EVALUATION OF THE CREEP OF ROCKS ON THE BASIS OF A RHEOLOGICAL MATERIAL MODEL

M. GÁLOS

Department of Mineralogy and Geology,
Technical University, H-1521 Budapest

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Abstract

The paper shows a possible method for the evaluation of creep experiments carried out in the rock-physical laboratory of the Department of Mineralogy and Geology of the Budapest Technical University, by which the rheological material characteristics needed for the dimensioning of constructions in rock environments built of elastic rocks can be determined.

1. The role of rock properties in projection

The interaction of engineering constructions and rocky environment has to be considered in both the placing and dimensioning of structures. Dimensioning requires a series of generalizations with which the material properties of the construction and the rock environment are idealized corresponding to the requirements of calculations. The generalization of the properties of the rock environment means an engineering geological modelling the fundamentals and practical usage of which have been summarized by Gálos and Kertész (1983).

The engineering geological model (geotechnical model) of the rock environment of the construction is built of model elements — rock bodies, rock blocks — that can be endowed with the starting conditions of dimensioning, namely, with the homogeneous isotropic condition system generally applied in engineering practice. To this, the elastic and plastic condition or their combination also make contribution, their consideration in calculations necessitates the use of material characteristics.

In dimensioning the rock environment interacting with constructions, the finite element method is very well applicable, in which case material characteristics are needed for setting up the rigidity matrix. This means, at the same time, that material properties should be provided for the design engineer, which are required by the chosen model, best representing reality. In engineering practice we are often satisfied with the elastic condition system, i.e. only two material characteristics, the modulus of elastiticy and the Poisson's number are considered in the construction of the rigidity matrix.

The rapid development of computer technique made it possible to construct more complex rigidity matrices and to compute them taking thereby into account also the rheological properties. In modelling the geological environment, while keeping the generally used elastic condition system valid, it is expedient to use an elastic-rheological material model in which the model elements expressing the elastic behaviour are unchanged and the time course of stresses and deformations are taken into consideration by using further model elements.

The above conditions are fulfilled for a series of rocks by the elastic-viscous material model of Kelvin and Voigt, which describes primary creep very satisfactorily (Kaliszky, 1975). The mechanical model of the material

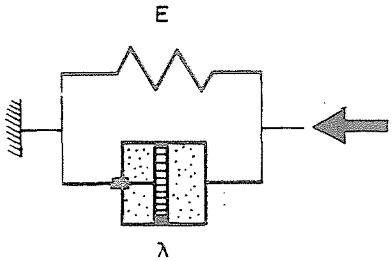

Fig. 1. The mechanical model of the Kelvin-Voigt equation

model consisting of a coupled elastic Hooke and a viscous Newton model element is shown in Fig. 1. Its well-known material equations are:

$$s_{ij} = s^e_{ij} + s^v_{ij}$$

$$s^e_{ij} = 2Ge_{ij}, \quad s^v_{ij} = 2\mu\dot{e}_{ij}$$

$$s_{ij} = 2Ge_{ij} + 2\mu\dot{e}_{ij}$$

$$\dot{\varepsilon}_{kk} = \frac{1}{A}\dot{\sigma}_{kk} \quad A = \frac{E}{1-2\nu}$$

where

ν is the Poisson's ratio

μ is the viscosity constant of the rock called internal damping coefficient,

A is the bulk modulus and

E is the modulus of elastiticy

The properties needed for dimensioning should be given in the form of numbers or functions for the experts, correspondingly to the model, so that they can be built into calculations as basic data. The time course of the stress and deformation properties of rocks became a very important group of properties owing to the above reasons. Attention has been focused on creep experiments, as the simplest technique for obtaining them.

2. Principles and practice of creep studies

Though creep experiments are, in principle, the simplest among rock mechanical studies, in practice, experts carrying out the investigations often have to fight with difficulties originating from the requirement that stresses should be kept at a standard level for a longer period and during this time deformation has to be continuously recorded.

Creep experiments have a long record for loose, sedimentary rocks (soils) and they are routine studies in laboratories dealing with soil mechanics. Creep studies for compact rocks were first made in the seventies, for which purpose the Department of Mechanics at the University for Agriculture at Gödöllő constructed a mechanical device applying small loads, and the Department of Mineralogy and Geology of the Technical University in Budapest built a hydraulic-rheological experimental device suitable for high loads having 4 measuring places, the so-called REOSZ.

