STOCHASTICALLY NONLINEAR ANALYSIS OF SHELLS, CONTAINING RIGID ELEMENTS AND ELASTIC JOINTS

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The paper gives a short review of the state equation and the statistical parameters of a shell containing infinitely rigid elements and elastic joints. The shell rests on elastic supports.

Introduction

In case of prefabricated shell structures we often find that the elements of the shell are much more stiff than the joints so the elements may be considered as infinitely rigid, while the joints are elastic [2]. The aim of this paper is to give a short review on the state equation of the structure under some particular conditions and describe principal formulas of the stochastically nonlinear problem. Numerical examples will be treated in a later contribution.

We make use of the following suppositions:

I) The middle surface of the structure consists of infinitely stiff triangular elements,

II) The elastic properties are represented by generalised spring elements situated in prescribed places along the edges of the shell elements,

III) The load vectors are considered as acting in the center of gravity of the shell elements,

IV) Initial strains are not dealt with.

Kinematic relationships, statical relationships and constitutive equations

The kinematic equations describe the relationships valid between the relative displacements of the joints and the absolute displacements of the gravity centers of the shell elements

$$\Delta \mathbf{w} = \mathbf{G}\mathbf{u}$$

where

u: displacement vector of the gravity centers, in detailed form at the gravity center bearing the subscript a

$$\mathbf{u}_a^* = [u_{ax}\, u_{ay}\, u_{az}\, \varphi_{ax}\, \varphi_{ay}\, \varphi_{az}]$$

and together at the elements (their number being l)

$$\mathbf{u}^* = [\mathbf{u}_1^* \, \mathbf{u}_2^* \dots \mathbf{u}_a^* \dots \mathbf{u}_l^*] .$$

Furthermore at the joint having the subscript c

$$\Delta \mathbf{w}_c^* = [\Delta w_{c,\xi} \, \Delta w_{c,\eta} \, \Delta w_{c,\zeta} \, \Delta \chi_{c,\xi} \, \Delta \chi_{c,\eta} \, \Delta \chi_{c,\zeta}]$$

and together at the joints (their number being k)

$$\Delta \mathbf{w}^* = [\Delta \mathbf{w}_1^* \dots \Delta \mathbf{w}_c^* \dots \Delta \mathbf{w}_k^*]$$

where x, y, z denotes the axes of the global frame while ξ, η, ζ denotes the local coordinates at the point c (we shall return to their positions later).

Introducing the hypermatrix of the orthogonal transformation

$$\mathbf{T}_c = \begin{bmatrix} \cos(\xi,x) & \cos(\xi,y) & \cos(\xi,z) & & & \\ \cos(\eta,x) & \cos(\eta,y) & \cos(\eta,z) & & & \\ \cos(\zeta,x) & \cos(\zeta,y) & \cos(\zeta,z) & & & \\ & & & \cos(\xi,x) & \cos(\xi,y) & \cos(\xi,z) \\ & & & \cos(\eta,x) & \cos(\eta,y) & \cos(\eta,z) \\ & & & \cos(\zeta,x) & \cos(\zeta,y) & \cos(\zeta,z) \end{bmatrix}_{(c)}$$

and the carry over matrix

$$\mathbf{B}_{ba} = \begin{bmatrix} 1 & & & & -(z_a - z_b) & y_a - y_b \\ & 1 & & -(x_a - x_b) & & z_a - z_b \\ & & 1 & -(y_a - y_b) & (x_a - x_b) & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

the blocks of $\mathbf{G}$ are generated by the following formulae

$$\mathbf{G}_{ca} = \mathbf{T}_c \, \mathbf{B}_{b_\beta a} \qquad a = a_\delta$$

if

$$\mathbf{G}_{ca} = -\mathbf{T}_c \, \mathbf{B}_{b_\gamma a} \qquad a = a_\varepsilon$$

else $\mathbf{G}_{ca} = 0$.

b denotes here the subscript of the joint at the shell element, a_δ refers to the element fitting into the joint with subscript c and possessing the higher number, while a_ε to that of the lower number at the boundary of each investigated element.

