UTILIZATION OF WATER RESOURCES OF SMALL CATCHMENT AREAS USING THE THEORY OF BAYES' ANALYSIS

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Abstract

The exact survey of water resources of small catchment areas is very important to plan the utilization of resources. The wide use of Bayes' thesis assists to solve this problem. This paper gives an example for the Tetves-brook at section Visz (A = 70.3 km²). The resulting diagrams (Fig. 4) are suitable for technical planning, e.g. for the design of utilization of water resources produced by a given small catchment area for agricultural — irrigaton — purposes.

Introduction

The exact survey of water resources of small catchment areas is very important for the planning of the utilization of resources. The wide use of Bayes' thesis permits to solve this problem.

Mathematical model structure

The thesis with the proportion of full probability can be written for continuous, discrete probability variables and also for the so-called "mixed" case.

The formulae are the following:

In the case of discrete events:

$$P(B_i \mid A) = \frac{P(A \mid B_i)\, P(B_j)}{\sum_{j=1}^{n} P(A \mid B_j)\, P(B_j)}$$

where $P(B_i) \neq 0$ is the probability of the investigated event B_i, $P(A) \neq 0$, in the probability of an optional event A (P is the sign of the probability)

In the case of continuous probability variables:

$$g(y \mid x) = \frac{f(x \mid y)\, g(y)}{f(x)} = \frac{f(x \mid y)\, g(y)}{\int_{-\infty}^{+\infty} f(x \mid y)\, g(y)\, dy}$$

where $g(y)$, $f(x)$ are the frequency functions of the probability variables η and ξ.

In the case of the composition of discrete and continuous probability variables:

$$g(y \mid x_k) = \frac{P(\xi = x_k \mid \eta = y)\, g(y)}{P(\xi = x_n)} = \frac{P(\xi = x_k \mid \eta = y)\, g(y)}{\int_{-\infty}^{+\infty} P(\xi = x_k \mid \eta = y)\, g(y)\, dy}$$

where ξ — is the discrete and η is the continuous probability variable.

In the case of discrete probability variables:

$$P(\xi = x_k \mid \eta = y_j) = \frac{P(\eta{=}y \mid \xi{=}x_k) P(\xi{=}x_u)}{P(\eta{=}y_i)} =$$

$$= \frac{P(\eta = y_j \mid \xi = x_k)\, P(\xi = x_k)}{\sum_{l=1}^{\infty} P(\eta = y_j \mid \xi = x_l)\, P(\xi = x_l)}$$

where on the analogy of the discrete events:

$$B_1 : \xi = x_1,\ B_2 : \xi = x_2, \ldots, B_n : \xi = x_n, \ldots \text{ and } A : \eta = y_j.$$

The use of the theses makes possible the investigation of water resources of small watercourses by a new method, as well as the calculation of utilizable water quantities.