In Figure 2, the device is shown by which creep measurements with a uniaxial stress state can be carried out for 4 samples simultaneously. It is also possible to have identical stresses for all the four places (i.e. the experiment can be "repeated" four times simultaneously), but different stress levels

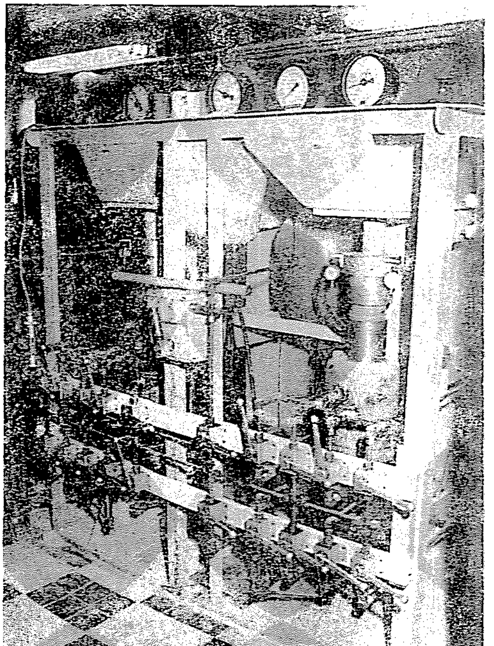

Fig. 2. The rheological measuring device operating at the Department of Mineralogy and Geology of the Budapest Technical University (REOSZ)

can also be applied. The maximum load at the individual measuring places may be 400 kN. Deformations caused by load are measured by high accuracy meters.

Experiments can be carried out on standard test pieces, i.e. cylindrical specimens with a height to diameter ratio of 2 : 1, with diameters of $d = 5.0 \pm 0.5$ cm, but cubic test pieces of a maximum edge length of 20 cm may also be tested in the device.

3. Evaluation of creep experiments by the Kelvin-Voigt model

In the evaluation of uniaxial compression measurements carried out in the rheological measuring device, the state equation of an elastic-viscous model is used, which, for a unidirectional stress state, may be written as:

$$\sigma = E\varepsilon + \lambda\dot{\varepsilon}$$

where σ is the stress,
ε is the deformation,
$\dot{\varepsilon}$ is the rate of deformation,
E is the elastic modulus and
λ is the linear viscosity or creep factor in GPa h units (MPa day).

The conversion between the linear creep coefficient and the internal damping coefficient in the general form of the material equation for this model can be performed by the use of Poisson's number (m) as follows:

$$\lambda = 2\eta \frac{m+1}{m} \qquad \eta = \frac{\lambda}{2} \cdot \frac{m}{m+1} \qquad m = 1/\nu$$

At a constant stress ($\sigma_0 = 0$), the material equation becomes

$$\dot{\varepsilon} + \frac{E}{\lambda}\varepsilon = \frac{\sigma_0}{\lambda}$$

This is an inhomogeneous differential equation whose solution — if time is measured from the point where the load reaches the value σ_0 and the deformation caused by it is ε_0 is:

$$\varepsilon = \frac{\sigma_0}{E} - \left(\frac{\sigma_0}{E} - \varepsilon_0\right) e^{-\frac{E}{\lambda}t}$$

The creep curve described by the above equation shows that according to the Kelvin-Voigt model, creep occurs via reversible deformations, i.e. if stress σ_0 is stopped, deformation decreases asymptotically to zero in the case of $t \to \infty$.

Within the elastic range, by using the value ε_∞ belonging to $t \to \infty$, the modulus of elasticity can be expressed as

$$E = \frac{\sigma_0}{\varepsilon_\infty}$$

By substituting this into the creep equation, the curve shown in Fig. 3 becomes

$$\varepsilon = \varepsilon_\infty - (\varepsilon_\infty - \varepsilon_0) e^{-\frac{E}{\lambda} t}$$

Thus, in creep experiments, we can determine from the creep curve

— the deformation in infinite time and

— the linear viscosity or creep coefficient.

Deformation in infinite time is sometimes very difficult to indicate, since the rock shows deformation for a long time even under constant stress, and thus only extrapolation can lead to a value for deformation at $t \to \infty$. If, during the time of the experiment, a deformation state is reached which can be regarded as the end of the creeping process, the largest deformation measured can be taken as equal to ε_∞. If, however, the creeping process exceeds the time of the experiment, t_∞ has to be given a probable value and the deformation at that time can be considered to be ε_∞. To this, the creep curve should be given by some function.

