INVESTIGATION OF ULTIMATE BEARING CAPACITY IN CASE OF AXIAL LOAD

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Summary

According to the new design standard concerning reinforced concrete structures, the examination of the load bearing critical condition has to be undertaken for axial load by taking into consideration plastic material features. The critical condition of cross sections has to be calculated either by supposing the critical fracture compression of concrete or the critical rupture elongation of the drawn reinforcement. In case of compressed rods the eccentricity increments due to the material inhomogeneity, measurement faults as well as the deformation occurring till the critical condition, have to be considered.

Introduction

Bending-, compressive- and tensile stress as well as their combinations belong to the concept of axial load. The term was first published in the 1971. Standard and its use proved to be expedient. The subject matter being reinforced concrete and critical bearing capacity the emphasis in the specifications is on the compressive- and bending load. The chapter touches upon two problem spheres: on the one hand the calculation of the arbitrary axial ultimate load of abstract cross sections and, on the other — in case of compressed elements — the consideration of effects endangering stability. The specifications concerning the examination of cross sections pertain both to compression and traction.

To begin with it should be stated that the chapter has been changed essentially only as regards the method of calculating the eccentricity increment, as compared to the former standard, other modifications are practically formal ones and came into being for a more uniform approach as well as a more compact construction. Thus the demonstrative character mention of the interaction curve and/or surface should be regarded as a formal innovation as the principle and method of their determination was regulated already earlier, in an implicit form.

The ultimate limit state of axially loaded cross sections

2.1. The critical condition

In our calculations concrete is generally regarded as a rigid-plastic material, at the limit of bearing capacity, while reinforced concrete as an elastic-plastic one (Figs 1a, 1c). Also the elastic-plastic material model (Fig.

1b) is permitted in the case of concrete. Based on the above the ultimate limit state — while adhering to the principle of plane cross section — has a deformation condition:

$$|\varepsilon_{b\,\max}| = \varepsilon_{bu}, \quad \text{and/or} \quad |\varepsilon_{s\,\max}| = \varepsilon_{su}, \tag{1}$$

viz. failure may occur accompanied by the ultimate compressive, strain of the most highly compressed fibre strand and/or the elongation at rupture of the exterior tensioned steel reinforcement. Figure 2 shows the possible range of cross section deformations satisfying the failure condition indicated in this way.

The limit state characterised by the balance of internal forces (pure bending) may occur depending on the measure of reinforcement, according to the deformation line 1., 2. and 3. shown in Fig. 2 (in case of normal-, slightly- and/or overreinforced cross sections).

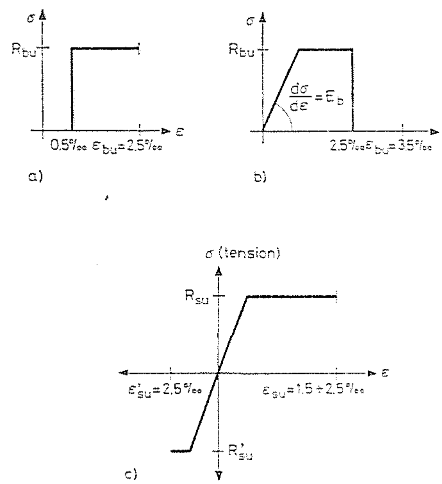


Fig. 1. Material models for calculation purposes. a. Concrete, rigid-plastic; b. Concrete, elastic-plastic; c. Reinforced concrete (drawing)

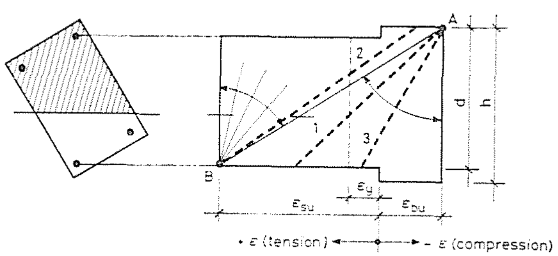


Fig. 2. Cross section deformations satisfying the conditions of bearing capacity critical state. Distribution ε through point "A"; concrete failures. Distribution ε through point "B": steel failures

2.2. The neutral axis and the compressed flange

It follows jointly from the condition of limit state and the material model concerning calculation of rigid-plastic concrete that the neutral axis of the cross section (line 0 of Figure 3) and the boundary of the compressed flange do not coincide.

