PLASTIC LIMIT ANALYSIS OF SPATIAL LARGE PANEL STRUCTURES

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Abstract

It is important that large panel buildings have a sufficient safety against progressive collapse. Recent researches [4] conducted in the Department of Civil Engineering Mechanics have shown that a good approximation for the resistance of the structure can be obtained if the dynamic analysis is replaced by a quasi-static plastic solution in which the damaged structure is subjected to the dead load increased by a dynamic factor $\mu = 1.10-1.30$. The present paper deals with this problem.

From the point of view of progressive collapse, experience shows that the weakest points of a large panel building are the joints. So the model adopted consists of rigid panel elements connected by rigid–perfectly plastic springs which represent normal and shear forces between the elements. Figure 1 shows the definition of the internal forces and their numbering on an element. The yield condition used is simply $-Q_{ip}^{-} \leq Q_i \leq Q_{ip}^{+}$ i.e. no interaction is taken into account between the spring forces.

According to the statical theorem of limit analysis the problem can be formulated as follows:

$$\mathbf{C}^* \mathbf{Q} + m\mathbf{F}_0 = \mathbf{0} \tag{1}$$

$$\mathbf{N}^* \mathbf{Q} \qquad \leq \mathbf{k} \tag{2}$$

$$m = \max!$$

Relation (1) represents the equilibrium equations and relation (2) is the yield condition [1], [2].

By appropriate partitioning of the matrices and vectors, the size of the problem can be reduced by matrix operations having only the redundant internal forces and the limit load parameter as unknowns.

