PARETO OPTIMA OF REINFORCED CONCRETE FRAMES

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Abstract

Optimal design techniques have been extensively applied to structural design in case of one objective function. They have hardly been used with several objective functions.

In this paper the multicriterion optimization of reinforced concrete frames is considered and the numerical method for determining the Pareto optimal set of the problem is presented. The criteria to be minimized are the weight of the frame, the volume of reinforcement for the structure and in certain cases the stability criteria. The solution is based on the vector optimization theory.

1. Introduction

During the 1960's and 1970's new directions were developed in structural design [9], [5] and the multicriterion optimization was one of these. The progress of vector optimization algorythms gave some possibilities to elaborate the methods in structural analysis and helped to develop the possibilities of automatic design. In mathematical programming the concept of Pareto optimality was first used in 1951 by Koopmans [10] and in 1955 by Gass [7]. Several applications of optimal structural design were presented in literature during the last decade and the multicriterion optimization is the newest direction in this special field [8], [15]. One can read about the mathematical technique in works by Bernau [1], Brosowski [3], [4], Koski [11] and Osyczka [14]. Koski [13] and Osyczka [14] gave a general description of multicriterion optimization in structural design. Koski and Silvennoinen [12] considered a truss structure problem where weight and some displacements were chosen as objective functions. A very interesting industrial application has been presented by Eschenauer [6] where the optimum design of a radiotelescope and shape optimization of beam structure were discussed. He described the strategy of structural design in this special field as well.

In this paper the multiobjective problem formulations are discussed first and second the application is presented in the domain of the optimal design of reinforced frames.

The cross-sectional dimensions and the volume of reinforcement are designed taking into consideration the stress and buckling criteria. The objective functions contain the volume of the structure and that of the reinforcement.

Finally, numerical examples are presented.

2. Mathematical Review

In this chapter the multicriterion mathematical programming problem is discussed briefly. First, the Pareto optimum is defined which generally gives a set of solutions. Secondly, some scalarization methods are described. With the help of these methods we can determine the Pareto optimal points. Each of these methods is a function scalarization in which the vector objective functions are transformed into a scalar objective function. Minimizing this scalar function we can obtain a Pareto optimal solution for the vector optimum problem.

A multicriterion optimization problem can be formulated as follows:

$$\min_{x \in L} f(x) \tag{2.1}$$

where $x = \{x_1, x_2, \ldots, x_n\}^T$ is a vector of decision variables in R^n, $f(x) = [f_1(x), \ldots, f_k(x)]^T$ is the vector of objective functions in R^k, L is the set of feasible solutions, given in the form

$$L = \{x\colon\ g_j(x) \leq 0,\ h_i(x) = 0,\ j = 1, 2, \ldots, m,\ i = 1, 2, \ldots, p < n\} \tag{2.2}$$

2.1 Pareto Optimum

The concept of this optimum was formulated by V. Pareto in 1896 and this is the most important part of multicriterion analysis at present.

Definition: A vector $\bar{x} \in L$ is Pareto optimal for problem (2.1) if and only if there exists no $x \in L$ that $f_i(x) \leq f_i(\bar{x})$, $i = 1, 2, \ldots, m$ and $f_j(x) < f_j(\bar{x})$ for at least one j.

This definition is based on the principle that the vector $\bar{x}$ is chosen as the optimal if no criterion can be improved without worsening at least one other criterion.

The Pareto optimum in general gives a set of solutions and not a single solution. In Fig. 1 the graphical illustration of a Pareto optimum can be seen. For this let F be the map of the feasible set, L, under the mapping of $f = (f_1, \ldots, f_k)$ defined by the objective functions.

One can see in Fig. 1 that the set of optimal solutions is non-convex, even in the simplest case, where the constraints and the objective functions are linear.

2.2. *Methods of Solution*

2.2.1. *Min-Max Optimum*

This method was introduced by Koski (1981). In his works one can find several numerical examples for this method [11], [12], [13].