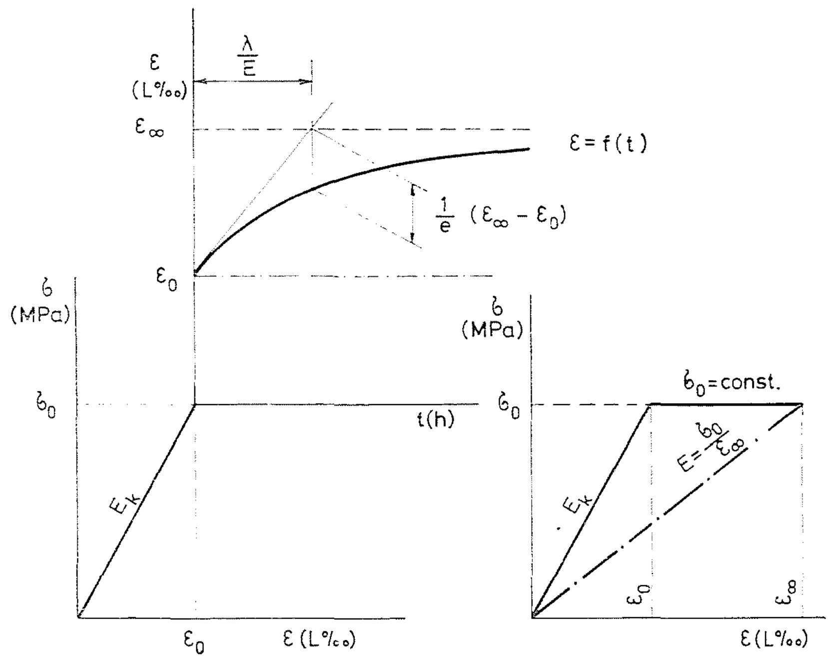

Fig. 3. Creep at constant stress, by the Kelvin-Voigt material model

Numerous correlations are given in literature for the description of the creep curves of rocks. Széki (1986) gives a good survey according to the character of the functions used, but he notes that the mathematical description of creep is only a tool for determining the motions in time to be expected.

According to our experience, the logarithmic functions containing one unknown — which was mentioned by Gálos (1982) — gives a well-usable correlation. Thus in the evaluation of our experiments, we use the creep equation

$$\varepsilon = \varepsilon_0 + r \ln(1 + t)$$

(where r is the rheological characteristic depending on the rock material) for the calculations for which Gálos (1982) provided a method.

We fit the creep curve to the points of time, t_i and deformations ε_i belonging to them by the least squares method and determine the values of ε_0 and r by the correlations

$$\varepsilon_0 = \frac{\sum_{i=1}^{n} \varepsilon_i \sum_{i=1}^{n} \ln^2 (1 + t_i) - \sum_{i=1}^{n} \ln (1 + t_i) \sum_{i=1}^{n} \varepsilon_i \ln (1 + t_i)}{n \sum_{i=1}^{n} \ln^2 (1 + t_i) - \left[\sum_{i=1}^{n} \ln (1 + t_i) \right]^2}$$

$$r = \frac{n \sum_{i=1}^{n} \varepsilon_i \ln (1 + t_i) - \sum_{i=1}^{n} \ln (1 + t_i) \sum_{i=1}^{n} \varepsilon_i}{n \sum_{i=1}^{n} \ln^2 (1 + t_i) - \left[\sum_{i=1}^{n} \ln (1 + t_i) \right]^2} .$$

Deformation ε_∞ at time $t \to \infty$ is determined by substituting a time t_∞ on the basis of equation

$$\varepsilon_\infty = \varepsilon_0 + r \ln (1 + t_\infty)$$

In the calculation of ε_∞, Széki's suggestion (1986) is followed, i.e. the rheological constant is determined from the first part of the creep curve and the goodness of the curve is checked on the last points. The above correlation can expediently be used in dimensioning the rock environment of engineering constructions, as the substitution of t_∞ may occur on the basis of the expected life time of the construction. In geotechnical specifications the life times of different constructions are given. They are summarized in Table 1.

If a deformation taking place in one year is considered to be 100%, the percental change of the increment of deformation in the chosen longer life time is shown in Fig. 4. This increment is negligible for provisional constructions, for half-permanent buildings or constructions such as e.g. mine drifts,

Table 1

Projected life-times of constructions

Nature of construction	Life-time
Provisional building	1.5 years
Half-permanent building for short periods	5 years
Half-permanent building for long periods	15 years
Permanent building	50 years
Building to remain	150 years

it is already 40%, whereas for permanent constructions (e.g. channels, tunnels, underground parking houses, storage rooms, etc.) it may reach even 60—80%.

The shape of the creep curve depends on the material quality of the rock, characterizable by its rock-physical state. This means, at the same time, that the texture determined by the binding between the rock-forming components is the carrier of creep properties. In this respect, the expression of rock-forming component is used in a wider sense. Rock-forming components are also pores and pore-filling materials, thus creep takes place in a three-phase system. Therefore the stability of the rock-physical state has to be ensured during creep experiments and in the evaluation and provision of the results this state has also to be given.

At a constant stress level, after having determined the deformation at infinite time, we can resubstitute the $t - \varepsilon_t$ pairs into the creep equation,

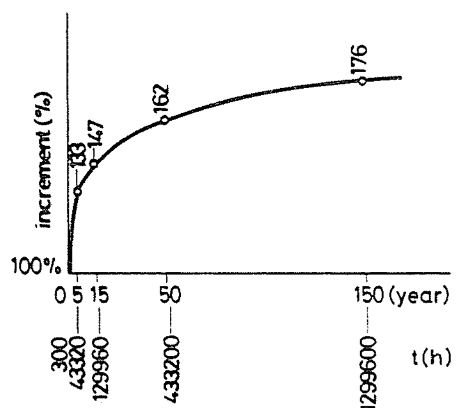

Fig. 4. Creep deformation increments as the function of life time

and then the linear viscosity or creep coefficient can be determined from equation

$$\lambda = \frac{\sigma_0 \cdot t}{\varepsilon_\infty[\ln(\varepsilon_\infty - \varepsilon_0) - \ln(\varepsilon_\infty - \varepsilon_t)]} = \frac{E \cdot t}{\ln \frac{\varepsilon_\infty - \varepsilon_0}{\varepsilon_\infty - \varepsilon_t}}$$

thus all the rock-physical characteristics needed for the calculation of the rock block are available.

