VISIBILITY DISPLAY IN FEM PROGRAMS

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Abstract

The goal of visibility display suiting finite element programs could be achieved by uniting surface elements into saddle surfaces, decomposed — in turn — to triangular surfaces displayed in a proper order.

1. Introduction

Actually, a preferred, generalized method of analysis is that by the finite element method. Since its arise and development not long ago, several important achievements have occurred in the numerical computer analysis of structures. Actually, there is hardly a structural element or effect inaccessible to this method. The resulting, increasingly complex procedures require ever more input data, while evaluation and analysis of the outputs demand ever more work and expert knowledge.

These problems are expected to be eased by graphic facilities of FEM. These graphic solutions are of a great help for data input, survey and use of outputs, and drawing of conclusions. Behold the graphic, interactive rendering of the finite element network, display of plane or spatial, mobile, perspectivic or axonometric strain or stress diagrams, or to draw figures for checking geometry data of the structure. Beyond a given degree of complexity, so-called transparencies are insufficient to examine e.g. foundations, or bones in biomechanical research, or even, for the analysis of complex stress patterns of a floor structure. These problems are eased by the visibility display.

2. Perspectivic display

To decide between perspectivic and axonometric display, it has to be pondered that axonometric display is simpler to perform, parallels remain such, but it is different from human vision. Axonometry does not offer an insight into the configuration. Perspectivic representation is much closer to human vision, hence it is more difficult to realize. Parallels do not remain so, while once penetrated into the configuration, to look around is no problem. Possibilities of perspectivic display will be examined below.

With symbols in Fig. 1, perspectivic image of a point is written as:

$$A' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}, \text{ where } \begin{aligned} x' &= x\frac{t}{t-z} \\ y' &= y\frac{t}{t-z} \\ z' &= 0 \end{aligned} \tag{1}$$

where t is the distance between looking point N and origin 0 of the coordinate system. Plane S_1 will be termed the image plane, and the parallel plane S_2 crossing N the looking plane. Domain T — to be termed image domain — is the part of the image plane appearing on the computer screen.

Points other than incident on the looking plane side nearer to the image plane must not be projected. Because of the finite number notation of computers, even points near the looking plane may cause difficulties, namely their projections may be outside the number display area. This is why (in assessing projectability of a point) the looking plane is numerically modelled by a so-called looking pyramid with the looking point as vertex, and a square cut out from the image plane by the boundaries of number display as basis.

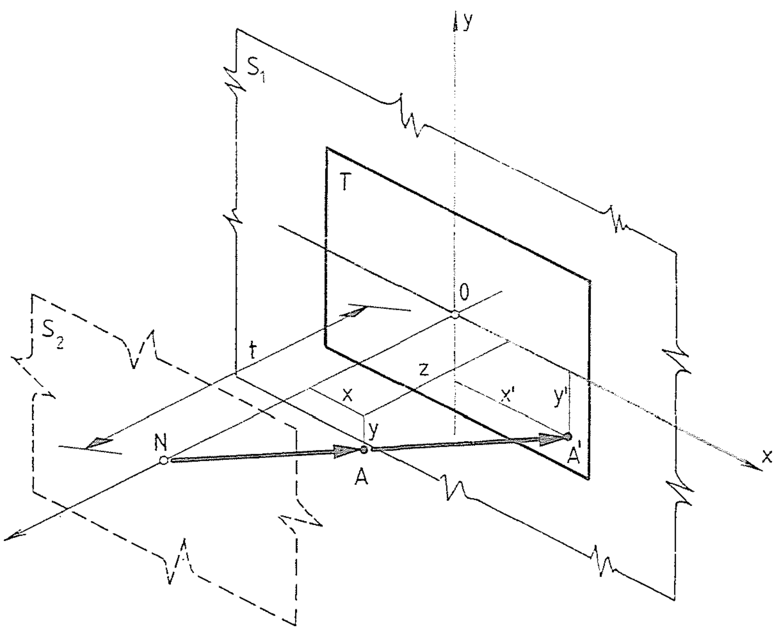

Fig. 1

After projection of the projectable points, the next problem is to display a straight line of which one end can be projected, while the other cannot because it is outside the looking pyramid, such as that in Fig. 2. End point A can be projected, while B cannot. The clue of the solution is to draw a half-line from A' parallel to the line connecting N and M, where M is the intersection between length $\overline{AB}$ and the looking plane.