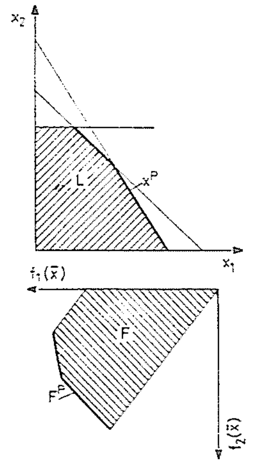

Fig. 1. Graphical illustration of the Pareto Optimum

First, a reference point, which is the so-called ideal solution, has to be defined.

$$f^0 = \left[\min_{x \in L} f_1(x), \min_{x \in L} f_2(x), \ldots, \min_{x \in L} f_n(x)\right]^T. \tag{2.3}$$

So one has to solve m scalar optimization problems. In general this ideal solution is not a feasible one $\left(f^0 \notin f(L)\right)$. The distance between these two points can be measured by the metric function:

$$d_\infty(\bar{f}, f^0) = \max_{i \in | I} |\bar{f}_i - f_i^0| \tag{2.4}$$

Minimizing this function we get an efficient point for the vector optimization problem. If one is interested in further efficient points, the possibilities are given to obtain them for appropriate chosen vectors $\bar{f}$ [15].

Numerical difficulties may occur when minimizing the function from (2.4) we propose for the problem the following scalarization.

The normalized vector objective function is:

$$\bar{f}(x) = [\bar{f}_1(x), \bar{f}_2(x), \ldots, \bar{f}_m(x)]^T \tag{2.5}$$

where

$$\text{where } \bar{f}_i(x) = \frac{f_i(x) - \min f_i(x)}{\max f_i(x) - \min f_i(x)}\,. \tag{2.6}$$

So the values of each normalized criterion are limited to an equal range $(\bar{f}_i(x) \in [0, 1])$. In this case the ideal solution is $f^0 = 0$. Our problem is formulated as follows:

$$\min_{\bar{f} \in f(L)} d_\infty(\bar{f}, 0)\,. \tag{2.7}$$

2.2.2 *Method of Weighting Objectives*

The method of weighting objectives is a well known approach to vector optimization problems. The basis of this method is summing all the objective functions using different weights for each. So the scalar problem has the following objective function:

$$f(\bar{x}) = \sum_{i=1}^{m} \lambda_i f_i(\bar{x}) \tag{2.8}$$

where $\lambda_i \geq 0$ are the weighting coefficients. It is usually assumed to

$$\sum_{i=1}^{m} \lambda_i = 1 . \tag{2.9}$$

Bernau [1], Koski [11], Osyczka [14] wrote about some disadvantages of this method. A main disadvantage of this technique is that it is not possible

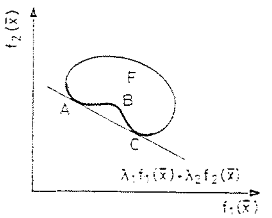

Fig. 2

to find all the Pareto-optimal points for non-convex problems in spite of varying the weighting coefficients [2]. Seeking the minimum of (2.8) depends not only on λ_i values but also on the scale of the objective functions.

Figure 2. Geometrical interpretation of the weighting method in case of two objective functions. In this case the efficient point, B, between A and C, cannot be determined by this method.

2.2.3 *Scalarization with Parametric levels*

For linear vector optimization problems Brosowski [3], [4] investigated a scalarization, which leads to the following scalar problem:

$$\begin{aligned}
&\min\ t \\
&t \in R \\
&g_i(x) \leq 0 && i = 1, \ldots, n \\
&h_j(x) = 0 && j = 1, \ldots, 1 \\
&f_k(x) - t \leq y_k && k = 1, \ldots, m
\end{aligned} \tag{2.10}$$

where

t — a scalar

x — vector of unknown

$g_i(x)$ and $h_j(x)$ — originally constraints of the vector optimization problem

n, l, m — number of inequalities, equalities and objective functions respectively

$f_k(x)$ — the k-th objective function

y_k — the demanded level of the k-th objective function.