Creep studies are designed in a way that the most possible material characteristics may be measured on each specimen and the reliability of data is increased by repeating the measurements several times. The stress program is built into the design of the experiment correspondingly to the objective to be achieved. In the study of the rock environment of a given construction, the constant stress in creep experiments is the load on the construction in normal function or that measured in the rock environment of the construction. If, however, our objective is to determine rock properties in general, creep experiments should be carried out by increasing the stress step by step. In this case, the measurements are continued in each step till evaluable creep is observed. The two types of stress programs are illustrated by Figs 5 and 6.

The simplest stress program (see Fig. 5) consists of five sections:

section "*a*": fast loading up to the desired level,

section "*b*": creep for a period corresponding to the specific density of the rock (according to experience for 1000—2000 h),

section "*c*": relaxation to a basic zero level stress,

section "*d*": keeping stress at the basic level for a predetermined period,

section "*e*": fracture caused by a rate of stress variation according to a standard (MSZ 18285/1).

From the corresponding stress-deformation and deformation-time values in individual periods, the deformation and creep curves can be constructed. Thus, from curves taken at a constant rate of stress variation (section "*e*"), the rock-physical modulus of elasticity (E_k) and Poisson's ratio whereas from the section taken at constant stress (section "*b*") the viscosity coefficient can be determined.

By using the above approach for the creep curve — i.e. by the determination of ε_∞ — the modulus of elasticity of the rock can be calculated.

By the application of the stress program illustrated in Fig. 6 the behaviour of the rock can be evaluated in the whole range of elasticity. Each step of stress consists of two characteristic parts:

section "*a*": fast loading up to the desired level,

section "*b*": creep study at a constant stress level for a predetermined period.

The steps of loading during the experiment are chosen on the basis of the stress-deformation curve in a way that within the linear section of the

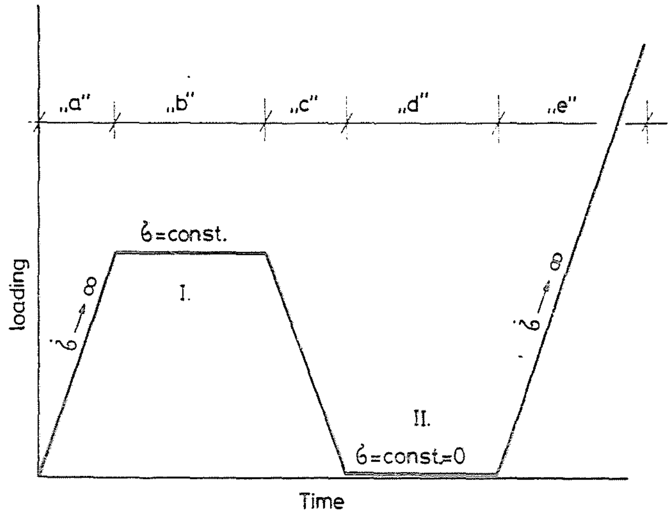

Fig. 5. Stress program of creep experiments for one-step loading

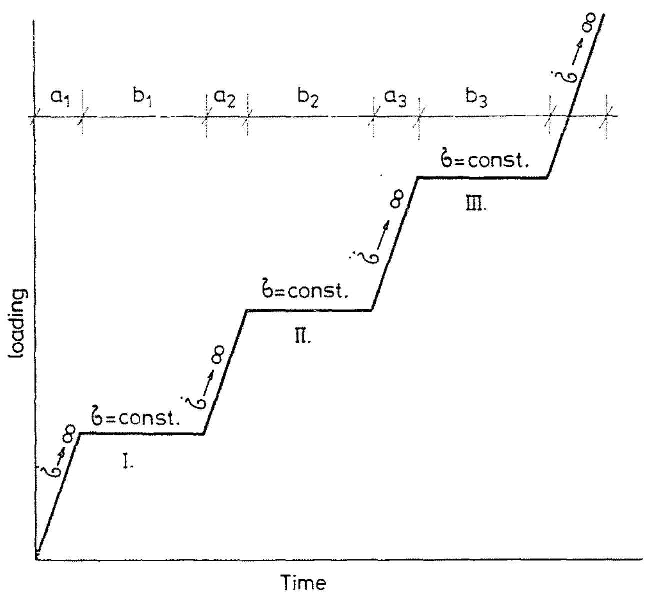

Fig. 6. Stress program of creep experiments for multistep loading

curve representing the elastic range, three-four levels of stress are pointed out.

Creep experiments carried out at different stresses, in addition to the determination of the rock characteristics corresponding to the material model, are also suitable for determining the limit of elasticity as well as for the explanation of certain structural observations, namely for proving the compression phenomena due to the microstructure of the rock.

The creep experiments carried out in the rock-physical laboratory of the Department of Mineralogy and Geology show that the deformation properties of rocks can be evaluated only as a function of the rock-physical state by constantly considering the binding between the rock-forming components. The process of creep is described by the creep curve whose shape reflects the changes in the internal structure of the rock in time caused by stress. These changes may be rearrangements at the borderlines of minerals in the rock processes connected with the internal structure of minerals (e.g. twinning), formation of microfissures, breakage or closing of pores.

