LOAD-DEFORMATION RELATIONSHIPS FOR SIMPLE STEEL FRAMES WITH UNSTABLE ELEMENTS

M. Iványi

Department of Steel Structures, Technical University, H-1521 Budapest
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Abstract

Procedures have been developed for investigating the plasticity of steel structures that retain the known, traditional steps of investigation but incorporate, instead of the traditional plastic hinge, the model of interactive hinges or interactive zones. In this way the joint consideration of quite different strength and stability phenomena is possible.

With the aid of the load-displacement diagram characteristic of the behaviour of steel structures an answer was sought to the question if the displacement capability of the given structure is sufficient and therefore both the "descending" and the "ascending" sections of the load-deformation diagram have been investigated both theoretically and experimentally.

1. Introduction

Questions relating to the effects of the plastic properties of steel material on the load bearing capacity of steel structures have long been of interest to researchers, a number of problems have been solved. The application of the theory of plasticity to designing has been enabled by the introduction of plastic load capacity investigations. These take into account that in steel structures an increasing number of extensive plastic zones are brought about by gradually increasing so-called "static" load until in the end, at the limit of load bearing, the structure, without further increase in load, is able to undertake continuous displacements. Calculations of plastic load bearing capacity are based on the research work of Gábor Kazinczy.

Several questions remained unanswered in connection with plastic load bearing capacity investigations. Thus, for instance, the difference between the behaviour of the ideally plastic model and of real steel material, as well as problems connected with loss of stability (plate buckling, flexural-torsional buckling). Extensive research, especially in the experimental field, tried to answer these questions determining the conditions the adherence to which allows the maintenance of the validity of plastic load bearing capacity investigations.

In present designing practice the plastic load bearing capacity investigation is carried out and thereupon complementary investigations are made to check the effects of loss of stability (plate buckling, flexural buckling and flexural-torsional buckling, etc.). The need for a separate investigation is due to the lack of a plastic hinge model that could have reflected these effects.

Our aim is a theoretical and experimental investigation that studies steel structures in steps known from traditional methods with the aid of a hinge model suitable to describe "more refined" properties, to embrace more phenomena (strain-hardening, residual deformation, plate buckling, flexural-torsional buckling, etc.).

2. Effect of local instability

The final collapse of steel structures is mostly caused by instability phenomena (Thürlimann [1]). These instability phenomena may be the following:

— disadvantageous change in the steel structure geometry,

— disadvantageous change in the cross section geometry.

The effect of disadvantageous changes in the steel structure geometry can be — traditionally — grouped in the field of plastic instability. It was Ottó Halász [2] who treated the problem in a doctoral thesis and, over and above theoretical studies, he also introduced a method suitable for practical design work.

The disadvantageous changes in the cross section geometry are mainly plate bucklings. Plate buckling causes a change in the behaviour of the plastic hinge, too, and thus for the plastic investigation of statically unstable steel structures, methods have to be elaborated that take into consideration also the effect of plate buckling.

The case of ideally plastic material is an assumed case and it should be taken while investigating the effects of the evolving plastic hinges, lest the previously formed plastic hinges should "close".

Studying the effect of strain-hardening and plate buckling one should keep in mind that the load-displacement diagram of the structure may be of an ascending type even if the characteristic curves of the given member section or sections are of a descending type in individual cross sections due to the effect of plate buckling.

The elements of the structure in Fig. 1 have dissimilar properties, the change in member 2 — though member 2 is in an unstable state — results in an increasing force-displacement at increasing load (Maier [3]).

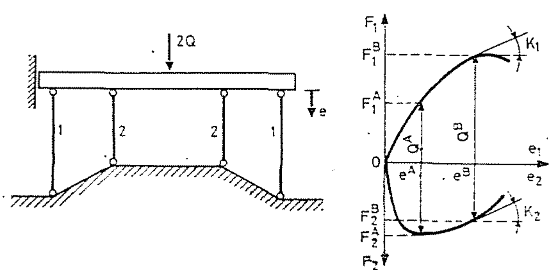


Fig. 1

The behaviour of the structure is characterized by the load-displacement diagram. Let $K_i = \mathrm{d}F_i/\mathrm{d}e_i$, i.e. the slope angle of the load-displacement diagram of the i-th member, and the case of $\mathrm{d}e_i > 0$ is investigated. As an effect of a given load the necessary and sufficient condition for the stable equilibrium state of the investigated structure is

$$K_1 + K_2 > 0.$$

The stability condition relating to individual structural elements is

$$K_1 > 0; \qquad K_2 > 0$$

which is a sufficient, but obviously not necessary condition of the stable equilibrium state of the entire structure. The entire structure may also be stable if, e.g.,

$$K_2 < 0.$$

In the course of our investigations we treated the load bearing critical condition of elastic-plastic structures where an unstable state occurs in some parts of the structure, so that the entire structure retains its stable state. It has been assumed that the unstable state occurs because of the material of the structure or the behaviour of individual structural units. (The effect of unstable state due to geometrical changes of structures has not been analyzed at the present level of investigations.)

In the theoretical plasticity, when deriving the condition of plasticity or some other physical relationships, Drucker's postulate for stability is applied, derived by assuming stable materials (Drucker [4]).

It should be noted that Drucker's postulate is not a natural law but a criterion of classification (Drucker [5]), the materials very often do not correspond to the stable material assumption, or structural elements may behave in an unstable way, while, at the same time, their material is of a stable state.

Maier [6] was the first to treat the problem of the effect of the unstable state of certain members on the behaviour of a triangulated structure. Again it was Maier who in 1966 re-introduced the subject and investigated a structure consisting of compressed members and rigid beams where the load-compression diagram of individual members contained stable and unstable parts.

Maier and Drucker [8] re-examined the original Drucker postulate applied when determining the condition of plasticity since the original postulate is suitable for the determination of the convexity and normality of the condition of plasticity in case of stable materials only.

The thermodynamical aspects of the postulate formulated by Maier and Drucker were elicited by Palmer, Maier, and Drucker, [9]. They found that the normality and convexity of the condition of plasticity are valid also in case of unstable material or structural elements with the exception of "sudden" discontinuous changes.

Maier extended his investigations to bending beams [10] and with co-authors investigated in details the problems of mathematical programming as applied in the field of plasticity (Maier, [7], [11]); Cohn, Maier [12]).

Wood [13] formulated certain problems that were, according to Drucker's postulate, connected with the investigation of unstable material or structural elements.

Ghosh and Cohn [14] undertook the non-linear investigation of continuous reinforced concrete beams with the aid of an experimentally determined moment-rotation relation.

