COMPUTER PROGRAM FOR STEEL FRAMES TAKING INITIAL IMPERFECTIONS AND LOCAL BUCKLING INTO CONSIDERATION

R. BAKSAI, M. IVÁNYI and F. PAPP

Department of Steel Structures,
Technical University, H-1521 Budapest

Received November 29, 1984

Abstract

Computer technique suits state change analysis of plane steel bar systems. Investigations considered several effects and influences such as plastic deformation, geometrical and mechanical imperfections, plate buckling. The analysis relies on modelling the structural member such as to involve several effects and influences.

1. Introduction

Recent development of engineering structures imposed to increase the accuracy of existing computing methods, to develop new methods for computing the novel structures — in fact, increasing dimensions and even complex structural design require lengthy, highly exact computations. The fast generalization of computers and rapid development of their technical parameters permitted to replace "manual computation" by computer analysis methods. Procedures such as finite element, finite strip, bar system programs exist for computing a wide range of structures: e.g. slabs, diaphragms, shells, lattices, trusses and frameworks. Application of these methods has the double advantage of making the "inaccessible" structures computable; and of increasing the accuracy of determining the stress pattern of structures computed earlier in a different way — generally approximated — improving the economy of design.

Analyses concerned more exact determination of the stress pattern of steel in-plane frameworks in the plastic range (by a more realistic approximation of the given technical parameters) are possible in either of the following ways:

a) Nonlinear model of the material and the bar

For structural analyses concerning static loads, essential strength and deformation characteristics of steel are supplied by static characteristic curves obtained in laboratory tests. σ—ε diagrams of steel are common knowledge but too intricate for practical computations, hence subject to different idealizations such as linear elastic, rigid-plastic, linear-elastic — linear-strain-hardening, etc. σ—ε curves. Selection of the material model is not determined by the required computation accuracy or the admissible computation complexity alone, since in several kinds of structures, certain circumstances prohibit the development of important deformations or yield. In such cases, the load capacity increase between the tensile strength and the yield point of steel is *a priori* to be ignored.

Based on theoretical and experimental research [1], introduction of the interactive plastic hinge model closer approximating the σ—ε diagram permitted to reckon with the elastic range, the effect of yield and strain hardening, of residual stresses of the structure, and buckling of member phenomena.

b) Increased accuracy of geometry description, i.e., applying the second-order theory

Also reckoning with geometrical nonlinearity increases the accuracy, though at a significant increase of the volume of computations.

The program writes equilibrium equations for deformed rather than undeformed members; thus it ignores the principle of stiffening. Computations according to the second-order theory permit to study how exactly the first-order theory describes the behaviour of the tested frameworks. Second-order computations are also justified by the important deformations of steel at and beyond the yield point, not to be omitted by applying the principle of stiffening.

Meeting these two kinds of requirements leads to a much more true image of the behaviour of structures.

2. The computer program

2.1 *Fundamentals of bar system computation*

Matrix methods are available for computer determination of stresses in plane bar systems [2]. These methods are relying either on the force or on the displacement method, this latter has been applied in making the program.

A method well-known and proven in the special literature has been followed. As a first step, the structure has been decomposed into nodes and rectilinear bars. Bar ends have three degrees of freedom in displacement: two-way displacements in the structure plane (u_x and u_y) and rotation in this plane (φ_z). Accordingly, in any bar, three different stresses can be determined, i.e., due to normal force N, shear force T, and bending moment M. In knowledge of bar stiffness matrices $\hat{\mathbf{K}}$, overall stiffness matrix $\mathbf{K}$ of the structure can be compiled. Also reckoning with supporting conditions followed the known procedure: cancelling from $\mathbf{K}$ block rows and columns corresponding to support numerals, thus, reckoning with a stiffness matrix of reduced size. For a given load, also load vector q is known, hence, after solving the equilibrium equation system $\mathbf{K} \cdot u = q$ to $u = \mathbf{K}^{-1} \cdot q$, bar end stresses are obtained from the system of nodal displacements.

To increase the accuracy of geometry description, the second-order theory has been applied. Second-order computation of bars loaded by normal forces may apply the known stability functions [3]. As a consequence, bar stiffness matrix $\mathbf{K}$ will not be constant (like in the first-order theory) but function of the bar force parameter $\frac{N}{N_E}$ (where N_E is the Euler critical load).