By minimizing t in (2.10) we determine a feasible point x for which the levels $y_k + t$ can be chosen as "minimal". A slight generalization of the problem (2.10) is the following scalar problem:

$$\begin{array}{lll} \min\ t & & \\ t \in R & & \\ g_i(x) \leq 0 & i = 1, \ldots, n & \\ h_j(x) = 0 & j = 1, \ldots, 1 & \qquad (2.11) \\ f_k(x) \leq y_k - t \cdot Z_k & k = 1, \ldots, m & \end{array}$$

where

$y \in R^m$ and $Z \in R^m$ (R^m is the space of objective functions)

$Z > 0$

y and Z are arbitrarily chosen.

Those models only give slightly efficient points [1]. A program system [1], [2] was elaborated at the Computer and Automation Institute of the Hungarian Academy of Sciences. The program system automatically produces the vectors y and z.

Here the point with a minimal functional is chosen for each objective function from calculated feasible points.

Let these points be $s_1, \ldots, s_k$ then the vectors y and z are chosen as

$$y_i = p_i(s_i) \qquad i = 1, \ldots, k \tag{2.12}$$

$$z_i = \frac{1}{k} \sum_{j=1}^{k} p_i(s_j) - y_i \quad i = 1, \ldots, k$$

This choice has two advantages:

— the scalarization can be used easily

— y and z vectors are not dependent on the scale of the objective functions.

Our examples show that this method is suitable in a non-convex case, as well [2], [17].

3. Optimal Design of Reinforced Concrete Frames

The optimization modelling is of primary importance. It includes the determination of the modelling of the structure (materials, supports) as mechanical criteria and the mathematical strategy, which contains the design variables and the formulation of constraints and objective functions. Our model is based on the Hungarian Standard, which is similar to the DIN.

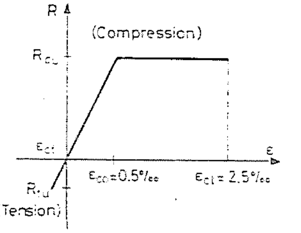

Fig. 3. $R - \varepsilon$ diagram for concrete

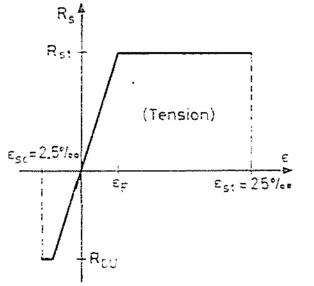

Fig. 4. $R - \varepsilon$ diagram for steel

It is supposed that:

— the stress-strain diagram for concrete can be seen in Fig. 3;

— the tensile strength (R_{tn}) of concrete is taken into consideration only in case of shearing;

— ultimate-load theory is applied;

— the stress-strain diagram for steel can be seen in Fig. 4;

— Generally, $R_{su} = R_{st}$ and $\varepsilon_{st} > \varepsilon_{sC}$ ($\sim$ ten times);

— the static loads are acting on the nodes;

— the frame is a planar structure;

— the cross-section is rectangular.

The general optimal frame problem may be stated as follows:

Given are: loading conditions (magnitude, location and type) support and joint conditions (type); frame configuration (number of spans and stories and, therefore, total number of members).

To find are: the cross-sectional dimensions and the volume of longitudinal reinforcement.

All the following constraints and objective functions were used at the optimal design. Generally, bending moment (M), axial force (N) and shear force act on the cross-section.

M and N are reduced into a normal force being outside of the cross-section center: Fig. 5

where:

$$e_{Mo} = \frac{M}{N}$$

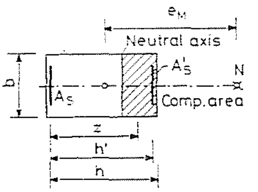

Fig. 5

Each cross-section has to satisfy:

$$A_{co} \cdot Z \geq \frac{N \cdot e_M}{k \cdot R_{st}} \tag{3.1}$$

where:

A_{co} — area of compressive-concrete
A_s, A'_s — area of longitudinal tensile and compressive reinforcement
Z — distance between the inner-forces
R_{st} — tensile strength of the steel
k — proposional factor ($1 \leq k \leq 2$)
e_M — depends on the stability criteria ($e_M > e_{Mo}$).

The longitudinal reinforcement is computed as follows. There are two cases.

a) No required tensile reinforcement.
b) Tensile reinforcement required.