Szatmári [15] introduced an investigation based on an energy-method suitable for the computerized simulation of static failure experiments of steel frames. In the frame of the investigation it is possible to take elastic-softening moment-rotation relations (elasto-softening plastic hinges) into account.

When studying the load bearing capacity of steel structures, the problem of unstable material or softening material, according to Drucker's postulate does not appear since the strain-hardening of the steel material may increase in a major way the plastic load bearing value of the steel structure. However, as it is long known, the final collapse of steel structures is caused — in a high percentage of cases — by instability (plate buckling, flexural-torsional buckling) phenomena that may occur in the cross section or in a structural unit. Concerning steel structures the properties of plastic hinges over and above the usual elastic-ideally plastic-hardening behaviour may be complemented with the effect of instability (flexural-torsional buckling) developing in the given structural unit (environment of the plastic hinge).

This type of inelastic or interactive hinge describes the behaviour of the structural unit and at the same time, also satisfies the criteria of unstable or softening structural unit, according to Maier—Drucker's postulate.

When determining the plastic load bearing capacity of steel structures the interactive hinge of softening character has so far not been considered or applied. The effects of the stability phenomena causing the softening character (flexural-torsional buckling, plate buckling) can be taken into account indirectly with the aid of construction rules. In principle, mathematical programming allows the investigation of more complex steel structures, too, however, it is less suitable for designing practice. The author (Iványi, [16]) has suggested a procedure that takes into account the softening character of the inelastic hinge in the form of an interactive zone. The softening character of the interactive zone is caused by the buckling of the component plates, a phenomenon that can be studied with the help of the yield mechanism.

In 1981, the author (Halász, Iványi, Szatmári [17]; Iványi [18]) investigated the plastic displacement capability of frame structures through the interaction of stability and strength phenomena, analyzed the effect of cross section elements containing buckling component plates on the behaviour of the entire structure. The model of the interactive plastic hinge has been introduced by the author in papers [19] and [20].

3. Investigation of planar frame structures with the effect of interactive plastic hinges taken into account

3.1. *Application of interactive hinges*

In engineering practice the plastic load bearing investigation of planar frame structures is carried out with the aid of the so-called plastic hinge. The investigation for computing statically undetermined structures of elastic-plastic material, using the principles of matrix-calculation has been elaborated — among the first — by Tassi and Rózsa [21].

In the theory of structures the wide application of matrix calculation was introduced by Szabó and Roller [22]. It gave the possibility to treat the theoretical and computational investigations of rod structures in a uniform way, allowed the application of computer technique. The results most important for our investigations are those concerned with the application of kinematic load. In their work Sándor Kaliszky and Márta Kurutz elaborated in detail the computerized computation of structures containing conditional joints with the kinematic load (Kaliszky [23], Kurutz [24]). The procedure was based on the solution of structures by the displacement method. The investigation imitates the effect of the hinges with a rigidity changing stepwise as a kinematic load without altering the original rigidity matrix of the structure continually softening due to the gradually developing interactive hinges.

The investigation of the change of state of the frame structure is carried out by matching two model-parts:

a.) an ideal linearly elastic member

b.) an interactive hinge: *R—O—S—L* model containing the effects of rigid — residual stress — strain hardening — plate buckling.

The ideally elastic member was studied by usual methods [22].

3.11. *The interactive hinge*

(i) In ref. [20] a description of the interactive hinge has been introduced that takes into consideration the effects of rigid — residual stress — strain — hardening — plate buckling and is called a *R—O—S—L* hinge. The model of the *R—O—S—L* hinge can be described with the aid of the "equivalent beam length". In the course of the investigations, the length "*h*" of the "equi-

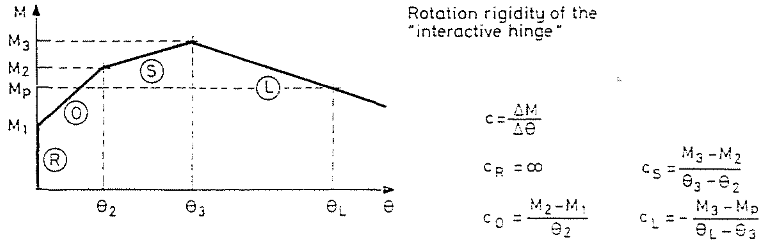


Fig. 2

Compatibility relations of rotation sections:
R section: $c = \infty$; $0 \leq M \leq M_1$; $\Theta = \emptyset$

kinematic load: $t = \emptyset$

O section: $c = c_O$; $M_1 \leq M \leq M_2$; $\emptyset \leq \Theta \leq \Theta_2$

$$\Theta = \frac{1}{c_O}(M - M_1); \quad \emptyset = -\Theta + \frac{1}{c_O} M + \left(-\frac{1}{c_O} M_1\right);$$

kinematic load: $t = -\frac{1}{c_O} M_1$

S section: $c = s_S$; $M_2 \leq M \leq M_3$; $\Theta_2 \leq \Theta \leq \Theta_3$

$$\Theta - \Theta_2 = \frac{1}{c_S}(M - M_2); \quad \emptyset = -\Theta + \frac{1}{c_S} M + \left(\Theta_2 - \frac{1}{c_S} M_2\right);$$

kinematic load: $t = \Theta_2 - \frac{1}{c_S} M_2$

L section: $c = c_L$; $M > M_3$; $\Theta \geq \Theta_3$

$$\Theta - \Theta_3 = \frac{1}{c_L}(M - M_3); \quad \emptyset = -\Theta + \frac{1}{c_L} M + \left(\Theta_3 - \frac{1}{c_L} M_3\right)$$

kinematic load: $t = \Theta_3 - \frac{1}{c_L} M_3$

valent beam" was determined from the moment diagram established during plastic load bearing investigations and this length h does not change — according to assumptions — with the increase or decrease of the load.

Figure 2 shows the model of the R—O—S—L hinge. The rotation rigidity of the hinge for different phases is expressed by the rigidity factor, $c = \Delta M/\Delta\Theta$.

Figure 2 also shows the kinematic loads valid for different rotation phases.

It was assumed that interactive hinges do not "close" with an increase in load, no elastic-type unloading occurs (Majid [25]).

(ii) Anchoring of the columns has not been developed in the form of a mechanical hinge or of complete clamping and therefore, primarily on the basis of experimental results, a "hinge" has been assumed in the cross sections of the anchoring of the column where an increase in moment involves a rotation increasing in a changing measure (Fig. 3).