Thereby all the computation becomes iterative, solutions $u_1, u_2, \ldots, u_n$ of the equilibrium equation system $\mathbf{K} \cdot u = q$ converge after a number of computation cycles to the exact second-order solution.

2.2 *The applied bar element*

Simpler cases involve the bar element in Fig. 1, permitting fast, easy computations mainly on an elastic material model. Bar stiffness matrix $\hat{\mathbf{K}}$ is common knowledge; stiffness values are obtained by solving basic problems of hyperstatic beams.

Our goal seemed to be better achieved by applying a complexer bar element (Fig. 2):

Two ends of the bar, of lengths l_1 and l_2, are infinitely rigid (maybe $l_1 = l_2 = 0$); the middle part is elastic. Rigid and elastic bar parts are connected by a screw spring each, able to rotation φ in the structure plane alone. Stiffnesses, i.e., spring constants of screw springs are c_1 and c_2.

This assumption of the bar element had the following motivations:

— A plastic second-order analysis was the goal. Applying the usual bar elements, development of plastic hinges would require intercalation of a spring of stiffness c instead of a plastic hinge each hence to increase the stiffness matrix $\mathbf{K}$ of the structure. This assumption of the bar element avoids this computational problem, by permitting simulation of the development of plastic hinges by changing the stiffness of the inner screw spring.

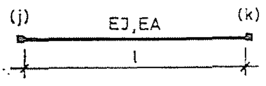

Fig. 1

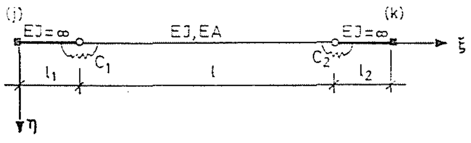

Fig. 2

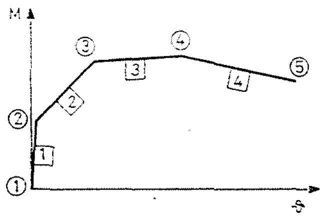

Fig. 3

— Assumption of an ideal clamping or ideal hinge is not exact enough for closer computations. This bar element permits also to reckon with elastic clamping.
— Structural design may produce infinitely stiff bar parts (gussets). Although it can be reckoned with by assuming EI sufficiently high for conventional bar elements, stiffness matrix $\hat{\mathbf{K}}$ becomes thereby poorly conditioned, adding to numerical errors. The presented bar element helps to avoid this problem.

Stiffness matrix $\mathbf{K}$ of the bar element can be written in knowledge of bar end stresses resulting from unit nodal displacements.

(Computation of bar end stresses is presented in Appendix.)

2.3 *Spring characteristics*

Spring characteristics are of the following general form (Fig. 3):

Sections have different spring constants $c = \dfrac{\Delta M}{\Delta \Theta}$ indicating the given section of the elasto-plastic behaviour or of the stability condition of the bar part. The characteristic is strictly monotonous for Θ but not for M. Namely there is a peak followed by a descending limb of the curve.

Application of spring characteristics lessens the validity of the theorem that a hyperstatic beam with n redundancies fails at the development of the $n + 1$-th plastic hinge.

It is true only for computations relying on rigid-plastic hence not strain-hardening models. It is both logically obvious and demonstrated by computations that plastic reserves after yield permit more than $n + 1$ interactive hinges to develop. Final failure depends on a complex interaction between structural design, loss of stability phenomena and yield mechanism.

2.4 *The computation method*

Also plastic analysis is set of computations to be considered as elastic where the structural problem is solved for successive load increments. In each load increment, the program performs the following computations:

— elastic-type computation for a load increment of assumed value (calculating with the actual c value for each spring), determining the moment at each spring;
— examination of each spring for eventual spring characteristic changes, hence for the need of turning to a section of reduced direction tangent;
— computation by linear interpolation or extrapolation of the least value of load increment (in second-order computation by several iterative cycles) leading exactly to the end of a straight section on a spring characteristic. Adding characteristics belonging to this load increment (e.g. moment level of springs, overall displacement vector of the structure, vector of normal load parameters of the bars, load increment value, etc.) to values obtained in the former step yields state characteristics for the complete load level;
— in the spring affected by the change, the program reduces the spring characteristic value according to the Θ—M function, underlying computation of the next load increment.

Details of the computation procedure have been described in [4].