The difficulty in rock mechanics is that the internal part of rocks cannot be investigated, hence in describing creep, the effect of the internal structure cannot be represented by individual coefficients.

Rocks are classified by Morlier (1966) according to their rheological behaviour under uniaxial compressive load as

elastic-rigid,
viscous-elastic and
viscous-plastic

groups. He states, on the basis of experiments, that there is a close correlation between the momentary behaviour and the time course of rocks. The Kelvin-Voigt model is suitable for describing the creep behaviour of rocks in the first two groups, whereas rocks belonging to the third group are characterizable e.g. by the Maxwell model. To the former groups belong fresh, slightly weathered rock variants with mineralic, glued and carbonatic textures, whereas to the viscous-plastic group rocks of high clay mineral contents, e.g. marls and strongly weathered decomposed rocks belong.

Several characteristic creep curves of rocks evaluated on the basis of the Kelvin-Voigt model are shown in the figures. They prove that these experiments can be used also for the determination of the desired material characteristics. The experiments were carried out in the region where the behaviour of the rocks can be considered elastic, correspondingly to the basic conditions of the model. This paper does not deal with creep phenomena outside this elastic range.

The most spectacular result of creep experiments is the creep curve which serves as a basis for evaluation. Within the elastic range, creep curves belonging to different stress levels theoretically form a series of parallel curves. In the logarithmic representation with respect to time, they are parallel straight lines for rocks of identical material quality. In this case, the slopes of the straight lines represent differences in rock properties.

The creep curve of rocks whose texture ensures that they dissipate the energy originating from the stress induced by a convective energy transfer

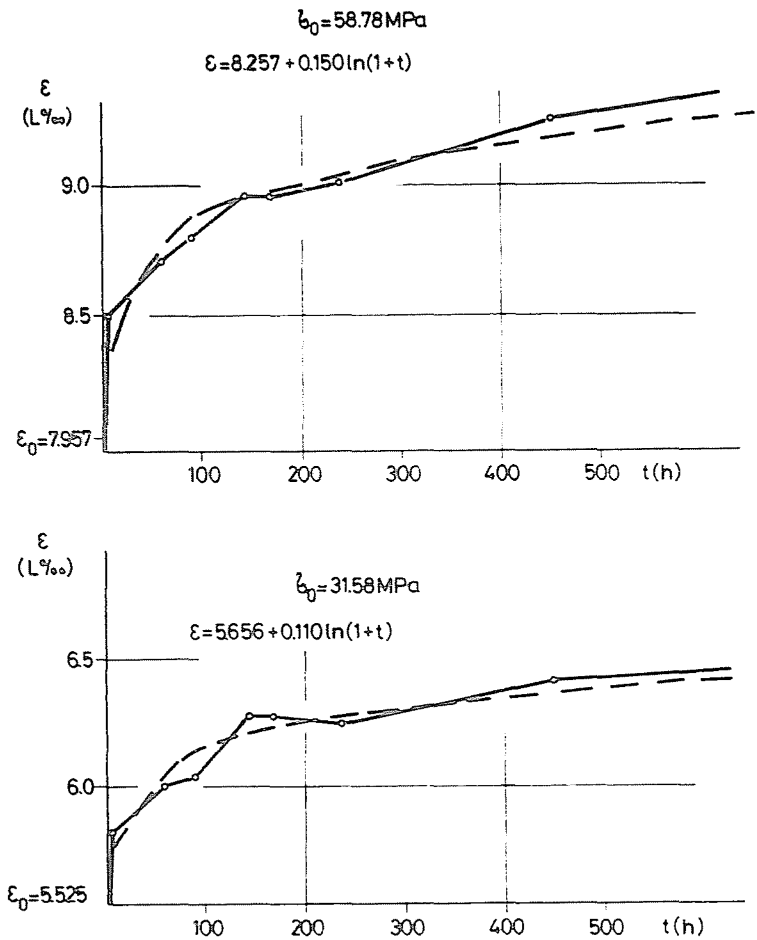

Fig. 7. Creep curves of granite (Emelianovsk, USSR) at constant stresses $\sigma_0 = 31.58$ MPa and $\sigma_0 = 58.78$ MPa (The rock is a reddish-brown colour, large particle, biotite-containing amphybole-granite. The quartz content is of irregular shape and wavey extinction. The potassium feldspar is of a microclinic character, the amount of typical ortoclase is smaller. The particle size of the components is 1—10 mm. The bulk density of the rock in an air-dry rock-physical state is 2660 kg/m^3, its apparent porosity is 0.40 V%, its uniaxial compression strength is 156.6 MPa, its rock-physical modulus of elasticity is 52.60 GPa)

either within the rock-forming components or at their boundaries, is steeper than for rocks in which this energy transfer occurs mainly via conduction.

Creep curves show characteristic differences for different textures, but they can also vary within the same rock texture due to the modifying effect of the particle size, pore distribution and the degree of weathering.

The differences can be observed well on the creep curves shown as examples in Figs 7—11. Textural characteristics are given in the captions.