$$\mathbf{B}^* \mathbf{x} + m\mathbf{b}_0 \leq \mathbf{k} \tag{3}$$

$$m = \max!$$

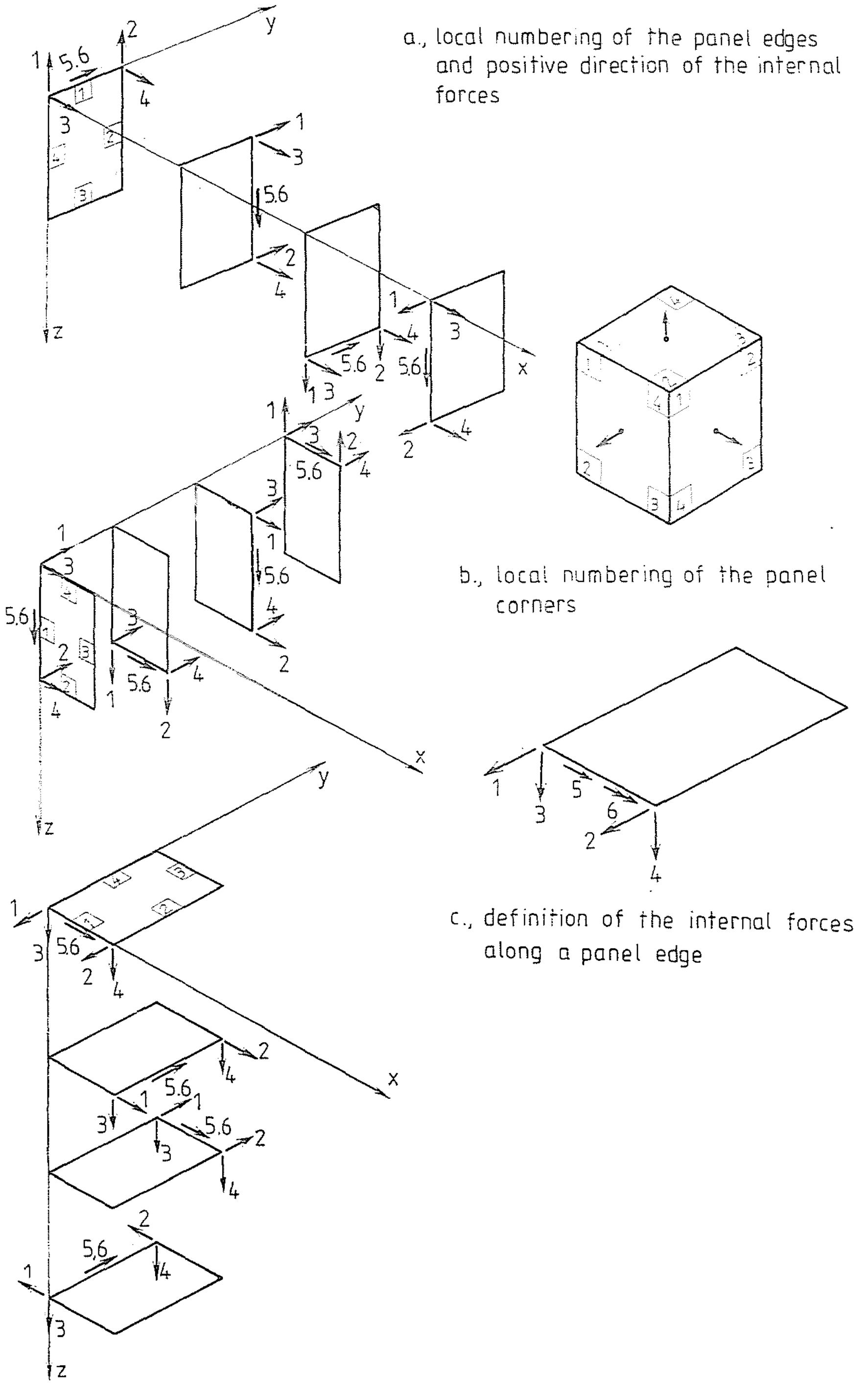

a., local numbering of the panel edges and positive direction of the internal forces
b., local numbering of the panel corners
c., definition of the internal forces along a panel edge

Fig. 1

where x represents the redundant internal forces. Due to the simple yield condition, (3) can be partitioned in the following manner:

$$\begin{bmatrix} \mathbf{A}^* & \mathbf{a}_0 \\ -\mathbf{A}^* & -\mathbf{a}_0 \\ \mathbf{E} & \mathbf{0} \\ -\mathbf{E} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ m \end{bmatrix} \leq \begin{bmatrix} \mathbf{k}^+ \\ \mathbf{k}^- \\ \mathbf{k}_r^+ \\ \mathbf{k}_r^- \end{bmatrix} \tag{4}$$

$$m = \max!$$

where $\mathbf{k}^+$ represents the positive and $\mathbf{k}^-$ the negative yield forces for the normal springs and $\mathbf{k}_r^+$, $\mathbf{k}_r^-$ are the same quantities for the redundant ones. $\mathbf{A}^*$ is the influence matrix corresponding to the redundant internal forces and $\mathbf{a}_0$ gives the internal forces due to the basic load $\mathbf{F}_0$.

The operations above correspond to choosing and solving a primary isostatic structure of the original one. In some cases it is possible to choose a primary structure such that no matrix operations and inversions are needed to derive the reduced problem. Our case belongs to this class. By appropriate choice of the redundant forces, the primary isostatic structure consists of a set of spatial cantilevers as shown on a small example in Fig. 2. Observing a convenient numbering rule for the panels and edges, it can be determined by logical operations which are the zero and non-zero blocks of matrix $\mathbf{A}^*$ and vector $\mathbf{a}_0$. Their structure is shown in Table 1. The elements of the non-zero blocks can be calculated by simple equilibrium equations. The calculations of the elements of the block marked in Table 1 are illustrated in Fig. 3. It can be mentioned that the present choice of the primary structure keeps the sparsity of the problem matrix.

For the solution of the linear programming problem in limit analysis, usually the kinematic (dual) formulation is used as the number of rows of matrix $\mathbf{B}^*$ is greater than the number of its columns. This is not necessary in this case because ranges can be applied on the constrained rows and bounds on the column variables and the problem tableau reduces only to matrix $\mathbf{A}^*$ in this way. This results in the fact that the size of the basis is only slightly greater in the static case than in the kinematic one. One can also mention that using the kinematic formulation the whole matrix $\mathbf{B}$ should be stored as data.

The method was coded in FORTRAN language on the IBM 3031 computer of the Hungarian Academy of Sciences. For the solution of the linear programming problem the MPSX program package was used [3].