This program lends itself to solve bar systems with single-parameter loads. It has generally the consequence that spring rotations monotonously increase with the load level. It can, however, be realized that in certain structures the developing interactive hinges change the stress pattern in the structure so as to unload certain springs, reducing the arisen rotation. If at that place no interactive hinge has developed to then, hence section 1. of the spring characteristic prevails, also unloading follows according to spring constant c_1 corresponding to this section (Fig. 4).

In sections 2, 3, maybe 4, however, unloading follows (Fig. 5).

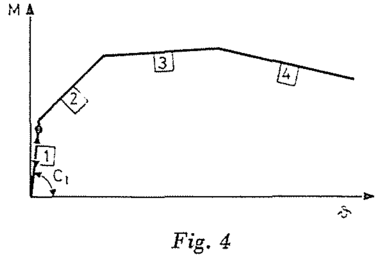

Fig. 4

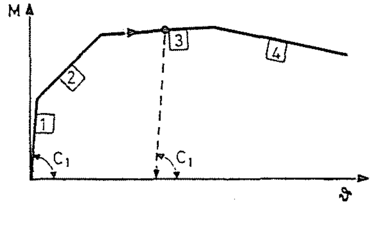

Fig. 5

Clearly, since an interactive hinge behaves elastically in opposite rotation. Thereby directions of new rotations in springs have to be observed, namely for a rotation opposite to the former one, the needed data have to be modified accordingly, and the load increment recomputed.

In course of loading, the bar can bear further loads, although with ever less increments, by developing further interactive hinges. Meanwhile, however, stability loss phenomena arise at these spots. After a given level, loading cannot be further increased, thus, increasing displacements arise at decreasing load levels. This phenomenon has to be reckoned with in computations, possible by examining the sign of the determinant of stiffness matrix **K**. Namely until det $|\mathbf{K}| > 0$, the beam can be further loaded, while for det $|\mathbf{K}| < 0$, there is unloading.

3. Survey of experimental and computational results

3.1 *Laboratory experiments*

In 1978, the Department of Steel Structures, TUB, launched a research program to underly plastic design specifications for steel frameworks.

The test series were made in the Building Laboratory on a framework to scale 1 : 1. Frames had high-strength prestressed bolts at rafter-to-column and midspan connections, with frame corners wedged. Frames were held in plane position by bars (U80 channel) with hinged support at the purlins, and by bars of variable stiffnesses and differently connected at wedgings, column top and at wall posts.

Frames were exposed to vertical loads and generally also to horizontal forces: vertical loads acted at designed purlin places while horizontal forces under the wedging pointed outwards to a column. Beam top flanges were loaded by uniform and conservative vertical forces by means of a so-called gravity force simulator, while the horizontal load was 5% of the overall vertical load level. Load was a single-parameter one. General sketch of the framework is seen in Fig. 6. For detailed description and results of the experiment see [5].

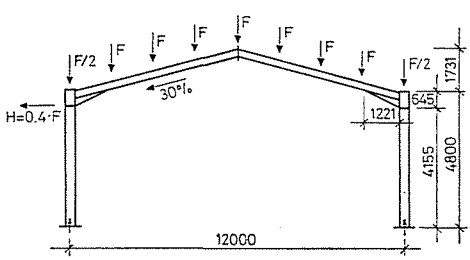

Fig. 6

3.2 *Comparison of computer outputs and test results*

First- and second-order analysis of framework *C*-3/2 has been compared with test results.

Computer analysis divided beams to 4 bars, columns to 1 bar each. On the one hand, wedging has been accounted for with bar ends of infinite stiffness be-

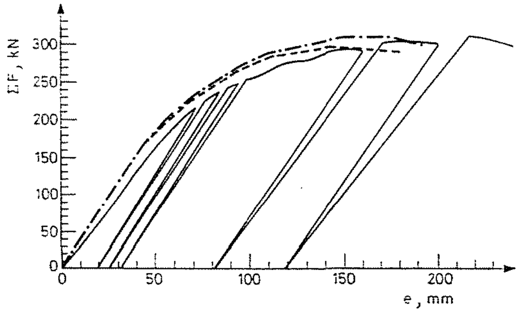

Fig. 7. Experimental and computed load-displacement diagrams. Legend: —— experimenta — . — first-order computation; ----- second-order computation

cause of structural design, on the other hand, varying inertia of wedging has been taken into consideration with an increased substitutive inertia. In assuming the theoretical bar axis, the wedging effect has been neglected.