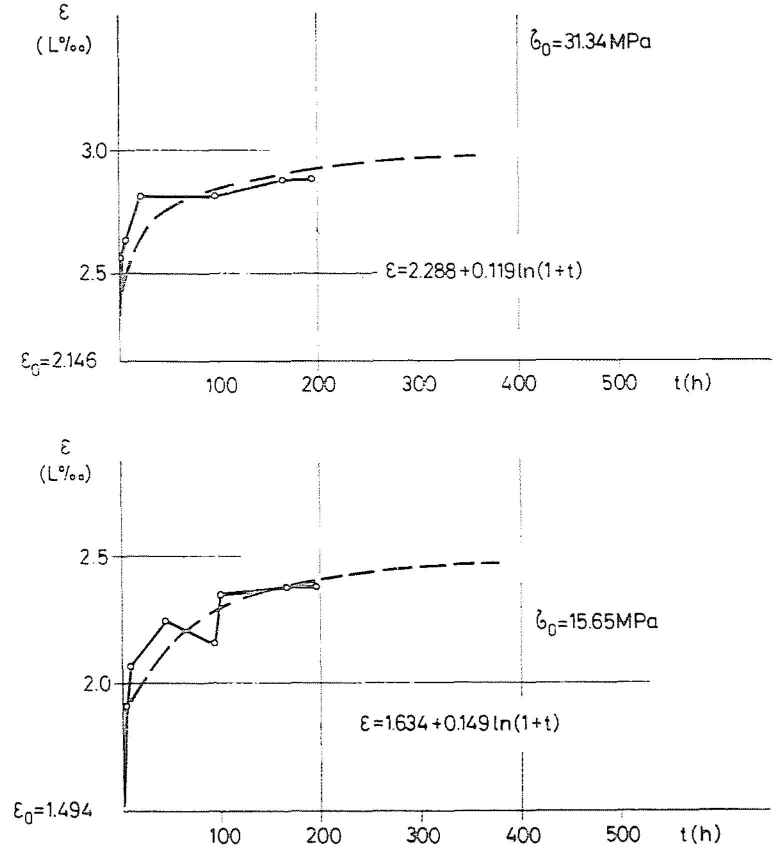

Fig. 8. Creep curves of andesite from Nógrádkövesd at constant stresses $\sigma_0 = 15.65$ MPa and $\sigma_0 = 31.34$ MPa (The rock is a pyroxenic andesite of greyish black colour with small particle sizes and porphyric texture. Plagioclases in it are isometric, twinplate structures, while pyroxenes have a lath nature. Its main mass is microcrystalline, with olivine slightly decomposed. The body density of the rock in the air-dry rock-physical state is 2730 kg/m^3, its apparent porosity is 2.9 V%, its uniaxial compression strength is 151.7 MPa, its rock-physical modulus of elasticity is 55.58 GPa)

Creep curves were taken at various stresses for individual rock variants. This enabled us to observe the compression phenomena under load. This compression manifested itself in the increase of the modulus of elasticity and the linear viscosity coefficient. The material characteristics calculated for the rocks are summarized in Table 2.

The data in Table 2 are not yet suitable for synthesizing generalization, but they exemplify the effect of geological variety. In the present phase of our work we carry out creep experiments at the level of constant stress exerted by the load and the effect of the construction on the rock environment. Our

