SEISMIC ANALYSIS OF STRUCTURES BASED ON THE BERLAGE IMPULSE

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Abstract

A relationship is given for the seismic analysis of structures in terms of the Berlage impulse. Resultant relationships yield displacements for zero initial displacement and velocity.

By the end of the last century, Japanese scientist F. Omori has developed the static theory of seismic analyses based on experiments (1889—1899). Accordingly, the building was considered as a rigid body, seismic force was reckoned with as a horizontal static force proportional to the building mass. This theory assumed the effect of the building's own deformation to be negligible in the analysis.

In serious earthquakes (such as that in Taskent, 1966) this theory practically failed for high-rise buildings, towers [1].

Development of computation methods has led to the examination of dynamic features of seisms. N. Mononobe was the first to reckon with dynamic characteristics, assuming earthquakes to feature harmonic vibrations. Zavriev (1927) described earthquake as cosine vibration:

$$\begin{aligned} y_0(t) &= a_0 \cos \omega t \\ \ddot{y}_0(t) &= -a_0 \omega^2 \cos \omega t. \end{aligned} \tag{1}$$

Accordingly, at time $t = 0$, the earth surface undergoes displacement a_0, at an initial velocity of zero, and the vibration process to be an undamped vibration.

Seismograms of real earthquakes showed earth surface motions to be describable by damped vibration components [3]:

$$y_0(t) = \sum_{i=1}^{n} a_{0i} e^{-\varepsilon_{0i} t} \sin (\omega_i t + \gamma_i). \tag{2}$$

It being rather intricate to determine the number of components and of parameters of every and each vibration component, in practical calculations,

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a single component is normally involved. According to several authors [3], surface motion is exacter described by function:

$$y_0(t) = a_0 t e^{-\varepsilon_0 t} \sin \omega_0 t \tag{3}$$

yielding zero at time $t = 0$ both for displacement and for velocity (a_0 being in cm/s units, and $a_0 \cdot t$ the amplitude).

Applying the above, so-called Berlage impulse, velocity and acceleration become, respectively:

$$\dot{y}_0(t) = a_0 e^{-\varepsilon_0 t}(1 - \varepsilon_0 t) \sin \omega_0 t + a_0 \omega_0 t e^{-\varepsilon_0 t} \cos \omega_0 t \tag{4}$$

$$\ddot{y}_0(t) = a_0 e^{-\varepsilon_0 t}(t\varepsilon_0^2 + \omega_0^2 t - 2\varepsilon_0) \sin \omega_0 t + 2\omega_0(1 - t\varepsilon_0) \cdot \cos \omega_0 t. \tag{5}$$

For the seismic analysis of real buildings, matrix differential equation of the system with many degrees of freedom may be written as:

$$\mathbb{M}\ddot{\mathbf{q}}(t) + \mathbb{K}\mathbf{q}(t) = -\mathbb{M}\ddot{\mathbf{y}}_0(t) \tag{6}$$

where $\mathbb{M}$ is mass matrix of the structure, $\mathbb{K}$ its stiffness matrix, $\mathbf{q}$ — vector of nodal elastic displacements, $\mathbf{y}(t)$ — vector of displacements of points contacting the soil.

In knowledge of eigenvectors and natural frequencies ω_i of matrix $\mathbb{M}^{-1}\mathbb{K}$, analysis of the system can be reduced to the analysis of systems with a single degree of freedom, and vector $\mathbf{q}$ can be obtained from components corresponding to eigenvectors (2).

Norming eigenvectors of eigenvalue problem

$$\mathbb{M}^{-1}\mathbb{K}\mathbf{v} = \omega^2 \mathbf{v}$$

as

$$\mathbb{V}^* \mathbb{M} \mathbb{V} = \mathbb{E}$$

and introducing another unknown as

$$\mathbf{q}(t) = \mathbb{V}\mathbf{z}(t) \quad \ddot{\mathbf{y}}_0(t) = \mathbb{V}\ddot{\mathbf{x}}_0(t) \tag{7}$$

the tested systems with a single degree of freedom can be written as:

$$\ddot{z}_i(t) + \omega_i^2 z(t) = -\ddot{x}_{0i}(t) \tag{8}$$

where

$$\ddot{x}_{0i}(t) = v_i^* \mathbb{M}\, \ddot{y}_0(t).$$

The solution conform to zero initial conditions:

$$z_i(t) = -\frac{b_{0i}}{\omega_i} \int_0^t e^{-\varepsilon_0 \tau}[(\tau\varepsilon_0^2 - \omega_0^2 \tau - 2\varepsilon_0) \sin \omega_0 \tau + \\ + 2\omega_0(1 - t\varepsilon_0) \cos \omega_0 \tau] \sin \omega_i(t - \tau)\, d\tau \tag{9}$$

where:

$$b_{0i} = a_0 \sum_{r=1}^{n} v_{ir}^{*} \, dr$$

$$dr = \sum_{k=1}^{n} mrk \tag{10}$$

$$z_i(t) = C_i R_i(t)$$

with:

$$C_i = \frac{b_{0i}}{\omega_i}. \tag{11}$$

After integration: (1)

$$R_i(t) = \frac{2\omega_i^4}{(\varepsilon_0^2 + 4\omega_i^2)^2} \left\{ \left[\frac{\varepsilon_0}{\omega_i}(1 + e^{-\varepsilon_0 t}) - \right.\right.$$
$$\left. - \frac{(3\omega_i^2 + \varepsilon_0^2)(\varepsilon_0^2 + 4\omega_i^2)}{2\,\omega_i^3} t e^{-\varepsilon_0 t} \right] \sin \omega_i t + \tag{12}$$
$$\left. + \frac{1}{\varepsilon_0^2} \left[(3\varepsilon_0^2 + 4\omega_i^2)(e^{-\varepsilon_0 t} - 1) + \varepsilon_0 t e^{-\varepsilon_0 t}(\varepsilon_0^2 + 4\omega_i^2) \right] \cos \omega_i t \right\}.$$

In knowledge of solutions for systems of one degree of freedom:

$$\mathbf{q}(t) = \sum_{i=1}^{n} \mathbf{v}_i z_i(t) = \sum_{i=1}^{n} \mathbf{v}_i C_i R_i(t). \tag{13}$$

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