3.12. *Model of planar frame structures*

The analysis of the Conder-system frame structure figuring in the experimental program has been selected as basis for our investigations. The selected model is shown in Fig. 4.

A potentially occurring interactive hinge has been assumed in the cross sections of possible maximal moment. Interactive hinges 1 through 7 have one degree of freedom while elastic member 8 through 17 have three degrees of freedom and thus the number of degrees of freedom of the model is 37. The state equation is:

$$\mathbf{G}^* \cdot s + q = 0$$

$$\mathbf{G} \cdot u + \mathbf{F} \cdot s + t = 0$$

where:

- $\mathbf{G}$ — geometric matrix
- s — vector or load actions
- q — load vector
- u — displacement vector
- t — vector of "kinematic load"
- $\mathbf{F}$ — flexibility matrix

The solution of the equations is possible in the usual form [22].

3.13. *Changes of state with a uni-parameter load system taken into account*

The load vector q is increased by a value Δq from a suitable starting value till the moment M in the cross section of a potential interactive hinge attains

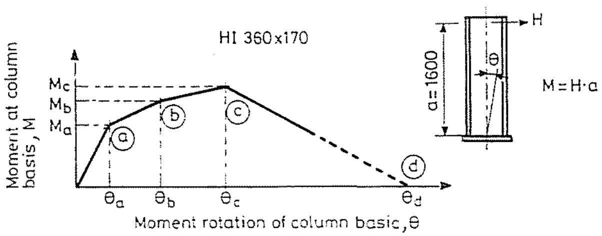


Fig. 3

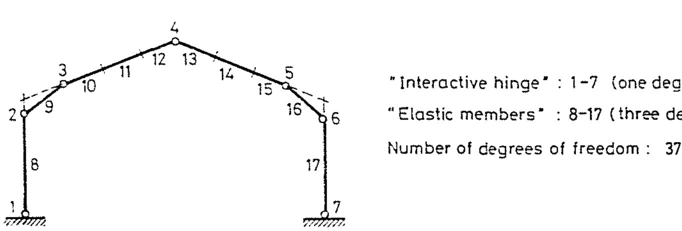


Fig. 4

the value M_1. At this point the relevant kinematic load t is inserted in the hinge and the hinge rotation rigidity is changed to the new value. Further increasing the amount of q, the modifications are undertaken until the determinant of the rigidity matrix becomes negative. This means that the size of the further steps is: $-\Delta q$.

In the course of the changes of state an interactive hinge may be encountered, where because of a plate buckling developing, a descending characteristic curve characterizes the hinge behaviour, however, the frame structure can take up further loads.

The computations have been carried out with the aid of a PDP 11/34 computer.

3.2. *Application of interactive "zones"*

It was Jezek [26] who first studied the role of inelastic "zones" while investigating the inelastic behaviour of cross sections; he pointed out — for simple frame structures — that the load bearing value, determined with the inelastic "zones" taken into account, differs but in a small measure from that determined with the aid of inelastic hinges.

Investigations concerning inelastic "zones" have not become widespread before the sixties, the first analyses being due to Roderick [27] who used experimentally obtained σ—ε diagrams to determine the moment-curvature of cross sections, and using these to obtain the elastic-inelastic load-displacement diagram of frame structures.

Uhlmann and Adam [28] analyzed the effect of load and of the frame structure geometry in case of structures studied with the plastic "zone" model. Their results indicated that in case of certain loads and geometries, a difference exceeding 10% may occur between the result obtained by the model of plastic hinge and that determined when assuming a plastic "zone".

In the course of our investigations

(i) the equilibrium conditions can be written on the one hand based on virtual displacements valid for solid bodies of any material regardless of the actual displacements being small or large and on the other hand, also on the basis of equilibrium equations,

(ii) the relationship between the frame structure displacements is written with the condition that the effect of axial compression of the member is negligible,

(iii) the relationship between the loads acting on the members and the displacements occurring at the member ends is determined with the form of the member element moment-relative rotation (curvature) curve and that of the moment diagram taken into account.

3.21. *Writing the condition of equilibrium*

In the course of our investigations we undertook the analysis of the frame structure figuring in the program with its model shown in Fig. 5.

(a) The conditions of equilibrium are written with the aid of the theorem of virtual displacements assuming that the virtual displacements occur at the corner points A, B, C and E of the frame structure. The assumed system of virtual displacements is shown in Fig. 6 enabling the writing of the condi-

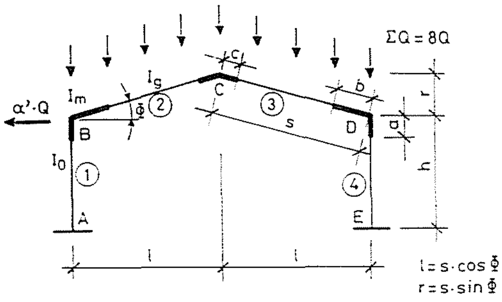
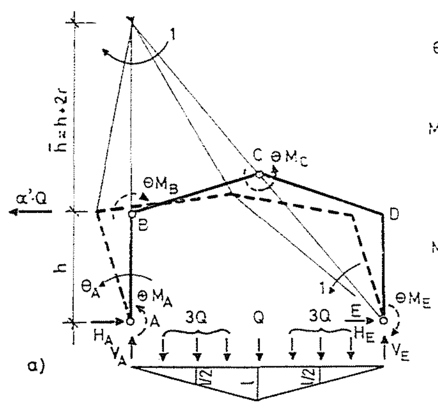


Fig. 5



$$\Theta_A = \frac{h+2r}{h} = 1 + \frac{2r}{h}$$

$$M_A\left(1+\frac{2r}{h}\right) - M_B\left(2+\frac{2r}{h}\right) + M_C \cdot 2 - M_E =$$

$$= 4 \cdot Q \cdot l + \alpha' \cdot Q(h+2r)\,;$$

$$M_D = -M_A + M_B + M_E - \alpha' \cdot h \cdot Q$$

Fig. 6.a

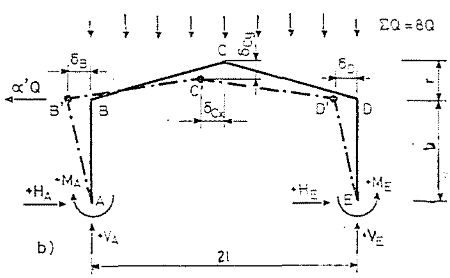


Fig. 6.b

tion of equilibrium relating to the given case. From the moments resulting for the four corner points the moment for point D, the fifth corner point can also be written.

