DYNAMIC ANALYSIS OF THE NONLINEAR BEHAVIOUR OF COMPOSITE STRUCTURES

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Summary

A method has been presented for the analysis of composite building structures under dynamic load, assuming an elasto-plastic material model. In final account, the problem can in any case be reduced to the solution of a second-order differential equation with nonlinear coefficients to be expressed by continuous functions.

Introduction

In examining the effect of dynamic loads on building structures, often it is justified to allow for the occurrence of stresses beyond the elastic limit, or else, of plastic deformations. For a simple structure, the load-deformation process may be described by the diagram of the so-called elasto-plastic material model. In the occurrence of several load-bearing members with different ultimate loads due to dimensions or position, this simple diagram is not valid for the structure as a whole, since members yield at different deformations. For a dynamic-type load, the situation gets still more complex because of the prevalence of the time factor in load bearing. Namely, single structural members reach ultimate capacity as a function of time. In structural systems with no coincident rotation center and centroid, "coupled motion" (simultaneous shifting and rotation) due to load causes further complications in locating elastic and plastic deformations.

All these point to the intricacy of the problem of analyzing dynamic effects, especially if also the possibility of plastic deformations is allowed for, and even international publications are only concerned with solutions oversimplifying the problem. This paper is intended to offer a handy method yielding results of practically adequate accuracy through admissible approximations. Besides, also realistic stress pattern of the building structure is taken into consideration.

Computational Model and System of Differential Equations for the Problem

The computational model has been assumed as a system of as many masses as there are storeys in the building, where diaphragms stiff in the horizontal floor planes are elastically interconnected masses. In the occurrence of horizontal forces, masses are assumed to be able to displacement in their plane alone. The lowermost mass is connected by elastic constraints to the ground, and the others to the overlying and underlying masses. In conformity with the general plane displacement system, constraints can develop forces and moments. In addition, also damping constraints proportional to the displacement and rotation velocity of the masses have been reckoned with.

Accordingly, the system of differential equations for the effect of a horizontal dynamic force on the complete building is, in matrix form:

$$\mathbb{A}\ddot{\mathbb{d}} + \mathbb{C}\dot{\mathbb{d}} + \mathbb{K}\mathbb{d} = \mathbb{p}(t) \tag{1}$$

where:

$\mathbb{A}$ — generalized mass matrix;
$\mathbb{C}$ — damping matrix;
$\mathbb{K}$ — stiffness matrix;
$\mathbb{d}$ — generalized displacement vector;
$\mathbb{p}$ — generalized load vector.

(Detailed definitions of the terms are found in (1), (2), (3).)

Reckoning with Stiffness of a Composite Structure according to an Elasto-Plastic Material Model

In matrix differential equation (1), $\mathbb{K}$ involves so-called stiffness coefficients proportional to displacements, or the following general definition based on the assumption of an elasto-plastic material model.

A single-mass system of one degree of freedom has been plotted in Fig. 1, assuming an elasto-plastic force-displacement relationship. For a mass supported on several, parallel connected springs, the force-displacement diagram will be polygonal, as seen in Fig. 2 for two springs of different characteristics, and identical x_1. Obviously, the polygonal diagram can be replaced by a polynomial fitting the polygon (accuracy may be increased with the number of degrees of the curve). Thereby a continuous function results, representing the correlation between the resultant of mass forces and the displacement.

The same principle may be applied for flexural beams. Cross section of a flexural beam is seen in Fig. 3, with its principal direction of inertia. In conformity with this figure, to replace cross section i of the homogeneous, prismatic bar of a material obeying Hooke's law by Δ_1 and Δ_2 parallel to principal directions 1 and 2, resp., forces F_1 and F_2 have to be applied in the shear centre of another cross section k, parallel to the respective displacements. In conformity with the elementary theory of strength, the forces are:

$$F_1 = HJ_2\,\Delta_{1ik}; \qquad F_2 = HJ_1\,\Delta_{2ik}$$

(H being a coefficient depending on the bar material, on supporting conditions and on the cross section place.)