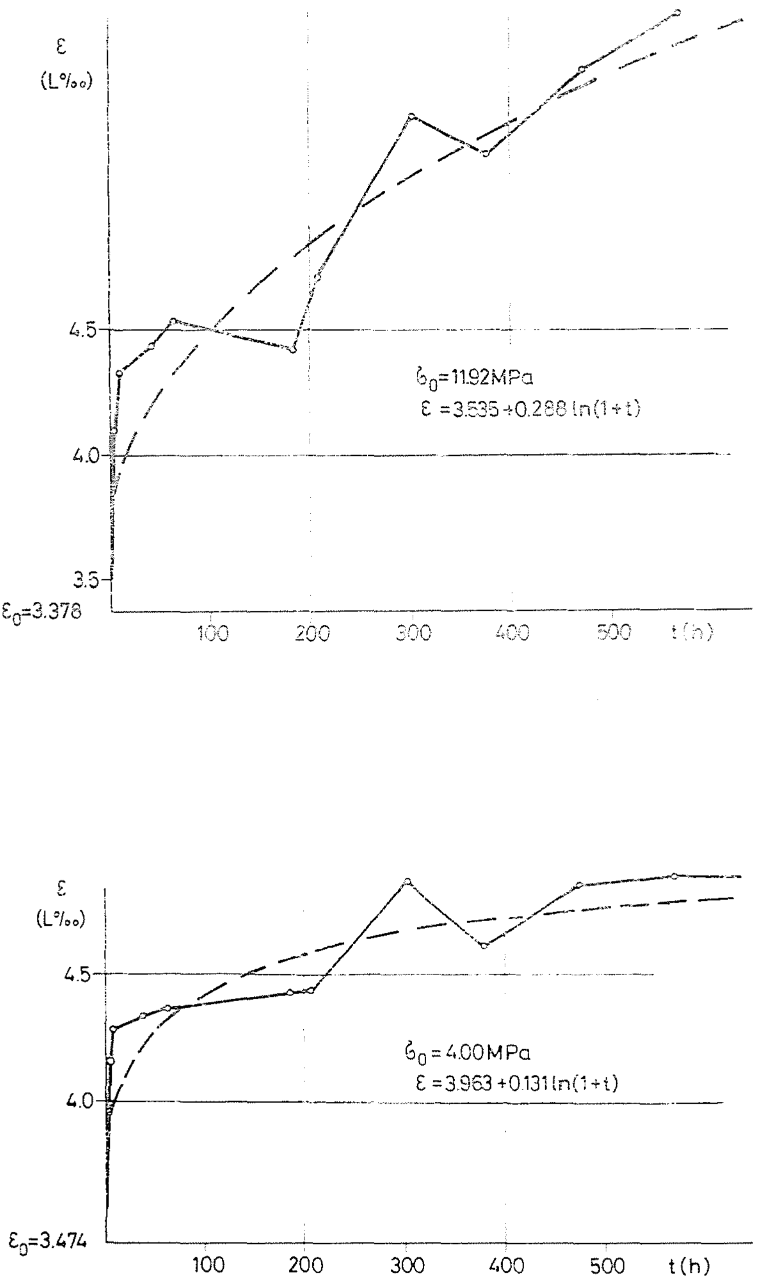

Fig. 9. Creep curves of the rhyolite tuff from Sirok at constant stresses $\sigma_0 = 4.00$ MPa and $\sigma_0 = 11.92$ MPa (The rock is a slightly siliceous sedimentary tuff of yellowish-grey colour with small particles and tuffeous texture. Pumices in it are isometric. In the light grey glassy material crystals of feldspars, biotite and quartz as well as pumice is embedded. The body density of the rock in air-dry rock-physical state is 1630 kg/m^3, its apparent porosity is 15.0 V%, the uniaxial compression strength is 43.50 MPa, the rock-physical modulus of elasticity is 6.20 GPa)

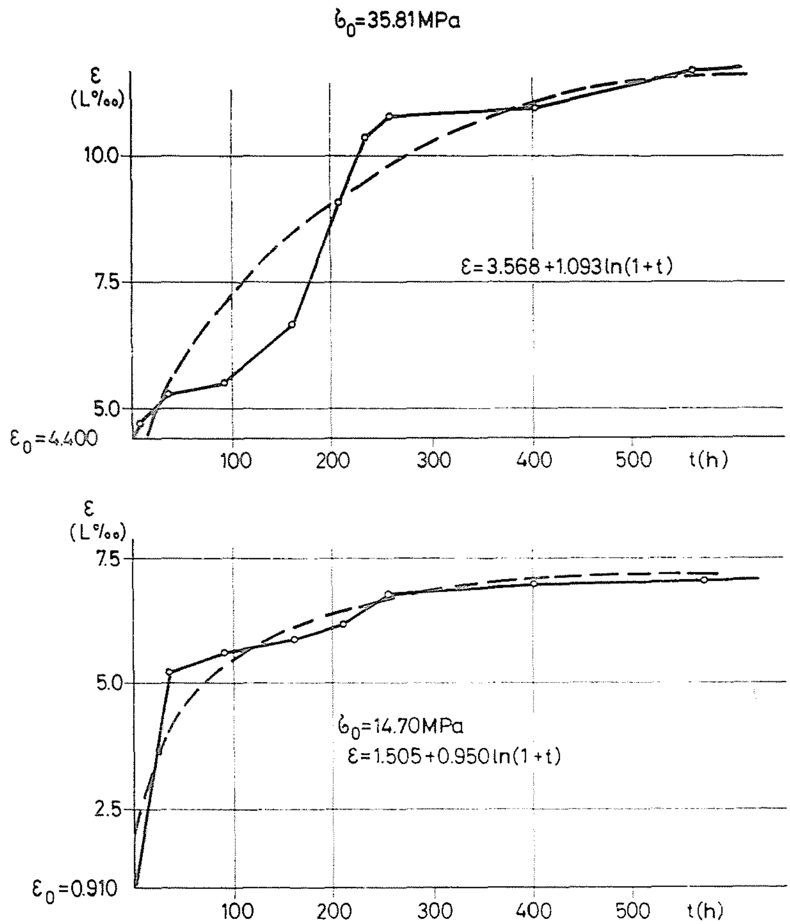

Fig. 10. Creep curves of sandstone from Balatonrendes at constant stresses $\sigma_0 = 14.70$ MPa and $\sigma_0 = 35.81$ MPa (The rock is a slightly porous sandstone of red colour, with small homogeneous particle sizes, siliceous binding and glued texture. The composition is: 60—65% quartz and 35—40% binding material. The quartz particles are edgy, only slightly rounded and of wavy extinction. The binding material has a basic material character, strongly coloured, amorphous or microcrystalline. The bulk density of the rock is 2480 kg/m^3 in an air-dry state, its apparent porosity is 9 V%, its uniaxial compression strength is 45.7 MPa, its rock-physical modulus of elasticity is 11.50 GPa)

aim is, of course, to elaborate the correlations between the rheological material characteristics in the whole region of load for the widest possible circle of rocks. However, numerous experiments are needed for this purpose, and since creep studies are very time-consuming, it is not possible to achieve spectacularly fast results.