The proportion of the distance of the two from the exterior compressed strand is:

$$\frac{x'}{x} = \frac{2.5\%}{2.5\% - 0.5\%} = 1.25$$

just as in the former Hungarian Standard. This, however, pertains only to the critical conditions characterized by the failure of concrete. In the cases accompanied by the rupture of steel, the concrete diagram in conformity with Figure 1a is taken as a basis, ratio x'/x will be a variable value concerning which the following formula can be derived: (Fig. 3)

$$\frac{x'}{x} = \frac{2\varepsilon_{su} + d/x}{2\varepsilon_{su} + 1} > -1.25; \quad [\varepsilon_{su}] = ‰. \tag{2}$$

Equation (2) has no too high practical value, thus the standard does not take it into consideration.

2.3. Reduced steel stress

Following from the above, steel reinforcements can be taken into consideration at any critical condition but according to a stress suitable to their compatible elongation, at the very most the limiting stress. This condition takes to the already known formula of reduced steel stress in the cases characterized by the failure of concrete ($\varepsilon_{b\,\max} = \varepsilon_{bu}$).

$$\sigma_s = \frac{400}{x} \cdot d - 500, \; [\mathrm{N/mm^2}] \qquad \sigma_s \leq R_{su} \qquad |\sigma_s'| \leq R_{su}'. \tag{3}$$

If, with a slight generalization, measurement d is understood as the distance of an arbitrary position steel reinforcement from the compressed exterior fibre then relation (3) is valid for both tension and compression (Fig. 4).

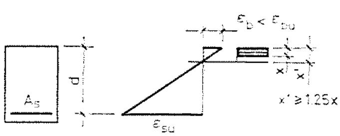
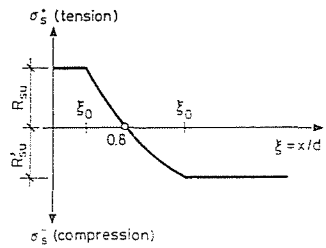


Fig. 3. Neutral axis and compressed flange in the critical condition accompanied by steel rupture



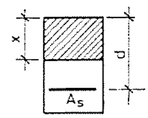


Fig. 4. Relationship between steel-stress and the relative position of the compressed flange

Equation (3) changed only in so much as compared to the hitherto standard, as necessitated by the introduction of the new measuring unit and the modification of the steel elasticity factor ($E_s = 2 \cdot 10^5$ N/nm2).

The relationships of compressed flange critical condition so important in practice for cross sections of simple- and/or double reinforcement can be derived from equation (3), (Fig. 5)

a) from the point of tensile reinforcement

$$\text{if } \frac{x}{d} \leq \xi_0 = \frac{400}{500 + R_{su}}, \quad \text{then} \quad \sigma_s = R_{su} \tag{4a}$$

b) from the point of compressed reinforcement

$$\text{if } \frac{x}{a'} \geq \xi_0 = \frac{400}{500 - R_{su}}, \quad \text{then} \quad \sigma'_s = R'_{su}. \tag{4b}$$

If necessary, in rather seldom cases, the reduced steel tension in the critical condition characterized by the rupture of the exterior compressed steel has to be calculated in a different way. Though the standard does not mention the problem, the mode of calculation can be derived from the basic principles.

Over and above the stress reduction regulation following from the mentioned compatibility principle, also the empirically indicated limitation is valid according to which no higher force should be supposed in the compressed steel reinforcement than taken into consideration in the compressed concrete flange.