(b) The conditions of equilibrium can also be written in the form of equations of equilibrium. The displaced state of the frame structure is shown in Fig. 6. The equation of equilibrium takes the corner point moments and the reaction forces into account.

The moment for points B' and D':

$$M_{B'} = M_A - H_A \cdot h - V_A \cdot \delta_{Bx}; \qquad M_{D'} = M_E + H_E \cdot h + V_E \cdot \delta_{Ex}$$

The horizontal projection equilibrium is:

$$H_A + H_E - \alpha' \cdot Q = 0$$

The perpendicular projection equilibrium is:

$$V_A + V_E = 8Q$$

The moment of forces acting from the left on point C':

$$M_{C'} = M_E + V_E(1 + \delta_{Cx}) + H_E(h + r - \delta_{Cy}) - 2Q(l + \delta_{Cx} - \delta_{Dx})$$

(bi) If moments M_A, M_B, M_C are assumed to be known the conditions of equilibrium of the second order investigation according to the foregoing are:

$$M_{E'} = M_{C'} + (M_A - M_B)\frac{h + r - \delta_{Cy}}{h} - Q[\alpha' \cdot (h + r - \delta_{Cy}) +$$

$$+ 6(1 + \delta_{Cx}) + 2\delta_{Dx}] + V_A\left(l + \delta_{Cx} - \delta_{Bx}\frac{h + r - \delta_{Cy}}{h}\right)$$

$$M_{D'} = -M_A + M_{B'} + M_E + Q(\alpha' h + 8\delta_{Dx}) + V_A(\delta_{Bx} - \delta_{Dx})$$

where

$$V_A\left(1 - \delta_{Cx} + \delta_{Bx}\frac{h + r - \delta_{Cy}}{h}\right) = M_{C'} + M_A + (M_A - M_{B'})\times$$

$$\times\frac{h + r - \delta_{Cy}}{h} + Q[2(1 + \delta_{Bx} - \delta_{Cx}) - \alpha'(r - \delta_{Cy})]$$

First order conditions of equilibrium:

$$(\delta_{Bx} = \delta_{Cx} = \delta_{Dx} = \delta_{Cy} = 0)$$

$$M_E = M_A\left(1 + \frac{2r}{h}\right) - M_B\left(2 + \frac{2r}{h}\right) + 2M_C - Q[4l + \alpha'(h + 2r)]$$

and

$$M_D = -M_A + M_B + M_E + \alpha' h Q$$

(bii) When assuming moments M_C, M_D, M_E to be known, the conditions of equilibrium of the second order investigations are according to the foregoing:

$$M_A = M_{C'} - (M_{D'} - M_E)\frac{h + r - \delta_{Cy}}{h} + Q[\alpha' h - 6(1 - \delta_{Cx}) + 2\delta_{Bx}] +$$

$$+ V_E\left(1 - \delta_{Cx} + \delta_{Dx}\frac{h + r - \delta_{Cy}}{h}\right)$$

$$M_{B'} = M_A + M_{D'} - M_E - Q(\alpha' h + 8\delta_{Bx}) + V_E(\delta_{Bx} - \delta_{Cx})$$

where

$$V_E\left(1 + \delta_{Cx} - \delta_{Dx}\frac{h + r - \delta_{Cy}}{h}\right) = M_{C'} - M_E - (M_{D'} - M_E)\times$$

$$\times\frac{h + r - \delta_{Cy}}{h} + 2Q(1 + \delta_{Cx} - \delta_{Dx})$$

First order conditions of equilibrium:

$$(\delta_{Bx} = \delta_{Cx} = \delta_{Dx} = \delta_{Cy} = 0)$$

$$M_A = M_E\left(1 + \frac{2r}{h}\right) - M_D\left(2 + \frac{2r}{h}\right) + 2M_C - Q(4l - \alpha' h)$$

and

$$M_B = M_A + M_D - M_E - \alpha' h Q$$

3.22. *Displacements of the frame structure*

The state of displacement of the investigated frame structure has been determined with the assumption that the effects of axial compressions are negligible, the development of so-called "small displacements" are reckoned with.

For the state of displacement shown in Fig. 7 the relationship between displacements (rotations, deflections), associated with the corner points can be written.

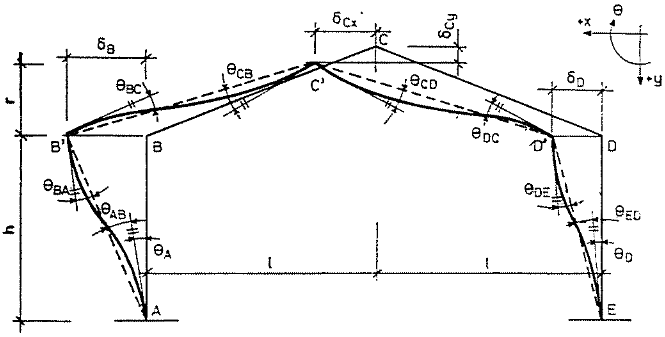


Fig. 7

ABC half-frame

$\delta_B = -(\Theta_A - \Theta_{AB})h;$

$\Theta_B = \Theta_A - (\Theta_{AB} + \Theta_{BA}) - \beta_B M_B$

$\delta_{Cy}^{\text{left}} = l\Theta_B - \Theta_{BC})$

CDE half-frame

$\delta_D = -(\Theta_E + \Theta_{ED})h$

$\Theta_D = \Theta_E + (\Theta_{DE} + \Theta_{ED}) + \beta_D M_D$

$\delta_{Cy}^{\text{right}} = -l\Theta_D + \Theta_{DC})$

① $\Delta_{yC} = \delta_{Cy}^{\text{left}} - \delta_{Cy}^{\text{right}} = \Theta_B - \Theta_{BC} + \Theta_D + \Theta_{DC}$

$\delta_{Cx}^{\text{left}} = \delta_B - (\Theta_B - \Theta_{BC})r$ $\qquad \delta_{Cx}^{\text{right}} = \delta_D + (\Theta_D + \Theta_{DC})\, r$

② $\Delta x_C = \delta_{Cx}^{\text{left}} - \delta_{Cx}^{\text{right}} = -\Theta_A + \Theta_{AB} + \Theta_{ED} + \Theta_E + \frac{r}{h}(-\Theta_B - \Theta_{BC} + \Theta_D + \Theta_{DC})$

$\Theta_C^{\text{left}} = \Theta_B - (\Theta_{BC} + \Theta_{CB}) - \beta_C M_C \qquad \Theta_C^{\text{right}} = \Theta_D + (\Theta_{DC} + \Theta_{CD})$

③ $\Delta\Theta_C = \Theta_C^{\text{left}} - \Theta_C^{\text{right}} = \Theta_B - (\Theta_{BC} + \Theta_{CB}) - \beta_C M_C - \Theta_D - (\Theta_{DC} + \Theta_{CD})$

It is assumed that the column anchoring developed at points A and E is loaded by a rotation increasing with moment at a varying rate.