As in the case of spatial structures the number of unknowns and inequalities increases very rapidly, a structure of small size shown in Fig. 4 was studied. The characteristics of the problem are the following:

number of panels:	32
number of global edges:	42

Table 1

Panel edge		Edge cut 1	2	3	4	5	6
		load					
1	3	.	.	.	.	.	.
1	2	.	.	.	.	.	.
1	4	.	.	.	.	.	.
1	1	.	.	.	.	.	.
2	1	.	.	.	.	.	.
2	7	.	.	.	.	.	.
2	9	.	.	.	.	.	X
2	5	.	.	.	.	.	X
3	2	.	.	.	.	.	.
3	8	.	.	.	.	.	.
3	10	.	.	.	.	.	.
3	6	.	.	.	.	.	.
4	5	.	.	.	.	.	X
4	11	.	.	.	.	.	X
4	6	.	.	.	.	.	.
4	3	.	.	.	.	.	.
5	7	.	.	.	.	.	.
5	12	.	.	.	.	.	.
5	8	.	.	.	.	.	.
5	4	.	.	.	.	.	.
6	11	.	X	.	.	.	.
6	10	.	.	X	.	.	.
6	12	.	.	.	X	.	.
6	9	.	X	X	X	.	.
7	9	.	X	X	X	.	X
7	15	.	.	.	.	X	.
7	17	X	X	X	X	X	X
7	13	X	.	.	.	.	.
8	10	.	.	X	.	.	.
8	16	.	.	.	.	.	.
8	18	.	.	X	.	.	.
8	14	.	.	.	.	.	.
9	13	X	.	.	.	.	.
9	19	X	X	.	.	.	X
9	14	.	.	.	.	.	.
9	11	.	X	.	.	.	X
10	15	.	.	.	.	X	.
10	20	.	.	.	X	X	.
10	16	.	.	.	.	.	.
10	12	.	.	.	X	.	.

"X" non-zero block
"." zero block

Edge cut								
7	8	9	10	11	12	13	14	15
load								
X	.	.	.	.	.	.	.	.
.	X	.	.	.	.	.	.	.
.	.	X	.	.	.	.	.	.
X	X	X	.	.	.	.	.	X
X	X	X	.	.	.	.	.	X
.	.	.	X	.	.	.	.	.
X	X	X	X	.	.	.	.	X
.	.	.	.	.	.	.	.	.
.	X	.	.	.	.	.	.	.
.	.	.	.	.	.	.	X	.
.	X	.	.	.	X	.	X	.
.	.	.	.	.	X	.	.	.
.	.	.	.	.	.	.	.	.
X	.	.	.	.	X	.	.	.
.	.	.	.	.	X	.	.	.
X	.	.	.	.	.	.	.	.
.	.	.	X	.	.	.	.	.
.	.	X	X	.	.	.	X	.
.	.	.	.	.	.	.	X	.
.	.	X	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
X	X	X	X	.	.	.	.	X
.	.	.	.	.	.	.	.	.
X	[X]	X	X	.	.	.	.	X
.	.	.	.	.	.	.	.	.
.	X	.	.	.	X	.	X	.
.	.	.	.	.	.	X	.	.
.	X	.	.	X	X	X	X	.
.	.	.	.	X	.	.	.	.
.	.	.	.	.	.	.	.	.
X	.	.	.	X	X	.	.	.
.	.	.	.	X	.	.	.	.
X	.	.	.	.	X	.	.	.
.	.	.	.	.	.	.	.	.
.	.	X	X	.	.	X	X	.
.	.	.	.	.	.	X	.	.
.	.	X	X	.	.	.	X	.