2.4. *Determination of load*

For calculating the axial ultimate load the standard contains no specification by formula, and that for two reasons. On the one hand as, after clarifying the science of material and compatibility principles the task is essentially limited to determine the internal forces, and the pertaining formulae and algorithms belong to a basic engineering knowledge and are valid without any change. On the other, the high number of concentric compression through pure bending to concentrie tension would not make a detailed discussion possible in the given limit. Here we only have the possibility to refer in short to the fact that determination of some axial ultimate load of a given cross section actually means determination of the given point determining the given

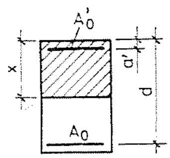


Fig. 5

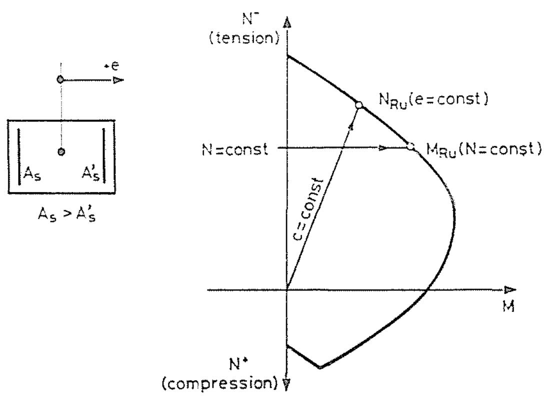


Fig. 6. Interpretation of the axial critical load on basis of interaction curve

condition of the interaction curve belonging to the cross section where the condition mostly is the fixed value of the size of force or of its eccentricity (Fig. 6). All this is true for the internal forces acting in a median plane. In a general, spatial case the load bearing capacity, the reference basis and the given conditions are all possible in a higher number of variations.

Over and above the methods accurate in principle, which can be characterized in the mentioned way, the Hungarian Standard also permits the approximating method that applies an approximate interaction line consisting of broken, straight phases and the one with an approximate interaction surface consisting of broken plains. This enables an important simplification especially in the range of small eccentricities where one would otherwise have to calculate with a reduced tension in case of tensioned (or less compressed) bars. Along the low eccentricity phase linear approximation leads to the following simple relations (Fig. 7):

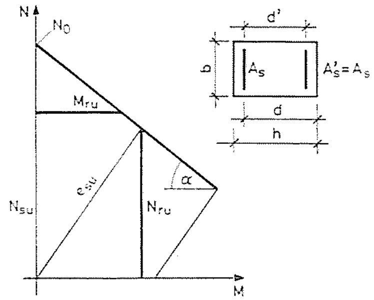


Fig. 7

a) if the limit moment is sought to a given compressive force

$$M_{Ru} = \frac{N_0 - N_{su}}{\operatorname{tg} \alpha}\,.$$

b) if the limit compressive force is sought to a given eccentricity:

$$N_{Ru} = \frac{N_0}{1 + e_{Su} \cdot \operatorname{tg} \alpha}\,,\ \text{respectively.}$$

In these equations:

N_0 — the concentric limit compressive force,

$\operatorname{tg} \alpha$ — the incline of the linear interaction span, concerning which the approximation

$$\operatorname{tg} \alpha \approx \frac{3{,}3}{d}$$

is generally accepted and which can be calculated more accurately in the knowledge of the reinforcement data. Concerning a symmetric reinforcement square cross section:

$$\operatorname{tg} \alpha = \frac{1}{d} \frac{\beta + 2\mu \dfrac{R_{su}}{R_{bu}} - \xi_0}{\xi_0 \left(1 - \dfrac{\xi_0}{2}\right) + \mu \cdot \gamma \dfrac{R_{su}}{R_{bu}} - \xi_0 \dfrac{\gamma}{2}}\,.$$

Where:

d — the useful height of the cross section

$\mu = \dfrac{A_s}{b \cdot d}$ the ratio of reinforcement

$\xi_0 = \dfrac{x_0}{d}$ the limit position of the height of the compressed flange

$\beta = \dfrac{h}{d}$,

$\gamma = \dfrac{d'}{d}$ the measurement parameters according to Fig. 7.

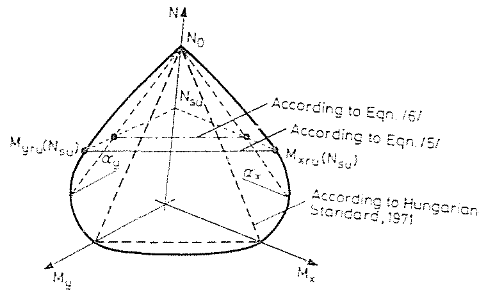


Fig. 8

In case of spatial eccentricity several stages of linear approximation of the load bearing surface are possible. The control-equation suggested by us:

$$\frac{M_{xSu}}{M_{xRu}(N_{Su})} + \frac{M_{ySu}}{M_{xRu}(N_{Su})} \leq 1 \tag{5}$$

is based on the linear approximation of the $N = N_{su}$ plane section of the surface, where the x and y direction limit moment pertaining to given normal force N_{su} can be determined with an accurate method or, according to the above with a linear approximation. Latter possibility leads, e.g. in the range of low eccentricity to the following control-equation:

$$\frac{M_{xSu} \operatorname{tg} \alpha_x + M_{ySu} \operatorname{tg} \alpha_y}{N_0 - N_{Su}} \leq 1 \tag{6}$$

where $\operatorname{tg} \alpha_x$ and $\operatorname{tg} \alpha_y$ means the incline of plain section $M_y = 0$, or $M_x = 0$ of the load bearing capacity approached by a straight line (Fig. 8). Both of the above methods give a better approximation than the equation suggested in the former Hungarian Standard:

$$\frac{N_{Su}}{N_0} + \frac{M_{xSu}}{M_{xRu}} + \frac{M_{ySu}}{M_{yRu}} \leq 1 \,.$$

Calculation of the eccentricity increment in case of compressed elements

3.1. Basic principles

Concerning the specifications of eccentricity increments of compressed elements, the draft of the standard follows the concept of the previous standard form 1971. Accordingly:

— Due to the inhomogeneity of the cross section and imperfections in form, the compressive force always has some original eccentricity. It would, however, be unjustified to make a difference between concentric and eccentric pressure.

— The deformation of the compressed element till the ultimate state has to be taken into consideration as a further, additive eccentricity-plus.

Latter effect was regarded by the previous standard as completely independent from the original (calculated and accidental) eccentricities supposing uniformly for each case that the yield of the tensioned reinforcement occurs in the critical condition always in the cross section of the highest bend. This supposition is in contradiction with both theory and practice (3), (4), (5). The new draft enables to take the relation between the original eccentricity and the failure increment into consideration with a simple correction factor.

When determining the buckling lengths and selecting the competent cross sections the former regulations are valid.

3.2. Eccentricity increment because of cross section inhomogeneity

As till now, its value is:

$$\Delta e_1 = 0.03\, d$$

where d is the useful height of the cross section.

3.3. The effect of form imperfection

This surplus eccentricity has to be taken into consideration with 1/300 of the buckling length

$$\Delta e_2 = l_0/300\,.$$

The till now usual formula

$$\Delta e_2 = 0.01 \left(\frac{l_0}{10\, d}\right)^2 \cdot d$$

viz. that Δe_2 depends quadratically from the slimness was to be explained with formal causes, only. It should be noted that the new value is, in general, not more favourable under $l_0/d = 33$ than the one according to the old formula. Despite this, the change is indicated by in principle suitability and the example of competent foreign specifications (2).

3.4. Effect of axis skewing

In the absence of more accurate data it can be supposed that the axis of the columns has a 1% skewing. A surplus load results, however, only in the columns of unbraced (sway) frames, that has to be considered when

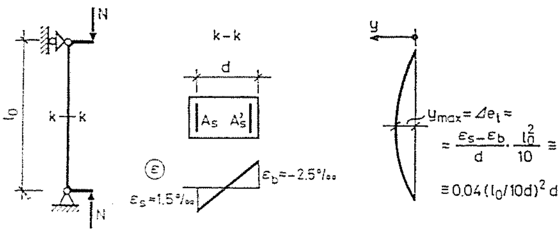


Fig. 9. Deformation at rupture of an eccentrically compressed road

determining the clearance of force. Thus the effect of axis skewing cannot be considered as an eccentricity increment only in case of the most simple sway structure, a column fixed at the bottom and free at the top. In case of groups of columns with identical measurement — and load data, there is a possibiliy to diminish the supposed skewness, starting from the supposition that it is 50% accidental and 50% a regular fault.