The equilibrium conditions describe the relationships being valid at each element between the load and the stress resultants. Referring to the principle of virtual work we obtain readily the formula

$$\mathbf{G}^* \mathbf{s} + \mathbf{p} = \mathbf{0}$$

where

$$\mathbf{s}_c^* = [s_{c,\xi} \, s_{c,\eta} \, s_{c,\zeta} \, m_{c,\xi} \, m_{c,\eta} \, m_{c,\zeta}]$$

is a stress vector of six dimensions and

$$\mathbf{s}^* = [\mathbf{s}_1^* \ldots \mathbf{s}_c^* \ldots \mathbf{s}_k^*]$$

(k denotes the number of the joints)

In addition we have to present the constitutive equations. While dealing with them, we have to fix the positions of the local frames: the ξ axis of the coordinate system fastened to the joint with subscript c is the common edge of the shell elements fitting together at c. Axis η is the halving line of the angle formed by both elements in the plane being perpendicular to the edge shaped by the elements. The axis ζ is perpendicular to both ξ and η and is positively directed towards the space covered by the structure.

The constitutive equation at point c is

$$\boldsymbol{\epsilon}_c = \mathbf{H}_c \boldsymbol{\sigma}_c$$

ϵ denotes the elements of the strain tensor, while σ those of the stress tensor. The independent elements of the symmetric tensor $\mathbf{H}_c$ are

$$h_{11} = \frac{1}{E_x} \quad h_{12} = -\frac{\nu_{xy}}{E_y} \quad h_{13} = -\frac{\nu_{xz}}{E_z}$$

$$h_{22} = \frac{1}{E_y} \quad h_{23} = -\frac{\nu_{yz}}{E_z} \quad h_{33} = \frac{1}{E_z}$$

$$h_{44} = \frac{1}{G_{xy}} \quad h_{55} = \frac{1}{G_{yz}} \quad h_{66} = \frac{1}{G_{zx}} .$$

E stands for Young's moduli, G denotes the shear moduli and ν for Poisson factors.

If the elasticity of those parts of the shell, represented by a joint, is substituted by a prism with area h^2 and thickness v, while being in homogeneous state, furthermore the principal axes of the orthotropy coincide with the local frames of the joints, the constitutive equations can be described in the form

$$\Delta \mathbf{w}_c = -\mathbf{A}_c \mathbf{H}_c \mathbf{A}_c^* \mathbf{s}_c$$

where

$$\mathbf{A}_c^* = \frac{1}{hv} \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & v/h & & & \\ & & & & & 1/h \\ & & & 1/h & & \\ & & & & 1/h & \end{bmatrix} .$$

Introducing the flexibility matrices

$$\mathbf{F}_c = \mathbf{A}_c \mathbf{H}_c \mathbf{A}_c^*,$$

$$\Delta \mathbf{w} = \mathbf{F} \mathbf{s}$$

holds. Considering the diagonal matrix $\mathbf{R}$ supporting the boundary elements at their gravity center and combining the equilibrium conditions, the kinematic relationships and the constitutive equations we obtain a hypermatrix equation analogous to the hypermatrix state equation of the bar structures

$$\begin{bmatrix} \mathbf{R} & \mathbf{G}^* \\ \mathbf{G} & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{s} \end{bmatrix} + \begin{bmatrix} \mathbf{p} \\ \mathbf{0} \end{bmatrix} = \mathbf{0}.$$

Random relationships

We will consider just those stochastically nonlinear problems where only the boundary spring coefficients and the components of the load vector are random, while the initial strain vector is supposed to be zero. First we determine the mean value and covariance matrices of displacements and load effects using the method of Taylor's expansions. Then we shall assume that the set of random variables located in the diagonal elements of $\mathbf{R}$ are independent of the random variables of the load vector. The spring coefficients of the supports are random variables of symmetrical to each axis common distribution, assuming strict quality control the "tails" (taken in the common distribution space) are cut and the distribution is redefined in such a way [1], the mean values of $\mathbf{R}$ are contained in matrix $\mathbf{W}$. Using this notation we can introduce a matrix $\mathbf{Y}$ as well,

$$\mathbf{Y} = -\mathbf{R} - M(-\mathbf{R})$$

where $\mathbf{Y}$ is a diagonal matrix, too, containing random variable elements. The diagonal elements of $\mathbf{Y}$ may be condensed in $\mathbf{y}$. Mean value of the vector $\mathbf{y}$ is equal to zero, the covariance matrix for the variables contained in $\mathbf{y}$ is $\mathbf{B}_{yy}$. The random load vector is also determined by the vector of mean values $M(\mathbf{p})$ and by the covariance matrix $\mathbf{B}_{pp}$.

