DISPLACEMENT ANALYSIS OF COMPOSITE BUILDING STRUCTURES UNDER MODELLED EARTHQUAKE ACTION BY DIRECT SECOND ORDER STATISTICAL MOMENT APPROACH

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Received July 10, 1984

Presented by Prof. Dr. S. Kaliszky

Summary

The applied model for earthquake action analysis of composite building structures — containing rigid floor platforms and the vertical load bearing structure substituted by visco-elastic bar connections between these platforms — is considered under the effect of stochastic horizontal dynamic loads replacing earthquake action. The components of this replacing force-system as instationary stochastic processes, are described by products of a periodic deterministic function and a non-stationary, stepwise stochastic process, latter can be given in each halfperiod by a in vicinity correlated Gauss-type random variable. The differential equation-system describing the linear elastic dynamic system with damping is solved with the aid of a linear transformation resulting in a set of equations for unknowns independent, so we can get formulas for mean value and covariance-functions of the displacement components of the rigid platforms corresponding to each floor-level of the structure. The elementary force components differing form each other in frequencies of excitation can be weighted with their probability and so we can follow the real earthquake spectra. In case of a simple excitation with 3 elementary components we illustrate our numerical results, which has been obtained by a computer program under development.

Introduction

In the actual civil engineering practice most buildings to be exposed to horizontal dynamic loads (replacing effect of earthquake) have vertical load-bearing structures of non-symmetrical floor plan. Structurally it means that the vertical load-bearing structures of the building are frameworks, columns, independent or connected bearing walls or combinations thereof. Deterministic analysis of such structures can be simplified by using the linear visco-elastic structural model suggested by the first author, which consists of rigid horizontal floor planes and of bar-connections substituting the vertical load-bearing elements [1].

Numerical method in case of deterministic loads

The behaviour of the previously described modelled structure can be characterised by the matrix differential equation of motion [2] (notations see at the end of the paper)

$$\mathbf{M}\ddot{\mathbf{f}} + \mathbf{C}\dot{\mathbf{f}} + \mathbf{K}\mathbf{f} = \mathbf{p}(t) \tag{1}$$

Here $\mathbf{f}$ represents a hypervector of 3 times n dimensions (n is number of floor planes) containing displacements in direction of floor plane axes and rotations in the plane of rigid floor platforms, $\mathbf{p}(t)$ contains the (in work expressions to $\mathbf{f}$ corresponding) time-dependent generalized forces. Time-dependence of coefficient matrices is neglected. The numerical method of solution [3] is based on the assumption $\mathbf{C} = \alpha_c \mathbf{K}$ on a linear transformation in the form $\mathbf{f} = \mathbf{Zq}$ and upon a multiplication of (1) by Z^* from the left side, respectively. It is possible to perform these after determination of eigenvalues and eigenvectors of the problem without damping. (Z contains these eigenvectors in a reduced form.) The obtained 3 times n, unconnected differential equations for the transformed generalized displacements can be solved and the last step of the algorithm needs a re-transformation to the originally unknown functions.

Investigation of the structure under effect of earthquake-like stochastic loads

Random excitation of a structure due to earthquake can be described mathematically by the aid of stochastic processes [4], [5], [6]. The solution of the problem was sought first for stationary parts of earthquake motion by the spectral method [6], [7]. The use of this method was extended for special instationary cases by use of envelope functions [8], [9], [10]. In the following we shall deal only with the solution of one differential equation of the previously described transformed system in case of stochastic loads:

$$\ddot{q} + d\dot{q} + \omega_0^2 q = G(t). \tag{2}$$

Herewith we omit to give indices, so the problem is treated as a one-dimensional case, but experiences of matrix solution for static random loads given in [11] might be used. According to the nature of loading the reduced random function $G(t)$ can be built up as the sum of products of a deterministic sinusoid function $g_1(t)$ and a stepwise random function $g_2(t)$. (See Fig. 1.)

If Q_i are chosen as mean values, then ξ_i are Gaussian variables with mean value 1, 0 and covariance matrix $\mathbf{B}_{\xi\xi}$, which generally is of banded nature. The principle of linear superposition is taken as valid and for one load component history as represented on Fig. 1. mean values and covariance functions of $q(t)$ can be evaluated as follows (q_0 and $\dot{q}_0$ are taken as normal variables independent of each other and of ξ_i values)

$$E[q(t)] = b(t)E[q_0] + h(t)E(\dot{q}_0] + \sum_{i=1}^{m+1} a_i(t)\, E[\xi_i]$$

here $t_m < t \leq t_{m+1}$ (formulas of $b(t)$, $h(t)$ and $a_i(t)$ are given in Appendix)

$$B_q(t_{\mathrm{I}}, t_{\mathrm{II}}) = b(t_{\mathrm{I}}) \cdot b(t_{\mathrm{II}})\, \sigma^2[q_0] + h(t_{\mathrm{I}}) \cdot h(t_{\mathrm{II}})\, \sigma^2[\dot{q}_0] + \mathbf{a}^*(t_{\mathrm{I}}),\ \mathbf{B}_{\xi\xi}\, \mathbf{a}(t_{\mathrm{II}})$$

here $t_{m_1} < t_{\mathrm{I}} \leq t_{m_1+1}$ and $t_{m_2} < t_{\mathrm{II}} \leq t_{m_2+1}$ order of vector **a** and of quadratic matrix $\mathbf{B}_{\xi\xi}$ is max $[m_1 + 1, \; m_2 + 1]$.