Load level and deflection at ridge point (midspan) have been plotted in Fig. 7.

Evaluation and comparison of first- and second-order computation results points to the following:

— load levels either at first yield or the maximum one are less in second-order computation, and also the second-order load-displacement diagram lies below the first-order one — in conformity with theoretical considerations;

— the two computations but slightly differ for long, and become first distinct at about 90% of the max. load level. Max. load levels in the two computations differ by as little as 5%;

— plastic hinges develop at the same spots in either computation, practically in the same sequence;

— the load-displacement diagram flattens gradually, although the second-order characteristic curve has no such an about horizontal plateau near the maximum load level as the first-order one; but also here, important deformations are seen to develop at slightly varying load levels.

3.3 *Effects of initial geometry imperfections and of residual stresses*

3.31 *On geometry imperfections*

The effect of initial geometry imperfections has been examined. Considering form and assembly technology of CONDER frameworks, the most likely geometry imperfections of the framework are due to a somewhat skew, rather than vertical, adjustment of the columns, involving cases (Fig. 8):

Cases 1: assumed values are: $\delta_1 = \delta_2 = \begin{cases} h/500 \\ h/100 \end{cases}$

Cases 2: assumed values are: $\delta_1 = 2\delta_2 = \begin{cases} h/500 \\ h/100 \end{cases}$

3.32 *On residual stresses*

Every structural member is known to have lower or higher residual stresses; that is, cross sections even unloaded are not exempt from stresses. Residual stresses arise in manufacture and assembly (e.g. rolling, welding, straightening, bar insertion, etc.). In welding an I-beam, stresses of $0.5\sigma_Y$ are no exception; the stress diagram may be assumed with a realistic shape. These assumptions permit to determine the load level causing yield in the extreme fibre of the cross section having residual stresses.

Superposing the two deformation systems in Fig. 9, moment M_{el} at first yield can be determined where there is a stress σ_Y at a point of the cross section, defining the elastic load capacity of the structure. Of course, the load can be further increased, thus, angle of rotation θ may increase, with ever larger areas plastified.

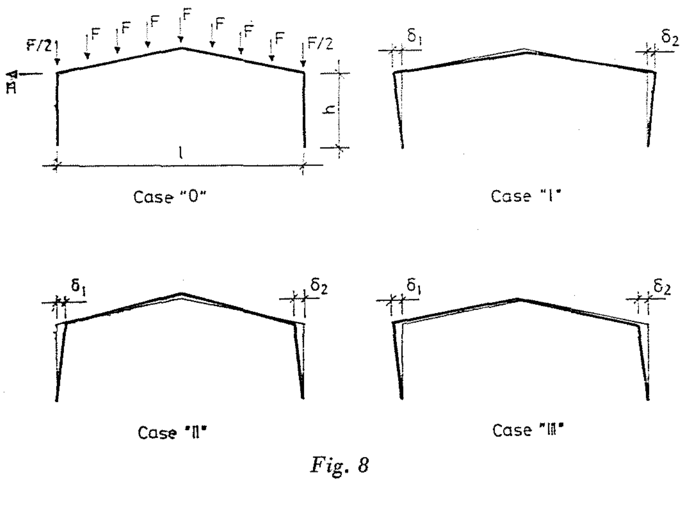

Fig. 8

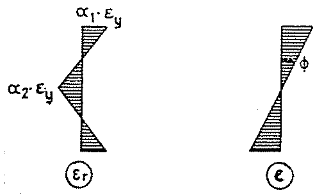

Fig. 9

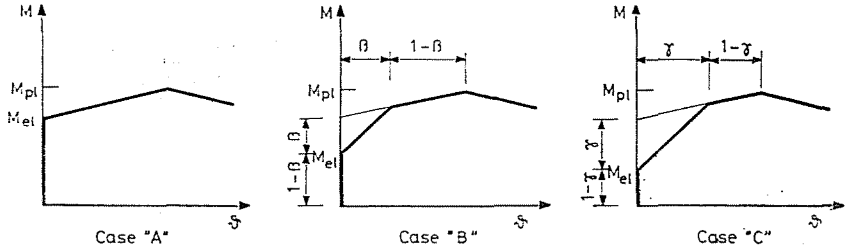

Fig. 10

Obviously, the higher the residual stress σ_r, the lower the moment M_{el}. For spring characteristics in the program (Fig. 10):

Computations involved in case B ($\beta = 0.4$), and in case C ($\gamma = 0.6$), simulating thereby the residual stresses.

