuous and
$w_0 = 1.5$ (m%) if the grading is gradual.

2. The fineness factor $t = m/\sqrt{S}$ was introduced with two important properties:

— the $V_p = V_p(m, S)$ is a function with two unknowns which is transferred into a function with only one unknown $V_p = V_p(t)$, thus it makes it easier to use the function.

— the fineness factor makes the function expressing the Gap-volume less ambiguous against the $V_p = V_p(m)$ or $V_p = V_p(S)$ functions.

3. Using the Abrams — Popovics fineness modulus (m) and the specific surface (S) of the sand aggregate an analytic function was developed to calculate the Gap-volume of dry sand supported by experimental data (Figure 7):

$$V_p = 450 - t^3 \cdot \exp\big(K(D, t)\big)\ (\mathrm{dm^3/m^3}).$$

If the grading is continuous then

$$K_f = \frac{156 \cdot t \cdot (D + 5.80) + 4D^3 - 21 \cdot D^2 - 1021 \cdot D - 1886}{D^3 - 12 \cdot D^2 - 140 \cdot D - 221}$$

if the grading is gradual then

$$K_l = \frac{12 \cdot t \cdot (D + 2.15) + 5.3 \cdot D^3 - 40 \cdot D^2 - 300 \cdot D - 290}{D^3 - 9 \cdot D^2 - 39 \cdot D - 31.4}$$

4. In the case of a given particle size, among identical gradings there is only one existing to fineness factor — the so called optimum grading — which has the lowest Gap-volume (V_{p0}). When D increases the optimal fineness factor also increases (Figure 7).

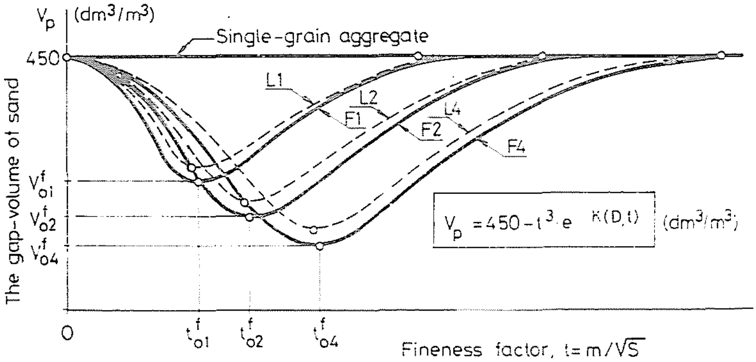


Fig. 7. Gap-volume of sand in the function of fineness factor (t)

The optimal gap-volume is calculated as follows:

$$V^f_{p0} = 450 - t^3_{f0} \cdot \exp\big(K_f(D, t_{f0})\big)$$

$$V^l_{p0} = 450 - t^3_{l0} \cdot \exp\big(K_l(D, t_{l0})\big)$$

where

$$t_{f0} = -0.0192 \cdot D^2 + 0.3375 \cdot D + 0.7317$$

$$t_{l0} = -0.0417 . D^2 + 0.4750 \cdot D + 0.6170.$$

Symbols

a, b, c, m, n, p, q — constant parameters of the functions
a_i — mass of aggregate passed through the i-th sieve (m%)
c_i — mass of aggregate which stayed upon the i-th sieve
d — diameter of the particles (mm)
d_i — particle diameter which belongs to the i-th sieve
d_{ai} — the average diameter of the i-th fraction
f, g, h — functions

m — fineness modulus from Abrams
m_0 — optimum fineness modulus from Abrams
s_i — the specific surface of the i-th fraction in the aggregate
t — the fineness factor
u — the inequality factor
w_a — the water need of the aggregate m%
w_0 — the water need of the aggregate (m%) depending on the grading
A, B — experimental factor
D — the maximal particle size (mm)
K — the power of the exponential function
S — the specific surface of the aggregate (m^2/kg)
V_p — the gap volume or the pulp content of the aggregate (dm^3/m^3)
V_{pHa}; V_{pHf} — the lower or upper limits of the Gap-volume (dm^3/m^3)
V — the optimum Gap-volume of the aggregate
Ψ — function
ϱ_a — the body density of the aggregate
ϱ_{av} — the body density of the wet aggregate.

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Nguyen Huu THANH H-1521, Budapest