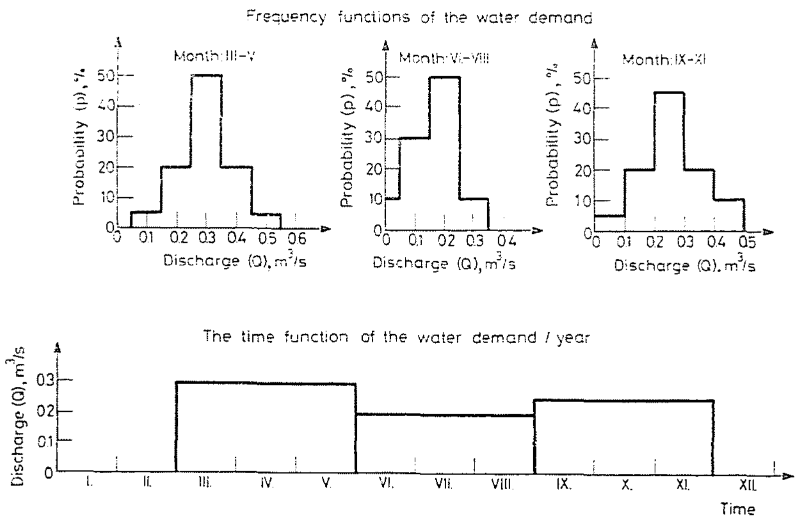

Fig. 1. **The function of water demand**

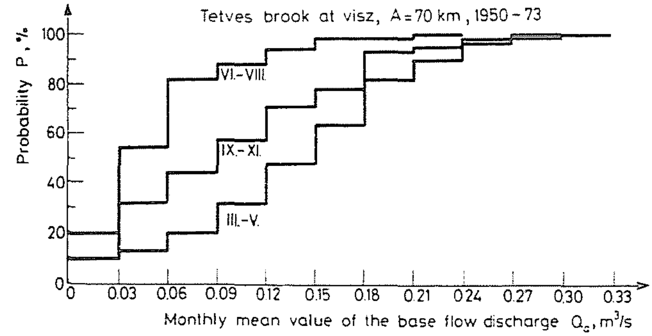

Fig. 2. Distributions of the base flow

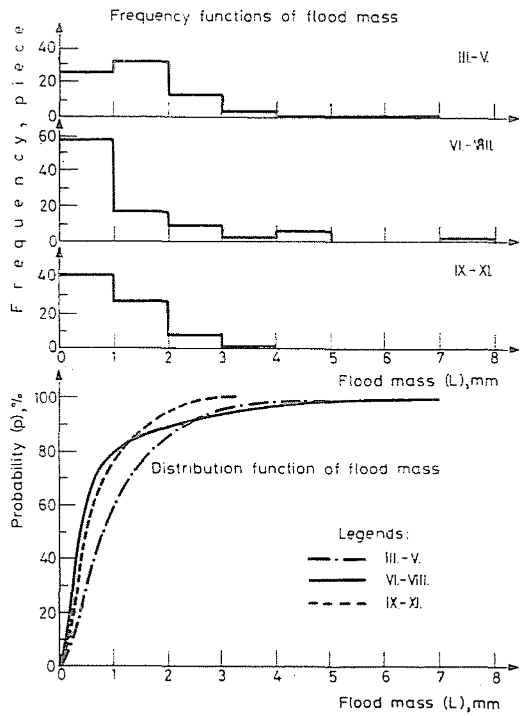

Fig. 3. The frequency and distribution functions of flood mass. Tetves brook at Visz. 1950—72.

Numerical method to calculate the theoretical model

To illustrate the utilization of the method the data related to section Visz of the Tetves-brook were used. (The dimension of the catchment area: $A = 70.3\ km^2$.)

The task is to utilize the water resources of the small watercourse using a small plantshop reservoir. The water demand and the probability distribution of water movement parameters are known (Fig. 1) The problem consists of determining the probability concerning the fulfilment of the given water demands. Water movement can be characterized after having determined the water discharge time series by a superposition of the base flow (Q_a) and the flood waves of the surface runoff (Q_Δ):

$$Q(t) = Q_a(t) + Q_\Delta(t)$$

Hereupon the distribution functions of the base flow (Fig. 2) and flood wave volumes (Fig 3) were calculated. The distribution of the number of monthly flood waves and intervals between them was described by Poisson, and/or an exponential distribution.

These were the following:

$$P(\xi = k) = p_k = \frac{1{,}47^k}{k\,!} e^{-1{,}47} \; (k = 0, 1, 2, \ldots)$$

$$f(x) = 0{,}0667 \; e^{-0.0067 \; x}$$

$$M(\xi) = 15.2 \text{ day}$$

$$D(\xi) = 15.0 \text{ day}$$

This means that the water discharge time series of the small watercourse was modelled by an independent, stationary, stochastic process. In the knowledge of water resources and water demands the probability was searched to satisfy the supposed water demand in case of a water discharge base flow, flood wave volume and monthly flood wave number of a certain likelihood. The calculations were performed with the aid of Bayes' analysis. As final result the families of curves related to probabilities of water demand fulfilment ($P(A)$) were obtained (Fig. 4).

$$P(A) = f(n, Q_{ap}),$$

where n — number of monthly flood waves with different, given flood wave volumes, Q_{ap} — the probable value, p, of the base flow.

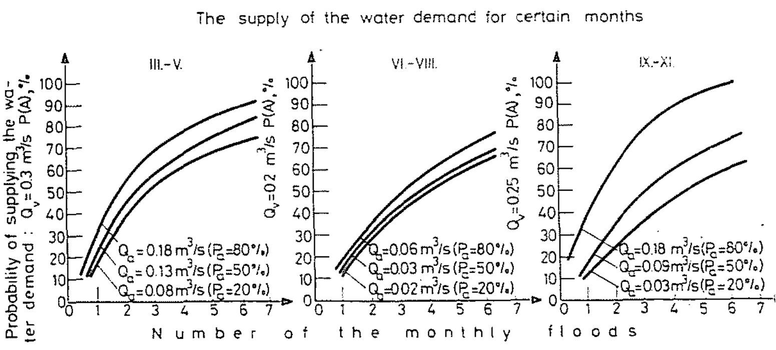

Fig. 4. **Diagrams concerning supplying the water demand. Tetves brook at Visz, A = 70.3 km²**

Conclusions

The final result diagrams are suitable for technical planning, e.g. for the design of utilization of water resources produced by a given small water-course for agricultural — irrigation — purposes.

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