Let us assume now an arbitrary orthogonal coordinate system xy at the shear center of cross section i, and determine the external force to be applied in cross section k for a displacement along the x-axis. Provided the x-axis is other than principal direction, the displacing force is not parallel to the

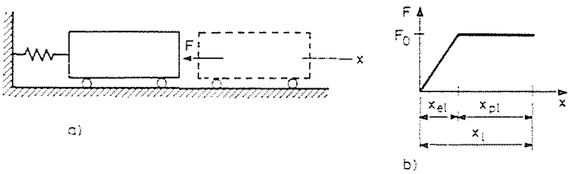

Fig. 1

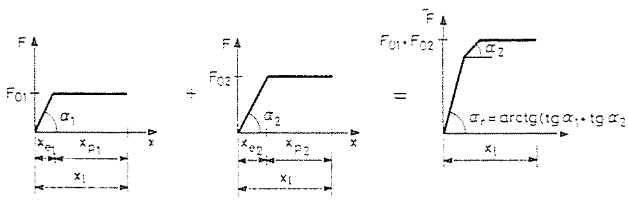

Fig. 2

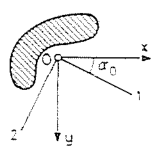

Fig. 3

x-axis, but it may be represented by components p_{xx} and p_{xy} in directions x and y, respectively, to be computed as:

Components in directions 1 and 2 of unit displacement along the x-axis including an angle with principal direction 1 amount to 1 cos, and —1 sin, respectively. In conformity with statements above, they are produced by forces:

$$Q_1 = HJ_2 \cos\alpha; \qquad Q_2 = -HJ_1 \sin\alpha$$

along axes 1, and 2, respectively. Force components p_{xx} and p_{xy} wanted are obtained as sums of projections along x and y, thus:

$$p_{xx} = Q_1 \cos\alpha - Q_2 \sin\alpha = H(J_1 \sin^2\alpha + J_2 \cos^2\alpha)$$

$$p_{xy} = Q_1 \sin\alpha - Q_2 \cos\alpha = -\cos\alpha \sin\alpha \, H(J_1 - J_2).$$

For unit displacement of the investigated cross section along y, forces p_{yy} and p_{yx} in directions y, and x, resp., are needed, to be determined as before, leading to:

$$p_{yy} = H(J_1 \cos^2 \alpha + J_2 \sin^2 \alpha)$$

$$p_{xy} = -\cos \alpha \sin \alpha (J_1 - J_2) = p_{xy}.$$

The obtained magnitudes $p_{xx}, p_{yy}, p_{xy} = p_{yx}$ are usually termed stiffness coefficients.

In the case of several interconnected beams, the overall stiffness coefficient is obtained as sum of single stiffness coefficients. Let us consider the case where the force-displacement diagram corresponds to the elasto-plastic material model. Now, according to Fig. 4 — in conformity with diagrams $F_1 - \Delta_1$, and $F_2 - \Delta_2$ — relationships p_{xx}, p_{yy}, p_{xy} equal the sum of tangents of the diagrams, to be plotted as a stepped diagram as seen in Fig. 4 for p_{xx}. In case of a composite cross section (of several distinct units), relationships for determining the rotation center, principal directions of stiffness, etc., involve sum of displacements for unit forces, to be plotted as a stepped diagram (Fig. 5a). Now, let us write relationships p_{xx}, p_{yy} and p_{xy} in form of a continuous approximate function. So, considering values for each step as tangents of sides of a polygonal diagram as integral of the stepped diagram (Fig. 5b), replaceable, according to the precedings, by a properly assumed polynomial of degree n. In its possession, continuous functions of the stiffness coefficients may be determined as derivatives of this polynomial.

Thereby relationships involved in elements of coefficients of differential equation (1) could be expressed as a continuous function of a generalized

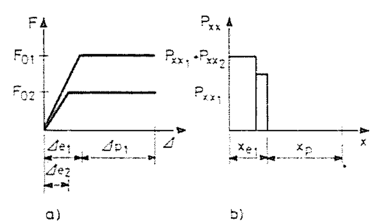

Fig. 4

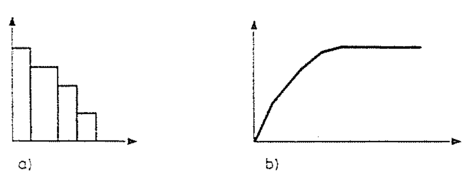

Fig. 5

vector $\mathbf{u}(x, y, \varphi)$ rather than as a constant, reducing the dynamic problem to the solution of a second-order differential equation system with nonlinear coefficients.

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