As various harmonic functions have different rates of occurrence we can use the method of weighted realizations [12], [13] to obtain final values of $E[q]$, $B_q(t_{\mathrm{I}}, t_{\mathrm{II}})$. We shall use the assumptions that series values of ξ_i belonging to different harmonics are independent, that the domain of frequency field of excitation is finite and width of it is rather small for active parts respec-

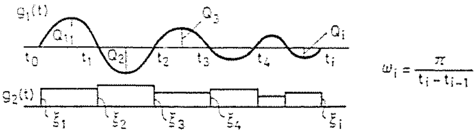

Fig. 1

tively. As earthquake excitation has been substituted in our research work [3], [4] by one random horizontal force in the height of the ground floor of the building, so when solving equations of type (2) for various elements of matrix, although appropriate additional indices must be used for values q_0, $\dot{q}_0$, d, ω_0, Q_i, the values ξ_i are common. Applying similar formulas as expression for $E[q(t)]$, $B_q(t_{\mathrm{I}}, t_{\mathrm{II}})$ (the above mentioned additional indices must be used in computations of results which can be found in our Appendix and instead of $\sigma^2[q_0]$ and $\sigma^2[\dot{q}_0]$ $B_{q_0 q_0}$ and $B_{\dot{q}_0 \dot{q}_0}$ must appear) the matrix $\mathbf{B}_{qq}$ and vector $E[\mathbf{q}(t)]$ be constructed, which contains all B_q-s and the cross-covariance functions, too. The covariance matrix and the expected values of the real displacements **f** can be gained by the following formulas in matrix form (see e.g. [14])

$$E\,[\mathbf{f}(t)] = \mathbf{Z}\,E\,[\mathbf{q}(t)]$$

$$\mathbf{B}_{ff}(t_{\mathrm{I}}, t_{\mathrm{II}}) = \mathbf{Z}\,\mathbf{B}_{qq}(t_{\mathrm{I}}, t_{\mathrm{II}})\,\mathbf{Z}^* .$$

Numerical example

A structure considered as particle with mass 10 t on a subgrade characterised with spring coefficient 31047 t/s^2 and damping coefficient 120 t/s (Fig. 2/a) shall be investigated numerically, if $f_0 = 0$, $\dot{f}_0 = 0$. Excitation is assumed as deterministically added of 3 random harmonics ($\omega_i = 20, 40$, 80 s^{-1}) each given in manner of Fig. 1. for $\omega = 80$ s^{-1} with different starting point on time axis with $E[\xi_i] = 1.0$ and various values of $\sigma^2[\xi_i]$ and Q_i for the different random processes. Correlation of ξ_i ($i = 1, 2, 3$) is neglected. Only one degree of freedom motion is considered, the mean value diagram of the resulting displacement field (Gaussian stochastic process) is drawn on Fig. 2/b too.

Our numerical results have been achieved with the aid of a computer program written in language FORTRAN for the CDC 3300 machine of the Hungarian Academy of Science. Double precision arithmetics has been used, the computer time for our example amounted to 5.5 minutes, so our method adopted here combined with the method of weighted realizations shall need rational computer times using our reduced-degree of freedom model of composite buildings for the case of earthquakes.

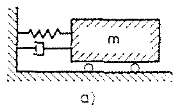

Fig. 2/a

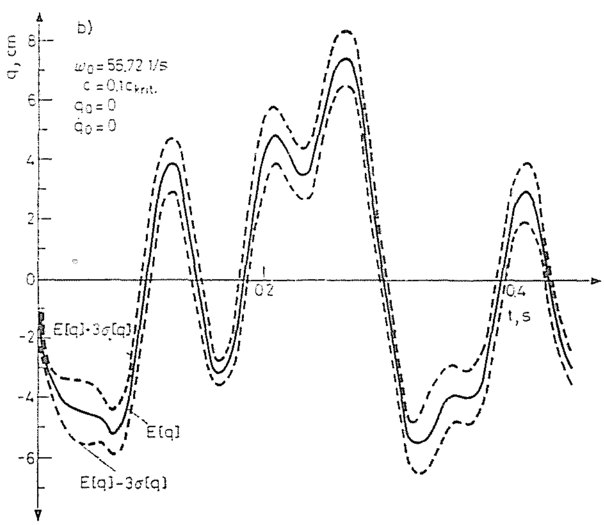

Fig. 2/b

Conclusions

Our results proved that the method for modelling earthquake caused excitations proposed in [15] is efficient. If vertical seismic loads must be taken also into account [16] our method can be extended without significant difficulties for this case. Computer time demand in real cases is rather high, so the possibility of extension given in [17] for treating problems of Level II of design for reliability does not seem realistic yet.