To be acquainted with the effects of geometry imperfections and of residual stresses, all combinations of the mentioned cases, hence those of

*AI*1, *BI*1, *CI*1, *AI*2, *CI*2, *AII*1, etc.

have been examined with both $\delta =$ h/100 and $\delta = h/500$, where A, B, C are for residual stress values (Fig. 10);

0, *I*, *II*, *III* indicate assumed forms of geometry imperfections (Fig. 9);

1, 2 refer to geometry imperfections of column tops. Reckoning with

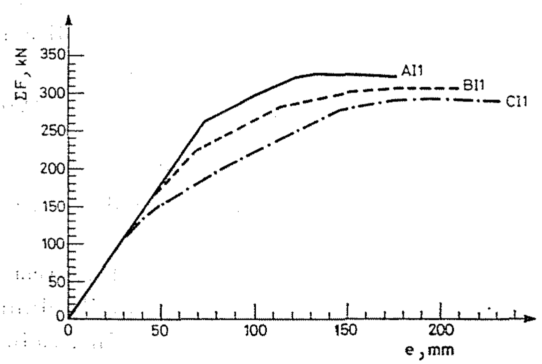

Fig. 11

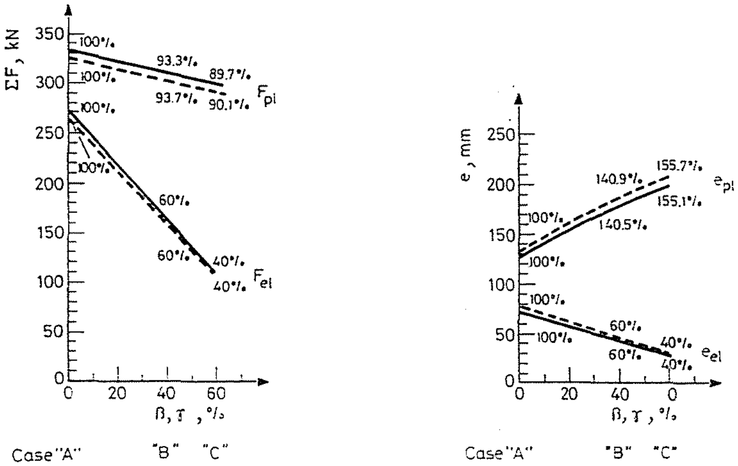

Fig. 12. Effect of residual stress in cases *I*1; Legend: —— for $\delta = h/500$; ----- for $\delta = h/100$

geometry imperfection I for $\delta = h/100$, load-deflection diagrams are seen in Fig. 11.

The effect of residual stresses has been illustrated on an example in Fig. 12.

3.33 *Observations*

Examination of the effect of geometry imperfections pointed out that:

— assuming a given, i.e. constant, system of residual stresses in the member, computations show both elastic and plastic load capacity to vary by at most 2—3% in either of cases *A*, *B* or *C*;

— load capacity varies by less than 0.5% for $h/500$, while for $\delta = H/100$, at the limit of coarse building mistake, load capacity decreases or increases by 2—3%;

— the trend of load capacity variation depends on whether the geometry change agrees with deformations under load or not. Obviously, load capacity decreases in cases I and III, while it increases in case II. Thus, if it is possible to choose among possible building mistakes, type II is advisably chosen.

Residual stresses may have the following effects:

— both elastic and plastic load capacities practically decrease irrespective of kind and degree of geometry imperfection;

— elastic load capacity much decreases for important residual stresses; but with increasing loads the residual stresses gradually diminish, moderating the plastic load capacity loss of the framework: for residual stresses in case *C*, elastic load capacity decreases by 60% but the maximum load capacity is only by 10% lower than that for an idealized beam exempt from residual stresses;

— deformations change is much more intensive in case C, deflection is reduced by 60% — obviously equal to the rate of elastic load capacity loss. On the other hand, deflections at maximum load have grown by 55%, thus, excessive residual stresses significantly "soften" the structure, reducing the framework rigidity in the plastic range.

4. Conclusions

The presented method for the complex analysis of frameworks takes several effects, influences into consideration. Its essential is the selection of the model of the structural member such as to accommodate the considered effects, factors.