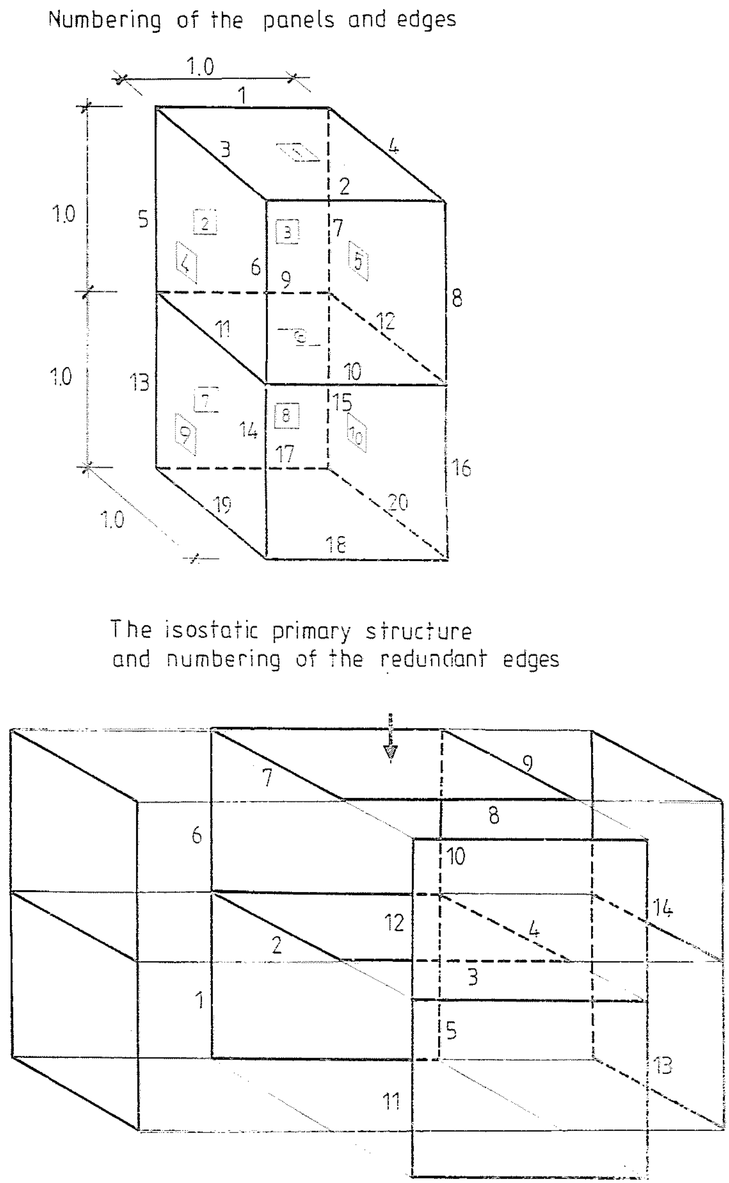
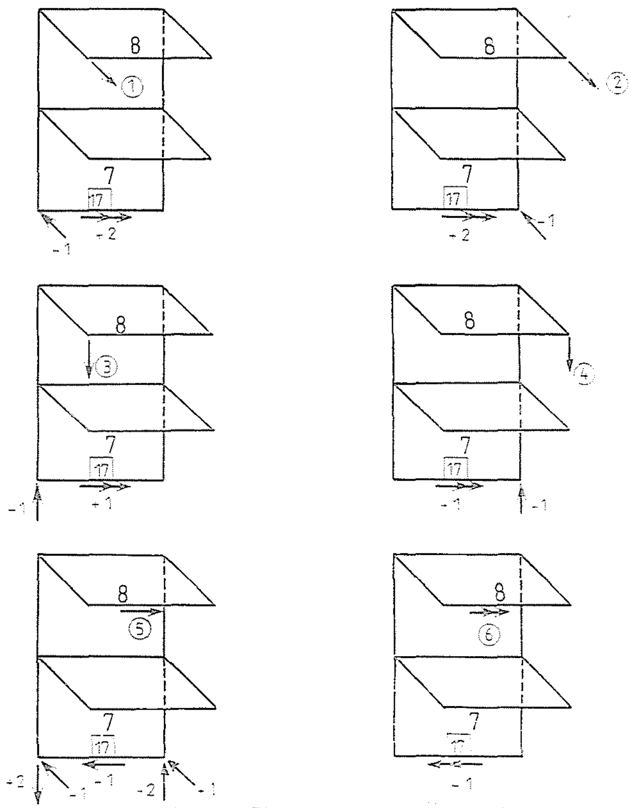