3.5. Failure eccentricity increment

According to the Hungarian Standard from 1971 the eccentricity increment developing in a compressed column till the point of critical condition can be calculated with the formula

$$\Delta e_t = 0.04 \left(\frac{l_0}{10\,d}\right)^2 d \tag{7}$$

where the symbols are to be interpreted according to Fig. 9. This supposition leads to the curvature size developing in the middle cross section of a bar with an l_0 buckling length, in the critical condition being:

$$\varrho_u = \frac{\varepsilon_s^{\text{yield}} + \varepsilon_b^{\text{rupturing}}}{d} \approx \frac{0.004}{d}$$

and the curvature changes according to the sinus curve.

Because of the causes mentioned in point 3.1 it was indicated to modify formula (7) in a way that it enable the assumption of a steel deformation less than the elongation due to yielding, depending on the initial eccentricity and/or such a limit curvature. Basing on Hungarian research results as well as on pertinent publications of CEB relation of critical condition curvature and initial eccentricity can be written with the following equation (3), (4):

$$\varrho_u = c\,\frac{\varepsilon_s^{\text{yield}} + \varepsilon_b^{\text{rupturing}}}{d}, \text{ where}$$

$$c = \left(1 + 0.15\,\frac{l_0}{10\,d}\right) 0.25 + 0.67 \sqrt{\frac{e_0 + \Delta e_1 + \Delta e_2}{d}} \leq 1$$

and the notations are to be understood according to the former. In this way eccentricity increment Δe_t can be modified with factor c:

$$\Delta e_t = \Delta e_3 = c \cdot 0.04 \left(\frac{l_0}{10\,d}\right)^2 d\,.$$

From the check-calculations it turned out that with the above reduction of member $\Delta e_t = \Delta e_3$ the effect of increment in member Δe_2 is generally balanced in case of the most often occurring column slimnesses ($10 < l_0/d < 35$).

3.6. The case of calculated eccentricity $e_0 = 0$

If the calculated load of the column is a concentric compression, viz. eccentricity is made up from increments Δe only, the more simple "φ factor" calculation can be applied. The changes in calculating terms' Δe made a slight modification necessary in the formula of factor φ, for a better harmony with the more accurate calculation method. In the new standard draft therefore:

$$\varphi_N = \frac{1}{1.1 + 0.11\left[\frac{l_0}{10\,d} + 0.8\left(\frac{l_0}{10\,d}\right)^2\right]}\,.$$

3.7. Complementary checks

According to the standard draft, e_{Su} competent eccentricity is, in general a vectorial resultant of e_0, the calculated eccentricity, as well as the sum of Δe increments. The vectorial summation comes up actually if the calculated eccentricity is outside the symmetry plane (Fig. 10.b, 10.c) or in the symmetry plane pertaining to the smaller l_0/d slimness (Fig. 10.a). In such a case the possibility of increments evolving in the direction of the bigger slenderness has to be investigated (Fig. 10). Following this investigation the cross section has to be controlled as to spatial eccentricity pressure, taking into consideration what has been said in point 2.4.

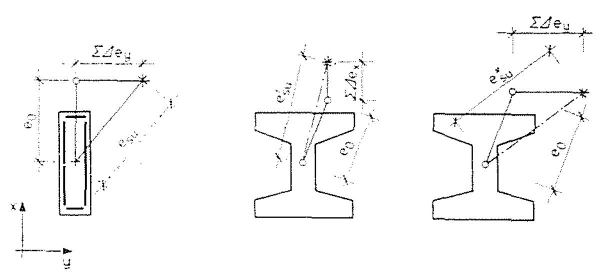


Fig. 10. Cases of vectorial summation of eccentricity increments

References

1. Szabványtervezetek Vasbetonszerkezetek tervezéséhez. (Standard drafts for designing reinforced concrete structures.) In Hungarian. Editor: Dr. Kálmán SZALAI, Budapest University of Technology. 1984.
2. CEB-FIP Model Code for Concrete
3. CEB-Buckling Manual. Bulletin d'Information No. 103. 1975.
4. SZALAI K.: A hajlékonyság szerepe a vasbetonoszlop teherbírásában. (The role of elasticity in the bearing capacity of reinforced concrete columns). (In Hungarian) Műszaki Tudomány *52*, 1976. 27—52.
5. KORDINA, E. h. K., QUAST, U.: Bemessung von schlanken Bauteilen — Knicksicherheitsnachweis (Measuring slim structural elements — Proving Buckling Safety) (In German) Beton-Kalender 1981.

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