Namely:

1. The effect of initial geometry imperfections both on load capacity and on deformability of the tested structures is by orders lower than that of the residual stresses.
2. Residual stresses affect moderately the plastic load capacity of the tested structure, and significantly the elastic load capacity, the first ultimate condition in yield.

Accordingly, analysis and design of welded structures are possible either by

a) analysis of the first condition in yield (elastic design), taking the reducing effect of residual stresses on load capacity and deformability into consideration; or

b) analysis of unconfined ultimate condition in yield (plastic design), with a low effect of residual stresses to reduce the load capacity. The trend of deformability needs a separate analysis to be known.

Appendix

Computation of bar end stresses

Assumptions were the following: normal force arises exclusively from displacement along the bar axis (u^j_ξ and u^k_ξ); displacements normal to it (u^j_η and u^k_η), and bar end rotations (φ^j and φ^k) produce shear force and bending moment. Thereby four computations are needed to see bar end stresses arising from $\varphi^j = 1$, $\varphi^k = 1$, $u^j_\eta = 1$ and $u^k_\eta = 1$.

At last, four stress vectors result:

$$S_{\varphi j} = \begin{bmatrix} \overline{M}_k[\varphi_j] \\ \overline{M}_j[\varphi_j] \\ F[\varphi_j] \end{bmatrix} \qquad S_{\varphi k} = \begin{bmatrix} \overline{M}_k[\varphi_k] \\ \overline{M}_j[\varphi_k] \\ F[\varphi_k] \end{bmatrix}$$

$$S_{uj} = \begin{bmatrix} \overline{M}_k[u_j] \\ \overline{M}_j[u_j] \\ F[u_j] \end{bmatrix} \qquad S_{uk} = \begin{bmatrix} \overline{M}_k[u_k] \\ \overline{M}_j[u_k] \\ F[u_k] \end{bmatrix}$$

where $\overline{M}_j$ and $\overline{M}_k$ are bending moments on bar ends j and k, and F is the shear force.

Physical purport of the stiffness matrix of the bar supplies matrix stiffness values true to sign:

$$\hat{\mathbf{K}} = \left[\begin{array}{c|c} \hat{\mathbf{K}}_{AA} & \hat{\mathbf{K}}_{AB} \\ \hline \hat{\mathbf{K}}_{BA} & \hat{\mathbf{K}}_{BB} \end{array}\right] = \left[\begin{array}{ccc|ccc} \frac{EF}{l} & 0 & 0 & \frac{-EF}{l} & 0 & 0 \\ 0 & F[u_j] & F[\varphi_j] & 0 & -F[u_k] & -F[\varphi_k] \\ 0 & \overline{M}_j[u_j] & \overline{M}_j[\varphi_j] & 0 & \overline{M}_j[u_k] & \overline{M}_j[\varphi_k] \\ \hline \frac{-EF}{l} & 0 & 0 & \frac{EF}{l} & 0 & 0 \\ 0 & -F[u_j] & -F[\varphi_j] & 0 & F[u_k] & F[\varphi_k] \\ 0 & \overline{M}_k[u_j] & \overline{M}_k[\varphi_j] & 0 & \overline{M}_k[u_k] & \overline{M}_k[\varphi_k] \end{array}\right]$$

Stress vectors $S_{\varphi j}$, $S_{\varphi k}$, S_{u_j} and S_{u_k} are obtained by solving four equation systems:

1. $$\begin{bmatrix} 1+\frac{s\cdot k}{c_2} & \frac{s\cdot c\cdot k}{c_1} & -\frac{s\cdot k\cdot l_2}{c_2}-\frac{s\cdot c\cdot k\cdot l_1}{c_1}-l_2 \\ \frac{s\cdot c\cdot k}{c_2} & 1+\frac{s\cdot k}{c_1} & -\frac{s\cdot c\cdot k\cdot l_2}{c_2}-\frac{s\cdot k\cdot l_1}{c_1}-l_1 \\ s(1+c)\frac{k}{l}\cdot\frac{1}{c_2} & s(1+c)\frac{k}{l}\cdot\frac{1}{c_1} & 1-s(1+c)\frac{k}{l}\left(\frac{l_2}{c_2}+\frac{l_1}{c_1}\right) \end{bmatrix} \begin{bmatrix} \overline{M}_k \\ \overline{M}_j \\ F \end{bmatrix}$$