On basis of experimental results the moment-end rotation (Fig. 3) is assumed to be characterized by the curve:

$$\Theta_A = f(M_A)$$

At points B, C and D at the so-called butt-plate connections, the moment-rotation relation is assumed to be linear on the basis of experimental results.

$$\Theta = \beta \cdot M$$

where

β is the spring parameter of the butt-plate connection.

Hence, the compatibility of the displacements at point C of the frame structure can be written, computed from the displacements of the half of frames ABC and CDE. Displacements Δy_C, Δx_C and $\Delta\Theta_C$ give the error of closure of the computation, in our investigations we wish to achieve a value below the prescribed limit of this error of closure.

3.23. *Relation between the member-end loads and rotations*

When developing the model of the "interactive zone", in the course of determining the moment-relative rotation (curvature) relation of the member element, in general both the effect of residual stresses and strain-hardening, as well as plate buckling can be taken into account. Presently, in order to facilitate manual computation, only the effect of strain hardening and of plate buckling is considered and thus the *E—S—L* model is applied (Fig. 8).

In the knowledge of the moment-relative rotation (curvature) curve the moment-rotation relations at the member ends can be determined for the sections between the so-called corner points of the frame structure (points *A*, *B*, *C*, *D* and *E*) with the aid of the "moment-area" method.

(i) If $M_p < |M| < M_H$ (section *S*), Fig. 9 shows member *JK*, bending by a linearly varying moment. The moment exceeds the value M_p characteristic of the cross section in the environment of cross section *J*, the hardening section develops.

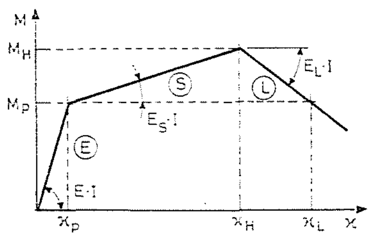
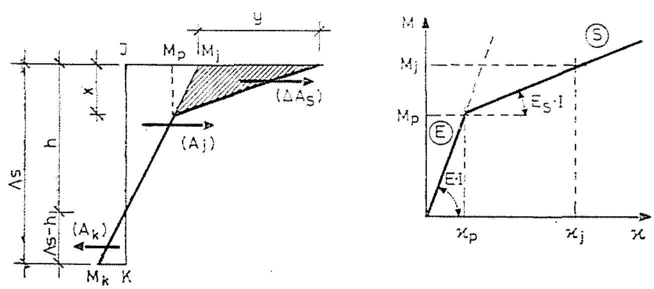


Fig. 8



$$x = \left(1 - \frac{M_p}{M_j}\right) h \; ; \qquad y = \frac{x}{E_S \cdot I} \; ;$$

$$(A_{jk}) = -(A_k) + (A_j) + (\Delta A_S) \; ; \qquad (\Delta A_S) = \frac{1}{2E_S \cdot I}\left[\left(1 - \frac{M_p}{M_j}\right) h\right]^2$$

$$(I_k) = \left[-(A_k) \cdot \frac{\Delta s - h}{3} + (A_j)\left(\Delta s - \frac{h}{3}\right) + (\Delta A_S)\left(\Delta s - \frac{x}{3}\right)\right]$$

Fig. 9

The moment diagram is modified by the rigidity factor $E_S I$ of the hardening section S and thus the member-end rotations can be determined:

$$\Theta_{JK} + \Theta_{KJ} = (A_{JK})$$
$$\Theta_{JK} = -\frac{(I_K)}{\Delta s}$$

(ii) If $|M| < M_H$ (section L), Figure 10 shows member JK. The moment in the environment of cross section J exceeds the value $\varkappa_H$, characteristic of the cross section, the plate buckling section develops, too. The moment diagram is modified by the rigidity factor $E_S I$ characteristic of the hardening section S and the rigidity factor $E_L I$ characteristic of the plate buckling section L and in this way the rod end rotations can be determined:

$$\Theta_{JK} + \Theta_{KJ} = (\bar{A}_{JK})$$
$$\Theta_{JK} = -\frac{(\bar{I}_K)}{\Delta s}$$

The values x_H, y_H pertain to the state $|M| = M_P$ and are constant when computing section L.

Let us note that at points B and D in the environment of wedging up (in a section of length b), the development of an elastic deformation only was reckoned with because of the anchoring.

3.24. *Investigation of the change of state of frame structures with the aid of interactive "zones"*

a) The change of state of the frame structure can be investigated by iteration.

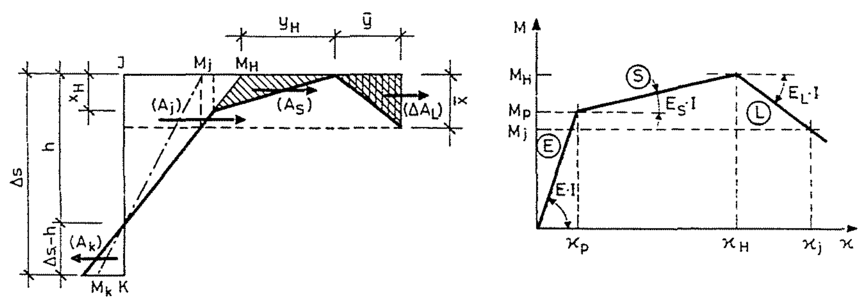
From among the six unknown values, Q, M_A, M_B, M_C, M_D, M_E the value one is assumed. This is usually the breaking point M_E or M_H of the relations M—Θ or M—$\varkappa$ pertaining to the support or the rods.

Further three values are selected to control the iteration and they are estimated:

$$X_1 = Q$$
$$X_2 = M_A$$
$$X_3 = M_C$$

The two remaining values, at present M_D and M_E are computed from the relations described in point 3.21.