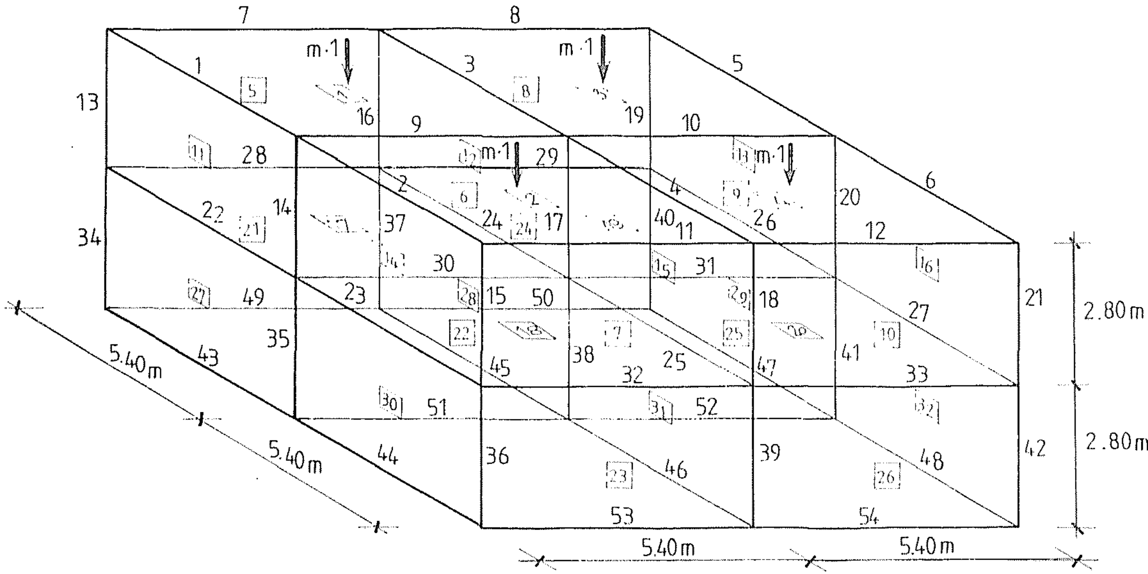
Fig. 2

number of internal forces:	$32 \times 24 = 768$
number of equilibrium equations:	$32 + 42 \times 6 = 444$
number of redundancies:	$= 324$
size of the problem matrix including one slack variable for each row:	445×770, density: 1.1%.



Edge cut N°8		1	2	3	4	5	6
panel 7, edge 17	1	0	0	-1	0	+2	0
	2	0	0	0	-1	-2	0
	3	-1	0	0	0	-1	0
	4	0	-1	0	0	+1	0
	5	0	0	0	0	-1	0
	6	+2	+2	+1	+1	0	-1

Fig. 3



Sample structure

Fig. 4

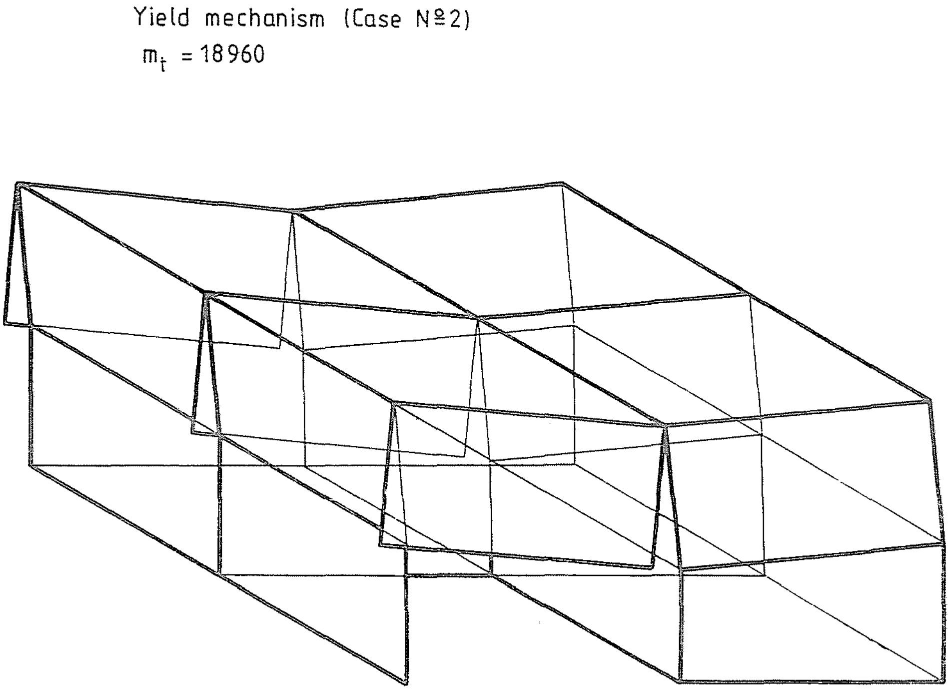


Fig. 5

Normal forces in the horizontal sections [kN]