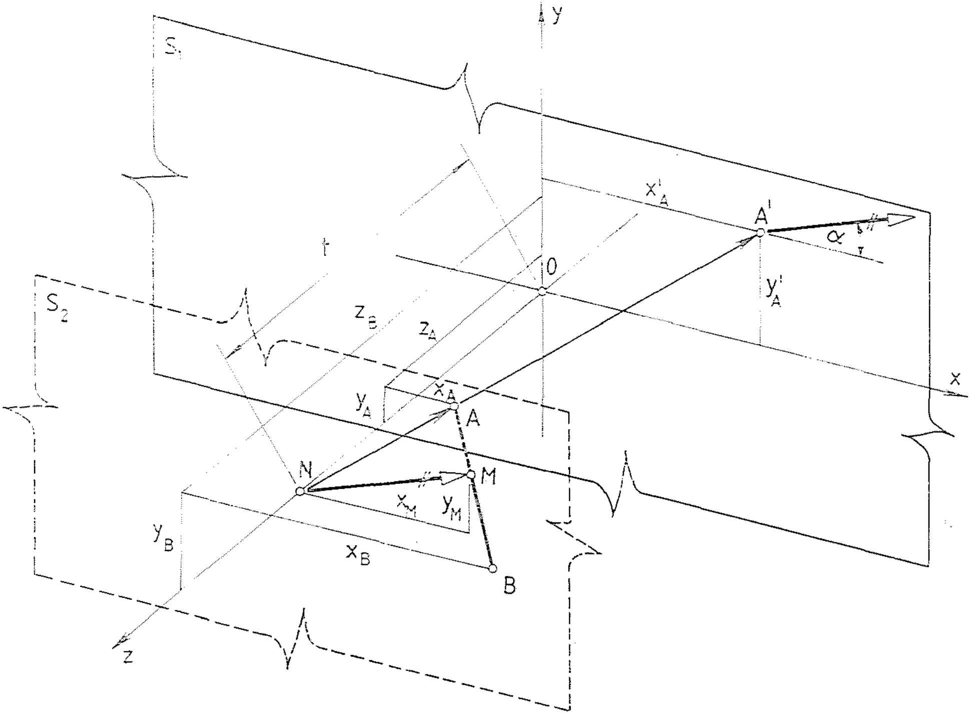

Fig. 2

3. The gist of display according to visibility

To now, points and length have been concerned with. Obviously, however, visibility is decided by surface elements. Graphic routines being always expected to help some finite element program, let the following stipulations be made for these surface elements:

- they are defined by four spatial points (not absolutely in the same plane) in a given order of connection;
- they join each other all along their sides;
- they do not intersect each other.

The idea underlying the visibility display is to determine the display order of these elements so that surface elements ever closer to the looking point are displayed consecutively. Of course, to be effective, interior of the just displayed element has to be cancelled or filled with some colour.

The four spatial points defining the element define in the general case a saddle surface probably requiring a rather complex analysis. Thus, let these elements be decomposed to triangles. These plane triangles in space are much easier to analyze and — in view of the approximateness of FEM — without detriment to the correctness of display.

Saddle surface elements can be decomposed along one diagonal to two triangle elements or — taking an internal point — to four ones. This internal point may be defined as interdiagonal middle point of the normal transversals of the two diagonals. The first variant requiring much less of running time has been adopted.

Now, visibility display of the triangles comprises two main problems:

1. Sequence of tracing the triangles (ordering);

2. among any two triangle elements, finding that nearer to the looking point.