Hence, the displacement state of the frame structure — on the basis of



$$x_H = \left(1 - \frac{M_p}{M_H}\right) h \; ; \quad y_H = \frac{x_H}{E_S \cdot I} \quad ; \quad \bar{x} = \left(1 - \frac{M_j}{M_H}\right) h \; ; \; \bar{y} = \frac{\bar{x}}{E_L \cdot I}$$

$$(\overline{A}_{jk}) = -(A_k) + (A_j) + (A_F) + (\Delta A_L) \; ; \quad (A_S) = \frac{1}{2} y_H x_H \; ; \quad (\Delta A_L) = \frac{1}{2 E_L \cdot I} \left[\left(1 - \frac{M_j}{M_H}\right) h\right]^2$$

$$(\overline{I_K}) = [-(A_k) \frac{\Delta s - h}{3} + (A_j)\left(\Delta s - \frac{h}{3}\right) + (A_S)\left(\Delta s - \frac{x}{3}\right) + (\Delta A_L)\left(\Delta s - \frac{\bar{x}}{3}\right)$$

Fig. 10

point 3.22 — can be determined and the compatibility error of closure of the corner point C can be written:

$$\Delta y_C \neq 0; \qquad \Delta x_C \neq 0; \qquad \Delta \Theta_C \neq 0$$

If the error of closure is less than the given bound of error, the unknowns satisfy the conditions of equilibrium of compatibility, as well as the physical requirements, thus they yield one point (the breaking point) of the load-displacement characteristic curve of the structure.

If the error of closure exceeds the error bound the investigation is to be repeated by assuming a further value. With sufficient practice the third trial usually yields a result — an acceptable error of closure.

The complete load-displacement characteristic curve is determined by assuming approximate moment diagrams and by linear extrapolation. It is therefore necessary to determine the characteristic curve starting from $Q = 0$ with Q gradually increasing and having achieved the maximum of load bearing, with Q gradually decreasing so that all characteristic curve points at any possible change in the relationship M—Θ and M—$\varkappa$ be considered one by one. The curve-sections between the breaking points of the characteristic curve can be computed in a relatively simple way.

(b) In the course of the investigation the possibility is given to take second order effects, too, into account, namely by writing the equilibrium statements for the displaced state of the frame structure.

In this case the investigations have to be first carried out in an initial state by assuming $\delta = 0$, thus carrying out a first order investigation and hence, the relations can be improved with the aid of the resulting corner point displacements and the iteration can be accomplished again.

4. Results of theoretical and experimental investigations carried out on frame structures

4.1. *Application of the model of the interactive hinge*

The effect of plate buckling has been studied by comparing the experimental and theoretical results of frame structures *C*-1, *C*-2, *C*-3/2; *B*-1/3 of the experimental program. The circumstances of experimental investigations, the mode of realization and the arrangement of the frame structure are introduced in ref. [29]. The results of the investigation C-3/2 are given in detail.

In the course of the theoretical computations the arrangements of the column bases have not been considered to be ideal hinges (interactive hinges 1 and 7) but (Figs 1—3) have been taken into account on the basis of experimental results.

When ascertaining the features of interactive hinges 2—6, the effects of normal force N developing in the rods can be taken into account, however, in the present case the effects resulting from finite deformations are small [29] and thus the reducing effect of normal force N as regards interactive hinges and elastic rods has been neglected.

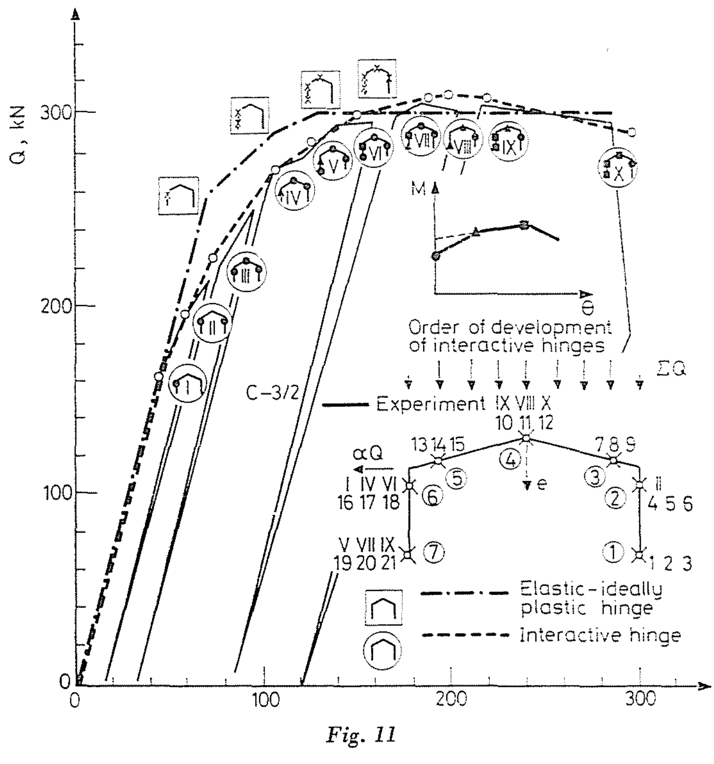


Fig. 11

The "equivalent beam" length necessary to determine the properties of interactive hinges 2—6 was found from the moment diagram determined with the aid of plastic load bearing study. Interactive hinges also contain the effect of residual stresses, during our investigations a ratio $M_r/M_p = 0.7$ has been assumed.

Concerning the experimental beam C-3/2, the relation of load-perpendicular displacement develops according to Fig. 11. At the side of horizontal load, the first inelastic hinge develops due to the residual stresses and deformations in the cross section beneath the frame structure wedging up and this hinge develops at 52% of the maximal frame load. At 97% of the maximal load, the zone L describing the effect of plate buckling develops also in this cross section, i.e. in the frame cross section an "unstable" state — a descending characteristic curve — develops.

Figure 11 introduces the characteristic load-displacement curve of the frame structure in the case, too, when the basis of the computations is the traditional plastic hinge.

The results well show that the presence of residual stresses influence in a major way the range of limited plastic deformation, however, mainly because of the cross section geometry of the experimental beam the maximal load bearing values computed with the traditional (elastic—ideally plastic) hinge as well as those obtained by the interactive hinge coincide with the experimental results.