$$= \varphi_j \begin{bmatrix} s\cdot c\cdot k\left(1-\frac{p\cdot l_1}{c_1}\right)+s(1+c)\frac{k}{l}l_1 \\ s\cdot k\left(1-\frac{p\cdot l_1}{c_1}\right)+s(1+c)\frac{k}{l}\cdot l_1-p\cdot l_1 \\ s(1+c)\frac{k}{l}\left(1-\frac{p\cdot l_1}{c_1}\right)+\frac{2s(1+c)}{m}\frac{k}{l^2}l_1 \end{bmatrix}$$

$$\mathbf{F}^*_{\varphi j}\cdot S_{\varphi j} = \varphi_j\cdot b_{\varphi j}, \qquad S_{\varphi j} = \varphi_j\cdot(\mathbf{F}^{*-1}_{\varphi j}\cdot b_{\varphi j})$$

that is, S_{φ_j} is expressible from the equation in dependence φ_j.

$$2.\begin{bmatrix} 1+\dfrac{s\cdot k}{c_2} & \dfrac{s\cdot c\cdot k}{c_1} & \dfrac{s\cdot k\cdot l_2}{c_2}+\dfrac{s\cdot c\cdot k\cdot l_1}{c_1}+l_2 \\ \dfrac{s\cdot c\cdot k}{c_2} & 1+\dfrac{s\cdot k}{c_1} & \dfrac{s\cdot c\cdot k\cdot l_2}{c_2}+\dfrac{s\cdot k\cdot l_1}{c_1}+l_1 \\ -s(1+c)\dfrac{k}{l}\cdot\dfrac{1}{c_2} & -s(1+c)\dfrac{k}{l}\cdot\dfrac{1}{c_1} & 1-s(1+c)\dfrac{k}{l}\cdot\left(\dfrac{l_1}{c_1}+\dfrac{l_2}{c_2}\right) \end{bmatrix}\begin{bmatrix}\overline{M}_k\\ \overline{M}_j\\ F\end{bmatrix}=$$

$$=\varphi_k\begin{bmatrix} s(1+c)\dfrac{k}{l}\cdot l_2+sk\left(1-\dfrac{p\cdot l_2}{c_2}\right)-p\cdot l_2 \\ s\cdot c\cdot k\left(1-\dfrac{p\cdot l_2}{c_2}\right)+s(1+c)\dfrac{k}{l}\cdot l_2 \\ -s(1+c)\dfrac{k}{l}\left(1-\dfrac{p\cdot l_2}{c_2}\right)-\dfrac{2s(1+c)}{m}\dfrac{k}{l^2}l_2 \end{bmatrix}$$

$$\mathbf{F}^*_{\varphi k}\cdot S_{\varphi_k}=\varphi_k\cdot b_{\varphi_k},\quad S_{\varphi_k}=\varphi_k(\mathbf{F}^{*-1}_{\varphi k}\cdot b_{\varphi_k})$$

that is, S_{φ_k} is expressible from the equation in dependence φ_k.

$$3.\begin{bmatrix} 1+\dfrac{s\cdot k}{c_2} & \dfrac{s\cdot c\cdot k}{c_1} & -\dfrac{s\cdot k\cdot l_2}{c_2}-\dfrac{s\cdot c\cdot k\cdot l_1}{c_1}-l_2 \\ \dfrac{s\cdot c\cdot k}{c_2} & 1+\dfrac{s\cdot k}{c_1} & -\dfrac{s\cdot c\cdot k\cdot l_2}{c_2}-\dfrac{s\cdot k\cdot l_1}{c_1}-l_1 \\ s\cdot(1+c)\dfrac{k}{l}\cdot\dfrac{1}{c_2} & s\cdot(1+c)\dfrac{k}{l}\cdot\dfrac{1}{c_1} & 1-s(1+c)\dfrac{k}{l}\left(\dfrac{l_1}{c_1}+\dfrac{l_2}{c_2}\right) \end{bmatrix}\begin{bmatrix}\overline{M}_k\\ \overline{M}_j\\ F\end{bmatrix}=$$

$$=u_j\begin{bmatrix} s\cdot(1+c)\dfrac{k}{l} \\ s\cdot(1+c)\dfrac{k}{l} \\ \dfrac{2s\cdot(1+c)}{m}\cdot\dfrac{k}{l^2} \end{bmatrix}$$

$$\mathbf{F}^*_{uj}\cdot S_{u_j}=u_j\cdot b_{u_j},\quad S_{uj}=u_j\cdot(\mathbf{F}^{*-1}_{uj}\cdot b_{uj})$$

that is, S_{u_j} is expressible from the equation in dependence u_j.