4. Ordering

Ordering is only possible if no triangular elements are related by so-called circular overlapping. Circular overlapping is that where the set of triangles comprises a partial set (of at least three elements) where drawing can be ended with none of the elements, as illustrated in Fig. 3. Since in displaying real structures, circular overlapping can hardly be realized, in the following, absence of circular overlapping will be another initial stipulation.

Among two triangles, one may be found to be nearer or farther from the looking point than the other, or, as concerns the drawing order, they may be independent.

The problem of ordering leads to the following mathematical model. Let us have a set H of elements with the following relations:

$a < b$; (a is nearer than b)

$a > b$: (a is farther than b)

$a \times b$ (a and b are independent of each other)

$(a, b \in H)$.

Statements can only be made in relation to two elements, since the basis of interrelation counteracts to assign some measure to set elements. Thereby the well-known triangle inequality fails. For

$$a < b \text{ and } b < c$$

it is not certain that $a < c$. Namely, cases of $a \times c$ or $a > c$ (circular overlapping) are possible.

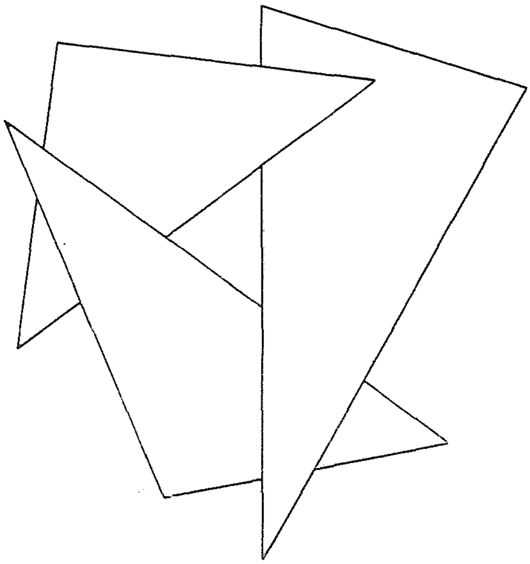

Fig. 3

What is more, ordering is not as unambiguous as usual among elements that can be assigned values. Let e.g. the following relations be known among four elements:

$$a < b;\quad a > c;\quad a \times d;$$
$$b > c;\quad b \times d;\quad c < d.$$

Sequences c, a, d, b ; c, d, a, b or c, a, b, d are equally correct but no solution were possible for $b > c$ instead of $b < c$.

Initially, ordering of elements of set H was attempted by realizing first n (numbered) elements of the set as ordered, and trying to insert anothes element in the order. But ranging such a new element may radically alter the (relative) order of the originally ordered n elements, too ! Let us consider e.g. triangles in Fig. 4. Previous to ranging triangle c, a correct order of display is b, a (since $a \times b$). After c has been ranged, it appears that the only correct display order is a, c, b changing the previous order, inducing to abandon this direct way of rearranging.

Thereafter an indirect way has been attempted. Obviously, if an element in the set is at place i (correct for the display order), then it is irrelevant for it if the relative order of elements following (or preceding) it changes. Its position at i remains correct. If the order up to (and including) element i is known to be correct, then the position of element i relative to subsequent elements is correct if:

$$a_i > a_j \text{ or } a_i \times a_j;\ (a_i, a_j \in H) \text{ for every } j > i.$$

This train of thought suits to check correctness of the instantaneous order by checking all the elements beginning with the first one. For n elements exactly in the proper order, this means evaluation of $(n-1) + (n-2) + \ldots + 1$ that is, $n(n-1)/2$ relations.