The computation was suitable to determine the moments developing on the interactive hinges. In the process of experimental investigations, the deter-

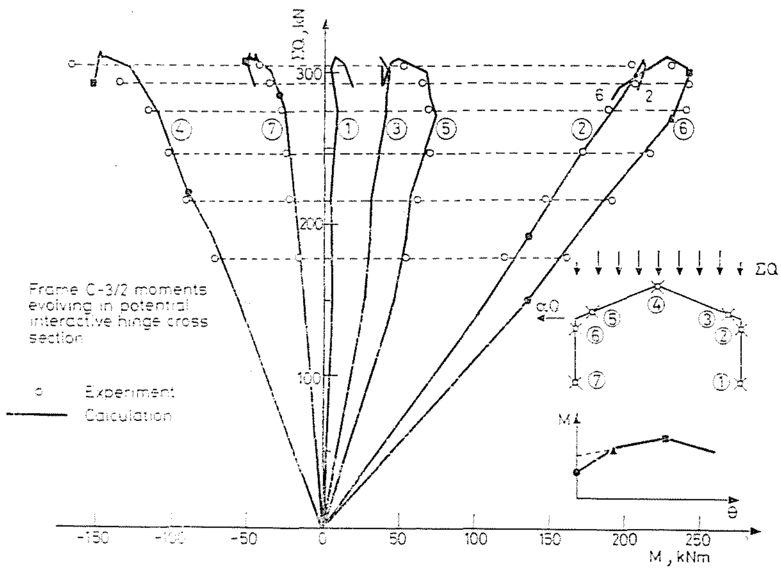


Fig. 12

mination of values of the moments in individual cross sections was only possible with the aid of strain gauges. When using an electric strain gauge the following has to be taken into account:

(i) the measuring range of strain gauges is limited, the strain hardening deformations of steel material can only be registered with high strain capability gauges;

(ii) due to the incidental nature of local deformations — because of the inhomogeneous behaviour of the steel material and of plate bucklings — acceptable results can only be obtained by using a very high number of strain gauges.

According to the foregoing, the extrapolation method has been employed. In different cross sections of the members that are expected to remain in elastic state during loading, the moments have been determined with strain gauges, so that the moment diagram of the relevant member could be drawn from these measurement results and this moment diagram served as a basis of comparison.

Figure 12 shows the results of the theoretical and experimental investigations. The values corresponding to the breaking points of the characteristic curve of the interactive hinge, have also been indicated. It can be seen that the tendency of the experimental results supports those of the theoretical ones.

4.2. *Use of the interactive "zone" model*

We introduce the results of investigations of the frame structure outlined in Fig. 13.a.

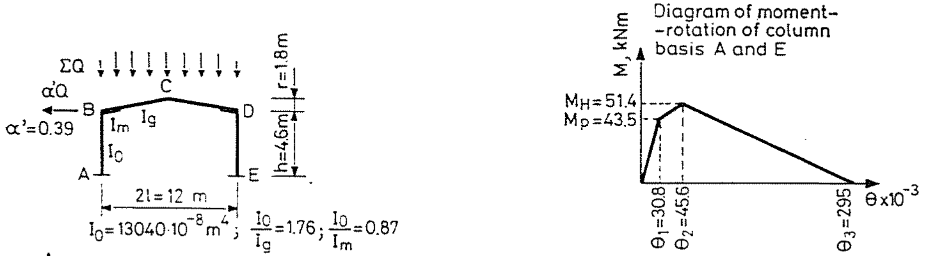
The investigation results are shown in Fig. 13. b. The diagram also gives the results of the first-order investigations carried out with the interactive hinge model *R—S—L* (and the interactive zone model *E—S—L*). The load displacement curves computed with the two kinds of models are practically coinciding. The investigation does not contain the effect of residual stresses and deformations.

The frame has been analyzed with the aid of second-order investigations. A load bearing capacity obtained by first-order investigation is higher by 6% than the one obtained with the second-order solution.

Assuming a partial clamping at the column, the first-order load bearing computed, based on the ideal plastic yield mechanism ($M = M_P$) is: $Q_t = 282$ kN.

This value — despite the fact that the moment load capacity $M_H > M_P$ in this case for individual members — is higher than for the load corresponding to the first-order solution computed with the interactive zone. The reason is that at the load bearing maximum (computed points 5—6) the moment load bearing of corner *B* has a highly diminished value.

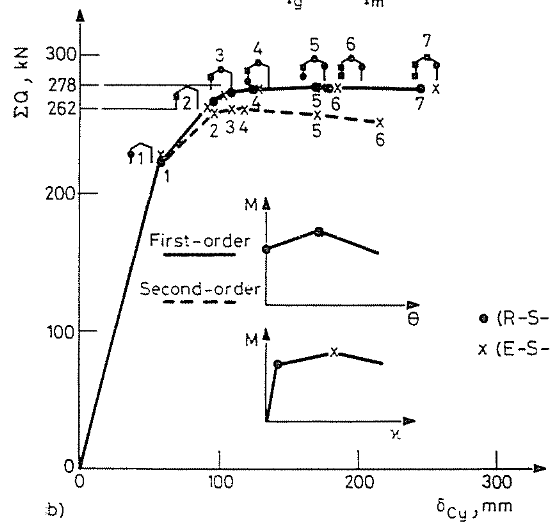
Figure 14 shows the development of loads determined with first-order and second-order investigations.



Moment-specific rotation diagrams of members

	M_p, kNm	M_H, kNm	$\frac{E}{E}$	$\frac{E}{E_L}$
Columns (AB & DE)	206	218.4	420	656
Beams (B & D)	135	147.7	420	494
Beams (C)	135	137.9	420	39

a)



● (R-S-L) interactive hinge
x (E-S-L) interactive zone

Fig. 13.a b

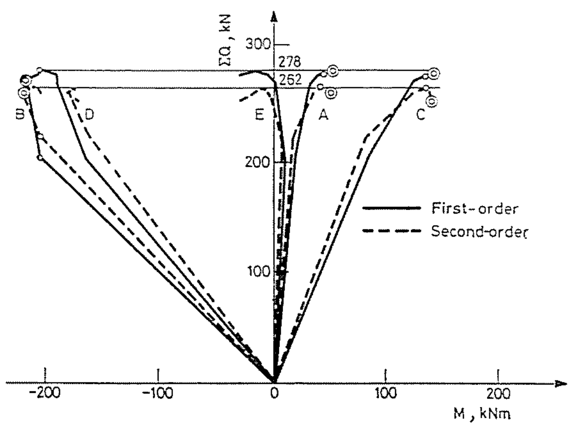


Fig. 14

4.3. *Comparison of investigation results*

When comparing the investigation results it has to be underlined that on the one hand the experimental investigations have been completed with a detailed experimental investigation of material and structural element characteristics, thus the experimental investigations could be tested in the different phases and on the other hand, the theoretical calculations endeavoured to take into consideration a number of influencing factors (e.g. strain hardening of steel material, residual deformation, plate buckling) thus the aim of these investigations has been to determine the complete load-displacement characteristic curve. We also studied the behaviour of frames with an engineering approximation method that did not analyze the load-displacement relation by the "step by step" method, but determined directly the maximum load bearing values and the displacement [19].