Introducing the matrix

$$\mathbf{A} = -\mathbf{W} + \mathbf{G}^* \mathbf{F}^{-1} \mathbf{G}$$

the vector of displacements can be written in the form

$$\mathbf{u} = (\mathbf{Y} + \mathbf{A})^{-1} \mathbf{p}$$

We expand this random function into Taylor series.

Expanding first the matrix $(\mathbf{Y} + \mathbf{A})^{-1}$,

$$(\mathbf{Y} + \mathbf{A})^{-1} = \{(\mathbf{Y}\mathbf{A}^{-1} + \mathbf{E})\mathbf{A}\}^{-1} = \mathbf{A}^{-1}(\mathbf{Y}\mathbf{A}^{-1} + \mathbf{E})^{-1} =$$
$$= \mathbf{A}^{-1}(\mathbf{E} - \mathbf{Y}\mathbf{A}^{-1} + \mathbf{Y}\mathbf{A}^{-1}\mathbf{Y}\mathbf{A}^{-1} \ldots)$$

holds, provided the norm of matrix $\mathbf{Y}\mathbf{A}^{-1}$ is less than 1. Let us introduce the operation $Di(\mathbf{c}) = \mathbf{C}$ where $\mathbf{c}$ is a vector of n element and $\mathbf{C}$ stands for a diagonal matrix with the dimensions $n \times n$. The elements of vector $\mathbf{c}$ are placed in the pivot elements of matrix $\mathbf{C}$.

Restricting ourselves to a linear approximation, we receive

$$\mathbf{u} = \mathbf{A}^{-1}\mathbf{p} - \mathbf{A}^{-1} Di\{\mathbf{A}^{-1}\mathbf{p}\}\,\mathbf{y}.$$

If the nonlinearity of the problem is not high we can get suitable approximations as follows

$$M(\mathbf{u}) = \mathbf{A}^{-1} M(\mathbf{p})$$

$$\mathbf{B}_{uu} = \mathbf{A}^{-1}\mathbf{B}_{pp}\mathbf{A}^{-1} + \mathbf{A}^{-1} Di\,\{\mathbf{A}^{-1} M(\mathbf{p})\}\,\mathbf{B}_{yy} Di\,\{\mathbf{A}^{-1} M(\mathbf{p})\}\,\mathbf{A}^{-1}.$$

With the aid of the expansion of $\mathbf{u}$ the vector of $\mathbf{s}$ is obtained as

$$\mathbf{s} = -\mathbf{F}^{-1}\mathbf{G}\mathbf{A}^{-1}\mathbf{p} - \mathbf{F}^{-1}\mathbf{G}\mathbf{A}^{-1} D_i\,\{\mathbf{A}^{-1} M(\mathbf{p})\}\,\mathbf{y}.$$

Besides

$$M(\mathbf{s}) = -\mathbf{F}^{-1}\mathbf{G}\mathbf{A}^{-1} M(\mathbf{p})$$

$$\mathbf{B}_{ss} = \mathbf{F}^{-1}\mathbf{G}\mathbf{A}^{-1}\mathbf{B}_{pp}\mathbf{A}^{-1}\mathbf{G}^{*}\mathbf{F}^{-1} +$$

$$+ \mathbf{F}^{-1}\mathbf{G}\mathbf{A}^{-1} Di\,\{\mathbf{A}^{-1} M(\mathbf{p})\}\,\mathbf{B}_{yy} Di\,\{\mathbf{A}^{-1} M(\mathbf{p})\}\,\mathbf{A}^{-1}\mathbf{G}^{*}\mathbf{F}^{-1}.$$

If the nonlinearity is strong, we must be satisfied with the comparison of limit values belonging to in advance specified probability level, obtained by the semi-probabilistic methods, otherwise the upper bound formula for the probability of failure derived in ref. Szentiványi B. (1980, 1981) can be directly used [6, 7]. In case when the random eccentricity of supports should be taken into account or if the randomness of spring coefficients is extremely high the method of realization weighted with its probability can be suggested (ref. Roller et al. 1976, Szentiványi 1977) [3, 5].

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