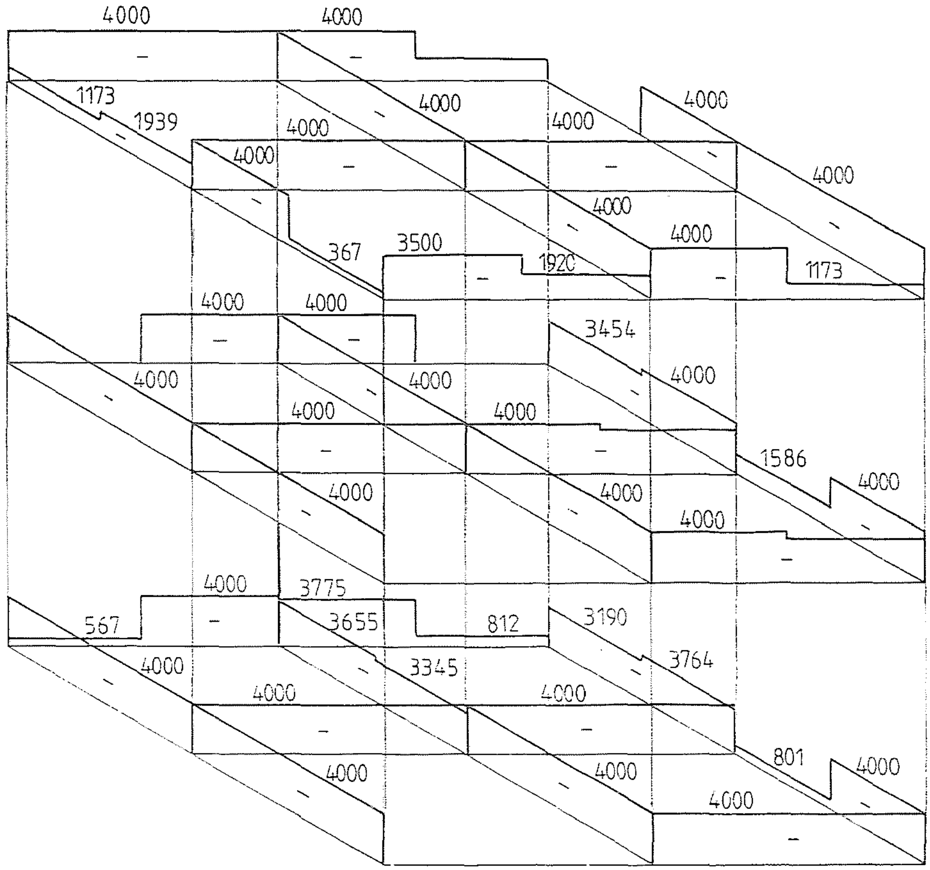


Fig. 6

The limits for the internal forces are shown in Table 2.

Table 2

Panel type	Local edge	Internal force 1 +	1 −	2 +	2 −	3 +	3 −	4 +	4 −	5 +	5 −	6 +	6 −
1	1	50	4000	50	4000	300	300	300	300	600	600	0	0
1	2	100	4000	100	4000	150	150	150	150	300	300	0	0
2	1	100	4000	100	4000	150	150	150	150	300	300	0	0
2	2	50	4000	50	4000	300	300	300	300	600	600	0	0
3	1	100	4000	100	4000	4000	4000	4000	4000	600	600	0	0
3	2	100	4000	100	4000	4000	4000	4000	4000	600	600	0	0

Yield values of the internal forces (kN)

The limit load parameter was calculated for three cases:

	Number of missing panels	Limit load	Solution time (min)
1	0	20000	1.09
2	1 (23)	18960	1.17
3	2 (30, 23)	16025	1.25

The yield mechanism for case 2 is illustrated in Fig. 5, and Fig. 6 shows the internal forces in the vertical springs connecting the vertical and horizontal panels.

References

1. MUNRO, J. and SMITH, D. L.: Linear programming duality in plastic analysis and synthesis. International Symposium on Computer-Aided Structural Design, Coventry, 1972, Proceedings
2. MAIER, G.—SRINAVASAN, R.—SAVE, M.: On limit design of frames using linear programming. International Symposium on Computer Aided Structural Design, Coventry, 1972, Proceedings.
3. IBM Mathematical Programming System Extended/370 Program Reference Manual.
4. KALISZKY, S., NÉDLI, P., TORNYOS, Á., WOLF, K.: Progressive collapse analysis of large panel structures. Scientific papers for the 70th birthday of Giulio Ceradini. Dipartimento di Ingegneria Strutturale e Geotecnica. Universita di Roma "La Sapienza", 1988.

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