In Table I the results of experimental and theoretical investigations have been summarized.

a) One of the bases for comparison is the determination of the ratio of the computed and measured values of the displacement corresponding to the maximum load bearing value.

On the basis of the results in the Table it can be ascertained that the computation methods determine the maximum load bearing and displacement values of the frame studied experimentally within an acceptable accuracy.

b) The other comparison basis is the so-called rotation-capability. When the design of steel structures, the rotation capability is an important requirement that can be measured with the "horizontal" length of the load-displacement curve.

Table I

Sign of frames	Results of experimental investigations		Plastic load bearing	Investigation with interactive hinge		Approximative engineering investigation	
	ΣQ_{exp} [kN]	$e_{r\,exp}$ [mm]	ΣQ_p [kN]	ΣQ [kN]	e_{r1} [mm]	ΣQ_2 [kN]	e_{r2} [mm]
C-1	285	145	289	282	158	291	132
C-2	312	218	313	313	197	305	166
C-3/2	311	209	309	310	193	318	174
B-1/3	322	197	315	317	184	310	163

$\frac{\Sigma Q_p}{\Sigma Q_{exp}}$	Investigation with interactive hinge		Approximative engineering investigation		Plastic displacement capability		
	$\frac{\Sigma Q_1}{\Sigma Q_{exp}}$	$\frac{e_{r1}}{e_{r\,exp}}$	$\frac{\Sigma Q_2}{\Sigma Q_{exp}}$	$\frac{e_{r2}}{e_{r\,exp}}$	e_r [mm]	e_{max} [mm]	$\frac{e_{max}}{e_r}$
1.014	0.989	1.068	1.021	0.910	80	180	2.25
1.003	1.009	0.929	0.978	0.783	74	260	2.50
0.994	0.997	0.923	1.016	0.852	78	290	3.70
0.978	0.984	0.934	0.963	0.827	98	280	2.95

It is necessary that the practically horizontal section be very long, the descending branch should intersect the horizontal straight line at $Q/Q_P = 0.95$ $e > 3e_r$, where e_r is the deformation of the structure considered elastic up to the computed load bearing capacity.

The evolving displacement capability of the frames under investigation is shown in Table I.

As it is to be observed, the computed and measured displacement capabilities coincide well. It should be noted that the displacement capability of frame *C*-1 is not sufficient as regards plastic dimensioning, first of all because of the development of the lateral supporting system.

A remark in connection with the character of loading seems rather important.

At mathematical (theoretical) investigations, virtual disturbances have been assumed for the equilibrium state analysis of the structure so that these disturbances do not influence loading (Hoff [30]). However, in case of experimental investigations, these disturbances are, quite naturally, real ones thus their effect does not only manifest itself on the structure but also in the loading system. Therefore the inter-effect of the structure and the loading system has also to be determined at experimental investigations.

The significant majority of steel structures in engineering practice is loaded with dead or gravity load, thus the highest point of the load-displacement diagram of the structure also indicates the loss of stable equilibrium state. However, this general observation has become a hindrance to the cognitive process, since, on the basis of the above observation, not only the complete structure but also individual structural elements have mainly been analyzed experimentally by gravity load. Therefore, according to the character of gravity load, the state after achieving the load-displacement diagram peak was not known in structural elements and this may have "given" basis to the statement that at the investigation of the entire structure no stability loss (evolvement of descending load-displacement) of the structural elements could take place prior to the development of global stability loss, or should it occur, that at the same time, means the collapse, stability loss of the complete structure. This train of thought eliminated major problem spheres from the program of theoretical and experimental investigations. In the course of our investigations and analyses it were first the experimental results that indicated and then proved very convincingly that this type of viewpoint simplifies the behaviour of the structure gravity load, type loading "covers up" the exhaustion of the load bearing capacity of individual structures and its effect on the structure.

A full recognition of the behaviour of the structure also involves the knowledge of the behaviour of structural elements and thus it is not only demanded from the viewpoint of "comfort" that when investigating the sup-

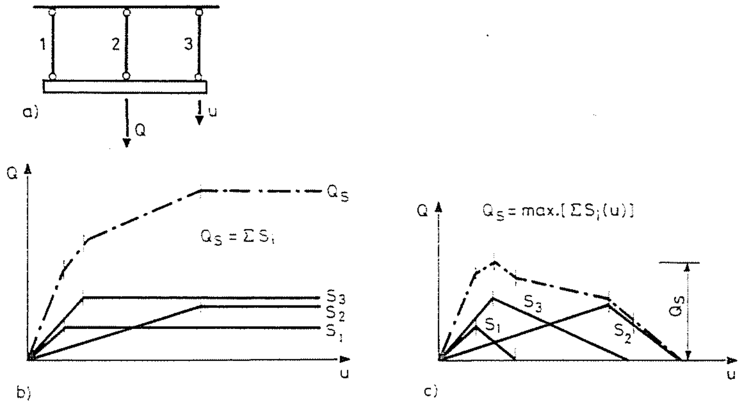


Fig. 15

porting structures also the descending section should be revealed, but this is also required by the demand of a complete knowledge.

The relationship of loading character and the model of supporting structure can be seen in the simple model in Fig. 15.a. The state indicated in Fig. 15.b develops as an effect of gravity load character loading. The state shown in Fig. 15.c — taking also into consideration the descending sections of the load-displacement curves of individual part-units — develops as an effect of deformation type loading. The maximum load capacity characterizes the behaviour of structures sufficiently well so the use of deformation-type loading is not absolutely necessary in this case. However, it is expedient to carry out the experiments usual for investigating structural elements, beams, columns, connections, column bases, etc., first and foremost with deformation-type loading if both the ascending and descending parts of the load-displacement curve are to be considered and if the displacement capability of the supporting structures is to be determined.

5. Conclusions

Theoretical and experimental investigations have been carried out in connection with the plastic load-bearing study of frame structures, with the steel material strain hardening, the residual stresses and plate buckling taken into account.

A method has been presented for the investigation of frame structures applying the steps of known, traditional methods so that the structure behaviour can be analyzed during the entire process of loading. Certain effects determining the structure behaviour (e.g. residual deformation, steel material strain hardening and plate buckling) have been taken into consideration with the aid of the interactive hinges. The interactive hinge was incorporated into an investigation method operating with the structure matrix-calculation method.

The plastic load bearing of the frame was studied also by applying the interactive zones. In the frame of the investigation the possibility was given to write the equilibrium equations relating to a deformed state, thus the investigation may be of the second-order character.

The results of the elaborated methods have been compared with the experimental investigation of full-scale structures.

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Prof. Dr. Miklós IVÁNYI H-1521 Budapest