It depends on whether $N \cdot e_M \gtreqless M_0$
where M_0 ultimate bending moment in ideal case based on the Hungarian Standards.
Reinforcement in case of
no required longitudinal tensile reinforcement

$$A'_s = 0$$

$$A_s = \frac{N \cdot e_M}{Z \cdot R_{st}}. \tag{3.2, a}$$

Reinforcement in case of
required longitudinal tensile reinforcement

$$A'_s = \frac{N \cdot e_M - M_0}{h' \cdot R_{su}} \tag{3.2, b}$$

$$A_s = \frac{M_0}{Z \cdot R_{st}} + \frac{N \cdot e_M - M_0}{h' \cdot R_{cu}} - \frac{N}{R_{st}}.$$

Each cross-section has to satisfy (3.3) in case of shearing.

$$0.25 \cdot A \cdot R_{cu} \geq T \tag{3.3}$$

where: A — the area of the cross-section
T — the acting shear force.
The shear reinforcement is computed on the basis of the Hungarian Standard.

The criteria of stability are taken into consideration at the computation of excentricity (e_M) and the internal resisting axial force (N_L) has to be greater than the acting axial force (N).

$$N_L - N \geq 0.0 \tag{3.4}$$

The above constraint (3.4) has to be satisfied in each section of the frame. Beside this, so-called technological constraints have to be used. There are specified regularities (story column, floor beam dimensions etc.) and permissible ranges of member sizes (clearances, minimum thicknesses etc.).

In the majority of the cases there are two objective functions: the volume of the structure (C1) and the volume of the reinforcement (C2) (3.5).

$$\text{C1:} \quad \sum_i A_{ci} H_i \to \min$$

$$\text{C2:} \quad V = \sum_i A_{si} \cdot L_i \to \min \tag{3.5}$$

where: A_{ci}, H_i — the cross-section and length of the i-th member, respectively

V — the volume of the reinforcement.

4. Numerical Example

Consider the reinforced concrete frame loaded as shown in Fig. 6.

Where: $F_1 = 200$ kN $\quad F_2 = 100$ kN $\quad p = 10$ kN/m
$L = 6.00$ m $\quad H = 4.00$ m

The redundant forces are determined according to the force method.

Each section of members has to satisfy 3.1, 3.2, 3.3, 3.4 conditions and the technological criteria. These are the constraints of optimality and the objective functions are 3.5.

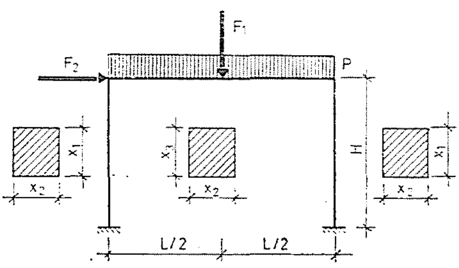

Fig. 6

For reinforced concrete frames, the significant aspect of this formulation is that it results in designs that automatically satisfy the basic requirements of any limit design [16].

Three Pareto optimal sets are calculated, each corresponding to certain values of parameters λ_1 and λ_2. The results are given in Table 1. The cross-sectional dimensions and the volume of the reinforcement can be seen in the Table I.

Table 1

	Number	$x1$ [cm]	$x2$ [cm]	$x3$ [cm]	Volume of reinforcement $E \times 1000$ [cm³]
	0	90	40	90	1.05571
$\lambda_1 = 0.25$	1	90.1	39.9	90.2	1.04935
	2	91.6	39.6	91.7	0.987142
$\lambda_2 = 0.75$	3	105.5	36.6	106.8	0.481877
	0	90	40	90	1.05571
$\lambda_1 = 0.5$	1	89.9	25.9	120.9	0.634289
	2	90.0	26.14	121.0	0.630073
$\lambda_2 = 0.5$	3	90.3	26.5	121.2	0.622922
	4	91.5	26.6	121.9	0.591739
	0	90	40	90	1.05571
Using parametric levels	1	91.5	34.3	104.5	0.774127
	2	91.4	32.5	102.8	0.798345
	3	91.7	37.8	108.8	0.723498
	4	91.5	27.8	98.6	0.862120
	5	91.5	27.8	98.6	0.861867

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