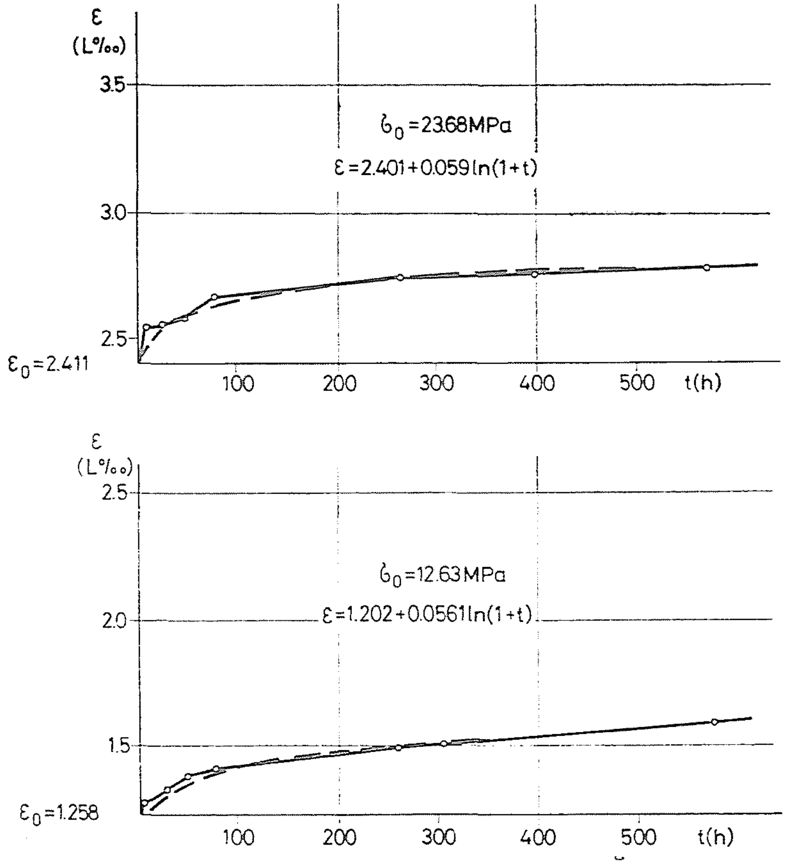

Fig. 11. Creep curves of dolomite from Gánt at constant stresses $\sigma_0 = 12.63$ MPa and $\sigma_0 = 23.68$ MPa (The rock is a slightly veined, compact, microcrystalline dolomite of a yellowish-pink-white colour, in which the dolomite crystals are isometric, their particle size is 0.02—0.04 mm in 60 vol.% and about 0.20 mm in 40 vol.%, slightly serrated and cracked. The bulk density of the rock in an air-dry state is 2760 kg/m^3, its apparent porosity is 1.5%, its uniaxial compression strength is 118.5 MPa, its rock-physical modulus of elasticity is 52.66 GPa)

Table 2

Rheological rock-physical material characteristics
(for t_∞ = 10 years)

Rock and occurrence	r (h^{-1})	λ (GPa · h)	E (MPa)	σ_0 (MPa)
Granite (Emeljanovszk)	0.1102	2889	4571	31.58
	0.1504	3724	5897	58.78
Andesite (Nógrádkövesd)	0.149	2972	4702	16.65
	0.119	5436	8608	31.34
Rhyolite tuff (Sirok)	0.1309	50	827	4.00
	0.2876	976	1608	11.92
Sandstone (Balatonrendes)	0.9502	755	1195	14.70
	1.0930	1415	2239	38.81
Dolomite (Gánt)	0.0559	14457	6873	12.63
	0.0592	37921	7703	23.68

Summary

The results of creep experiments show that rheological processes cannot be neglected when studying the behaviour of rocks. From the results of creep experiments rock characteristics can be determined which are necessary for to investigate the interaction between constructions and rock environment, i.e. for dimensioning work.

The mechanical material model most suitable for calculations can be chosen on the basis of laboratory experiments. In the majority of rocks — with crystalline, porphyritic, glued and carbonatic texture — the elastic-viscous material model (Kelvin-Voigt model) is applicable well, the real creep measured experimentally can be described with it suitably. It can also be established both by calculation with this model and experimentally, that deformation at a given level of stress is the greater, the slower is the change of stress.

The shortcomings of the elastic-viscous material model are pointed out by Asszonyi and Richter (1969), most significant being that the model cannot include the phenomenon of relaxation.

The rheological measuring device (REOSZ) described in this paper can be made suitable for measuring relaxation by building in appropriate automatics. Our further goal is to carry out this development and to perform creep studies on rocks at constant stress complemented by relaxation studies.

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Dr. Miklós GÁLOS H-1521, Budapest