Notations

$\mathbf{a}(t)$	auxiliary vector-function
$b(t)$	auxiliary function
$B_q(t_I, t_{II})$	covariance function of q
$\mathbf{B}_{qq}(t_I, t_{II})$	covariance matrix of $\mathbf{q}$
$\mathbf{B}_{\xi\xi}$	covariance matrix of random variable $\xi_1 \ldots \xi_i \ldots$
c	damping coefficient
$\mathbb{C}$	damping matrix of the structure
d	damping coefficient after reduction
$E[.]$	expected value of random expression in the brackets
$f, \mathring{f}, \ddot{f}$	displacement hypervector, first, second time derivatives, respectively
$g(t), G(t)$	given time dependent function
$h(t)$	auxiliary function
i, j, k	integer numbers (indices)
$\mathbb{K}$	stiffness matrix of the structure
$\mathbb{M}$	generalized mass matrix of the structure
max [ ]	maximum value of constants in the brackets
n	number of floor planes of the structure
$\mathbf{p}(t)$	hypervector of time dependent generalized forces
$q(t)$	generalized displacement after transformation
$\mathbf{q}(t)$	generalized displacement vector
q_0	initial value of q
$\dot{q}_0$	initial value of first time derivate of q
Q_i	absolute value of reduced amplitudes (generally mean value)
t	time parameter
t_0	starting time of loading
$\mathbb{Z}$	transformation matrix built up of reduced natural mode shapes of the structure
α_c	constant
ξ_i	correlated sequence of normally distributed random numbers
$\sigma[\ ]$	standard deviation of random expression in the brackets
Σ	summation convention
ω_0	associated natural frequency
ω_i	changing frequency of force system of excitation in period i $(i \neq 0)$

Appendix

Computation of $\mathbf{a}(t)$, $b(t)$, $h(t)$

If $t = t_k$, $m = m_k$ and $t_{m_k} < t_k \leq t_{m_k+1}$ respectively then for $i = 1, \ldots, m$

$$a_i(t) =$$

$$=(-1)^{i-1}Q_i\left\{\omega_i \frac{e^{-\frac{d}{2}(t-t_{i-1})}}{(\omega_i^2-\omega_0^2)^2+d^2\omega_i^2}\left(\frac{\frac{d^2}{2}+(\omega_i^2-\omega_0^2)\sin\left(\sqrt{\omega_0^2-\frac{d^2}{4}}(t-t_{i-1})\right)}{\sqrt{\omega_0^2-\frac{d^2}{4}}}+\right.\right.$$

$$\left.+d\cos\left(\sqrt{\omega_0^2-\frac{d^2}{4}}(t-t_{i-1})\right)\right)-\omega_i\cdot\frac{d\cos(\omega_i(t_i-t_{i-1}))}{(\omega_i^2-\omega_0^2)^2+d^2\omega_i^2}+$$

$$\left.+\frac{(\omega_i^2-\omega_0^2)\sin(\omega_i(t_i-t_{i-1}))}{(\omega_i^2-\omega_0^2)^2+d^2\omega_i^2}\right\}$$

$$a_{m+1}(t) =$$

$$=(-1)^mQ_{m+1}\left\{\omega_{m+1}\frac{e^{-\frac{d}{2}(t-t_m)}}{(\omega_{m+1}^2-\omega_0^2)^2+d^2\omega_{m+1}^2}\left(\frac{\frac{d^2}{2}+(\omega_{m+1}^2-\omega_0^2)\sin\left(\sqrt{\omega_0^2-\frac{d^2}{4}}(t-t_m)\right)}{\sqrt{\omega_0^2-\frac{d^2}{4}}}+\right.\right.$$

$$\left.+\mathrm{d}\cos\left(\sqrt{\omega_0^2-\frac{d^2}{4}}(t-t_m)\right)\right)-\omega_{m+1}\frac{d\cos(\omega_{m+1}(t-t_m))}{(\omega_{m+1}^2-\omega_0^2)^2+d^2\omega_{m+1}^2}+$$

$$\left.+\frac{(\omega_{m+1}^2-\omega_0^2)\sin(\omega_{m+1}(t-t_m))}{(\omega_{m+1}^2-\omega_0^2)^2+d^2\omega_{m+1}^2}\right\}$$

$$a_{m+2} = a_{m+3} = \ldots = 0$$

$$b(t) = e^{-\frac{d}{2}t}\left\{\cos\left(\sqrt{\omega_0^2-\frac{d^2}{4}}\,t\right)+\frac{d}{2}\,\frac{\sin\left(\sqrt{\omega_0^2-\frac{d^2}{4}}\,t\right)}{\sqrt{\omega_0^2-\frac{d^2}{4}}}\right\}$$

$$h(t) = \frac{\sin\left(\sqrt{\omega_0^2-\frac{d^2}{4}}\,t\right)}{\sqrt{\omega_0^2-\frac{d^2}{4}}}$$

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