In checking element i, suppose $a_i < a_j$ to be found for some $j > i$. Thus, element i is in a wrong position, the order has to be corrected. This is advisab ly

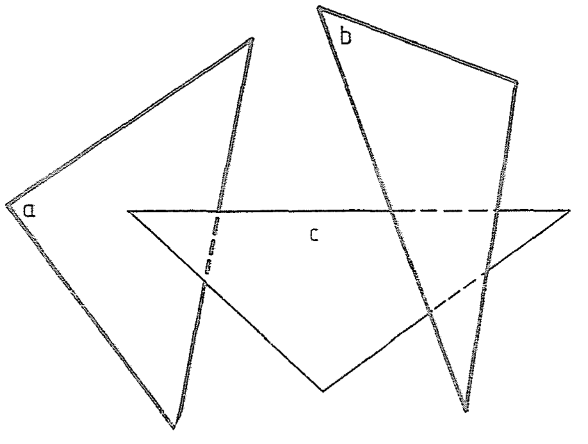

Fig. 4

done by exchanging a_i for a_j, while elements from i to $(j-1)$ get shifted toward subscripts higher by one. Thereby the original element a_j gets to i, and element a_i to $(i+1)$. To shift the other elements is convenient by saving the relative order of the original element i and the subsequent elements $(j-i-1)$ deemed to be correct. Once the order has been corrected, position of the new element a_i has to be checked again. Correction and checking have to be iterated until checking shows the order to be correct. In the case of circular overlapping, this method obviously results in an endless cycle. Assuming a total of n triangles, in checking element i, max. $(n-1)$ corrections may be assumed. Corrections more than that point to circular overlapping, helping to recognize the endless cycle (hence, the phenomenon of circular overlapping). Thus, it is clear that ordering means evaluation of max.

$$n(n-1)/2 + (n-1)(n-2)/2 + \ldots + 3\cdot 2/2 + 2\cdot 1/2 =$$
$$= \big[(n-1)n^2 - n\big(1+3+5+\ldots+(2n-3)\big) + \big(0\cdot 1 + 1\cdot 2 + \ldots +$$
$$+ (n-2)(n-1)\big)\big]2 = n(n-1)/2 + \big(0\cdot 1 + 1\cdot 2 + \ldots + (n-2)(n-1)\big)/2$$

relations.