4. $$\begin{bmatrix} 1+\frac{s\cdot k}{c_2} & \frac{s\cdot c\cdot k}{c_1} & \frac{s\cdot k\cdot l_2}{c_2}+\frac{s\cdot c\cdot k\cdot l_1}{c_1}+l_2 \\ \frac{s\cdot c\cdot k}{c_2} & 1+\frac{s\cdot k}{c_1} & \frac{s\cdot c\cdot k\cdot l_2}{c_2}+\frac{s\cdot k\cdot l_1}{c_1}+l_1 \\ -s(1+c)\frac{k}{l}\cdot\frac{1}{c_2} & -s(1+c)\frac{k}{l}\cdot\frac{1}{c_1} & 1-s(1+c)\frac{k}{l}\cdot\left(\frac{l_1}{c_1}+\frac{l_2}{c_2}\right) \end{bmatrix} \begin{bmatrix} \overline{M}_k \\ \overline{M}_j \\ F \end{bmatrix} =$$

$$= u_k \begin{bmatrix} -s(1+c)\frac{k}{l} \\ -s(1+c)\frac{k}{l} \\ \frac{2s(1+c)}{m}\cdot\frac{k}{l^2} \end{bmatrix}$$

$$\mathbf{F}^*_{uk}\cdot S_{u_k} = u_k\cdot b_{u_k}, \quad S_{u_k} = u_k(\mathbf{F}^{*-1}_{uk}\cdot b_{u_k})$$

that is, S_{u_k} is expressible from the equation in dependence u_k.

Legend for the equation systems:

l, l_1 and l_2	bar element length data;
c_1 and c_2	inner screw spring stiffnesses;
$k = EI/l$	bar stiffness value;
p	normal force on the bar;
$s = s(\varrho)$	stiffness function

$$s(\varrho) = \frac{(1-2\alpha\cdot\text{cotan}\,2\alpha)\alpha}{\tan\alpha-\alpha} \quad \text{where } \alpha=\frac{\pi}{2}\sqrt{\varrho} \quad \text{if } \varrho=\frac{N}{N_E}\geq 0$$

$$= \frac{(1-2\gamma\,\text{cth}\,2\gamma)\gamma}{\text{th}\,\gamma-\gamma} \quad \text{where } \gamma=\frac{\pi}{2}\sqrt{-\varrho} \text{ if } \varrho=\frac{N}{N_E}<0 \quad \left(N_E=-\frac{\pi^2 EI}{l^2}\right)$$

$c = c(\varrho)$ transfer function

$$c(\varrho) = \frac{2\alpha-\sin 2\alpha}{\sin 2\alpha-2\alpha\cdot\cos 2\alpha}$$

$$= \frac{2\gamma-\text{sh}\,2\gamma}{\text{sh}\,2\gamma-2\gamma\,\text{ch}\,2\gamma}$$

$$m = m(\varrho) = \frac{2s(1+c)}{2s(1+c)-\pi^2\varrho}$$

References

1. IVÁNYI, M.: Interaction of Stability and Strength Phenomena in the Load Carrying Capacity of Steel Structures. Role of Plate Buckling*. Doctor Techn. Sci. Thesis, Hung. Ac. Sci., Budapest, 1983.
2. SZABÓ, J.—ROLLER, B.: Theory and Analysis of Bar Systems*. Műszaki K., Budapest, 1971.
3. HORNE, M. R.—MERCHANT, W.: The Stability of Frames. Pergamon Press, Oxford, 1965.
4. BAKSAI, R.: Plastic Analysis by Theoretical Methods of the State Change of Steel Frameworks*. Diploma Work, TUB, Budapest, 1983.
5. HALÁSZ, O.—IVÁNYI, M.: Test with Simple Elastic-Plastic Frames. Periodica Polytechnica, C. E. *23*, 157 (1979).

Róbert BAKSAI
Prof. Dr. Miklós IVÁNYI
Dr. Ferenc PAPP
H-1521 Budapest

* In Hungarian.