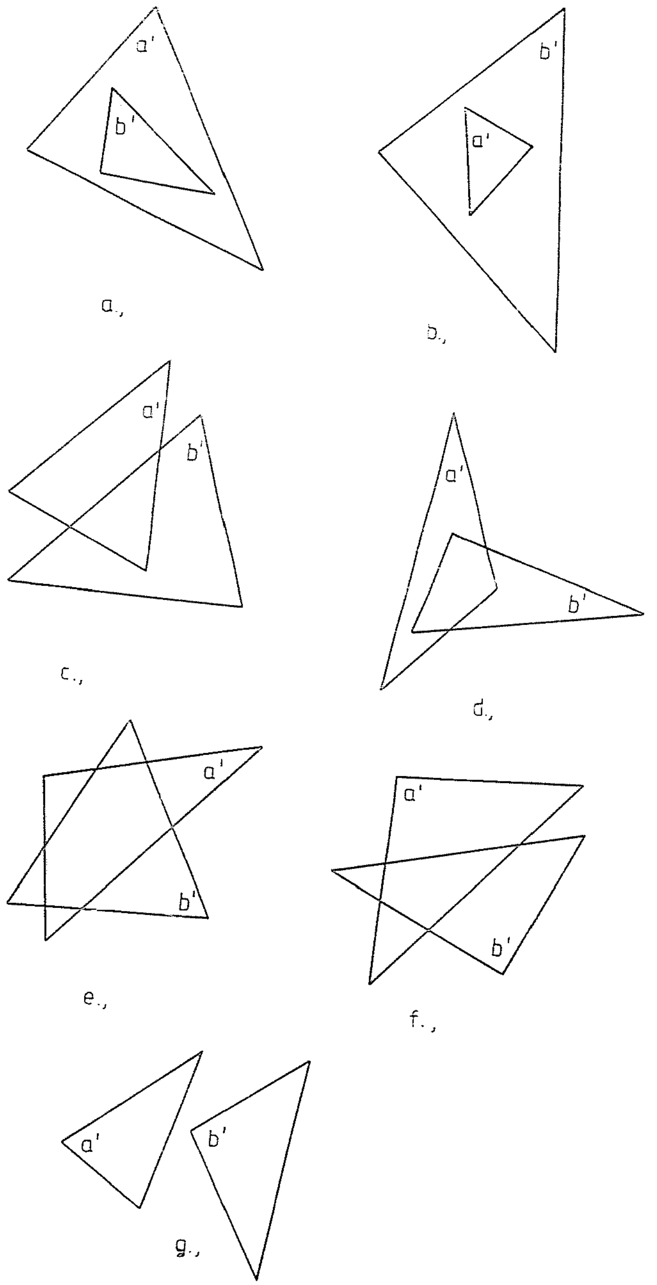

Fig. 5

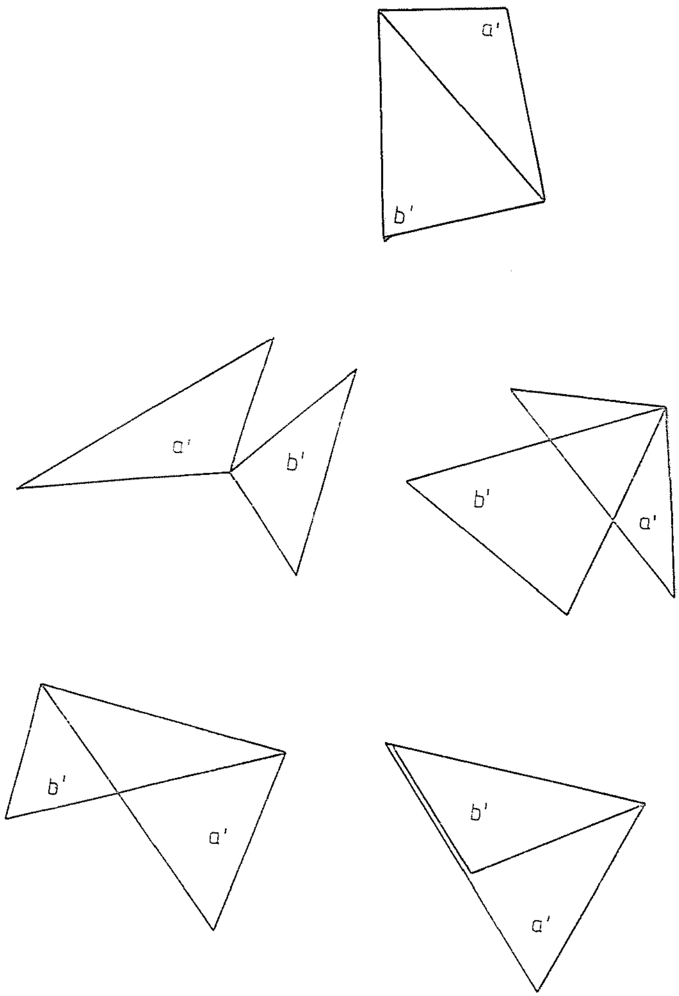

Fig. 6

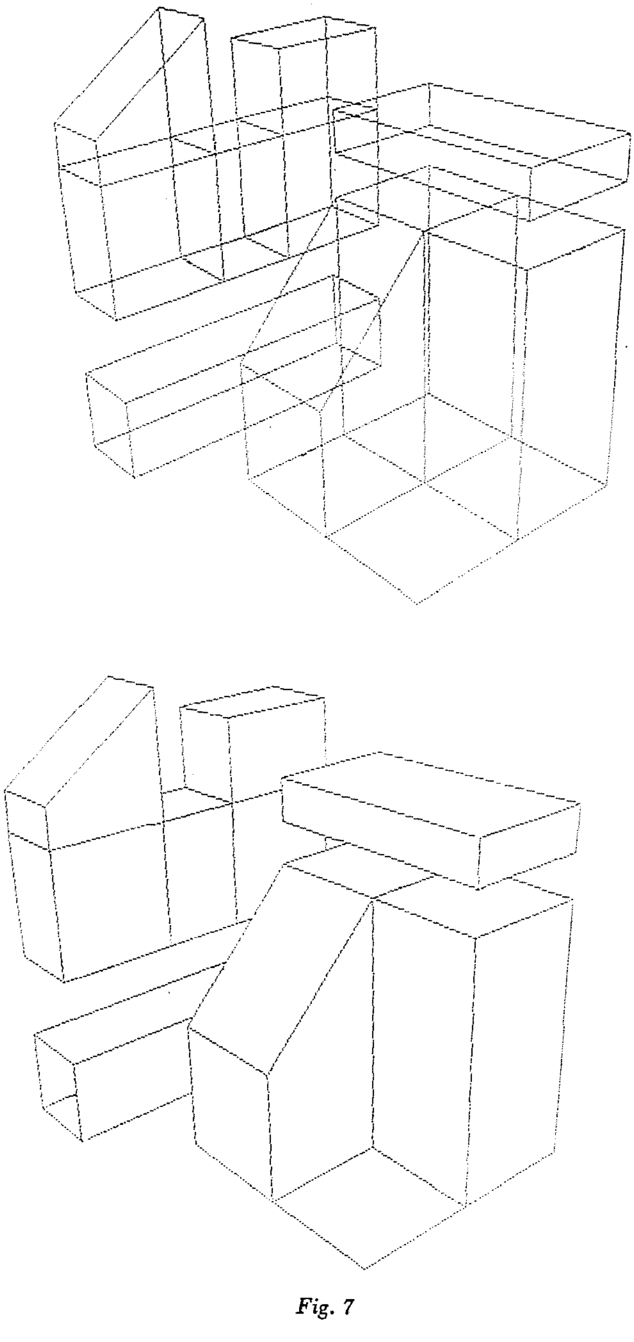

Fig. 7

5. Evaluation of proximity

To evaluate relations between pairs of elements in set H, it has to be decided, which one of each two elements is the nearer to the looking point. To that, let us consider relative arrangement possibilities of projected images of two triangles in Fig. 5. Figures a, b, c, d show cases where one or more corners of a triangle lie on the area of the other triangle, while in Figures e and f, only triangle edges intersect. At last, Figure g shows the case of independent elements.

A procedure for finding the triangle the closer to the looking point has first been developed for positions in Figures a, b, c, d, let us consider, e.g., whether corners of triangle a' can be expressed as convex linear combinations of corners of triangle b'. (In the following, projected images of triangles in space will be marked by ' as a distinction from the original ones.) In the positive case, let us connect the corner of triangle a' inside triangle b' with the looking point, and find the intersection between this line and triangle b. The relation can be unambiguously evaluated from this intersection and the distance of the looking point from the corner of the triangle on this line. If none of the corners of triangle a' is inside triangle b', then this procedure has also to be performed for triangle b'. If none of the triangle corners are inside the other triangle, then only cases e, f or g in Fig. 5 are possible.

To recognize cases e and f, a cycle should be organized to compute intersections of edges of a' and b'. If a computed intersection is an internal point of the examined edges of both triangles a' and b' (that is, convex linear combination of the two corners defining the edge) then there is certainly a case e or f. Now, connecting this intersection to the looking point, then finding intersections between this line and the spatial edges corresponding to the intersected projected edges, distances between these two intersections from the looking point unambiguously show which of the triangles is closer to the looking angle.

At last, let us point to the handling of marginal cases shown in Fig. 6. Stipulations have to be written so that these marginal cases (corresponding to correct visibility) belong to one of the cases shown in Fig. 5.

To illustrate the outlined procedure, Fig. 7 shows an overall view and a visibility picture of a structure formed of tetragons.

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