mondofisair.
$\dot{s}$
MGRNÖKIIASZNALATRA
KÜLÖNÖS TEKINTETTEL.
IIAZAI VISZONYAINKRA
furn
KRUSPER ISTVÁN,



XVI. Könyomafí táblával.


## PEST,

kifan grörar m. k. equert köyvabus maminta.
1860.

## SUPVEYING

## § 344 Stampfer's method for trigonometric levelling and distance measurement

1) Let $A$ and $G$ be two terrain points, the height difference of which has to be determined (Fig. 1). Let us set up an instrument in $A$ for measuring ele-


Fig. 1
vation angles, and a vertical rod in $G$, that has a double target ( $C, D$ ). Let us measure vertical angles $\alpha$ and $\beta$ included
between the two lines-of-sight ( $C$ and $D$ ), and the lower target and the instrument horizon ( $D$ and $E$ ), respectively.
Knowing $\alpha, \beta$ and section $C D$, distance $B E$ and height $D E$ can be calculated. This way, the measurement of height differences is traced back to the indirect measurement of elevation.
2) Denoting $C D=d, D E=H, B E=D$, then from triangle $B C D$ we get:
$C D: B D=\sin \alpha: \sin C$.
But from triangle $C B E$ it follows that

$$
\begin{aligned}
\mathrm{C} & =90^{\circ}+\beta-\alpha, \text { which substituted yields } \\
B D & =\frac{d \cdot \cos (\beta-\alpha)}{\sin \alpha} .
\end{aligned}
$$

Now from triangle $D B E$ :

$$
\begin{aligned}
& D E=H=B D \cdot \sin \beta \\
& B E=D=B D \cdot \cos \beta
\end{aligned}
$$

and substituting the value of $B D$ :

$$
\begin{align*}
& H=\frac{d \cdot \cos (\beta-\alpha) \sin \beta}{\sin \alpha} \\
& D=\frac{d \cdot \cos (\beta-\alpha) \cos \beta}{\sin \alpha} . \tag{1}
\end{align*}
$$

Formula for $H$ gives the vertical distance of the lower target from the horizon of the instrument. Adding the distance of the lower target from the lower end of the rod results in the height-difference $l$ earlier obtained by direct measurement.

Formula for $D$ yields the horizontal distance between the axis $B$ of the instrument and the rod. This method of measuring height differences has the advantage to permit determination of differences forward and backward from the station greater than the rod length; slopes of $5-6^{\circ}$ or, if no excessive precision is needed, even of $30-40^{\circ}$ may be handled at once.

It is to be mentioned that the value of $l$ may also be negative, inaccessible to earlier methods. The work in hilly regions is accelerated by Stampfer's method. Another advantage is that the work is concentrated in the hands of the surveying engineer, the staffman must only hold firm the rod. The safety of sightings and readings increases compared to earlier methods, where the surveyor depended on the skill of the staffman. Besides, chain measurement of horizontal distances is therefore unnecessary, because they can be accurateIy determined from measured data, for the knowledge of the earth curvature and for construction.

## § 345

Angles $\alpha$ and $\beta$ are determined with Stampfer's instrument by means of a slow-motion screw. Setting up this instrument, aiming at the rod with the telescope, this screw is turned until the bubble of the level is adjusted just in the centre, then the screw position $h$ is read off. Without moving the instrument, the screw is turned until the cross-hair fixes the targets $C$ and $D$ and then positions $o$ and $u$ are read off. Let $B K$ be the position of the sighting-axis corresponding to screw position 0 , then vertical angles $K B E, K B C$ and $K B D$ will be some function of screw positions $h, o, u$. Without a geometrical examination of the form of this function (published in my paper in "Poggendorf Annalen", Vol. CXXX), series expansion yields the general form after Stampfer

$$
\begin{aligned}
& K B E=a h+b h^{2}+c h^{3}+\ldots \\
& K B C=a o+b o^{2}+c o^{3}+\ldots \\
& K B D=\mathrm{au}+b u^{2}+c u^{3}+\ldots
\end{aligned}
$$

where $a, b, c=$ constants.
Subtracting them from each other, we get:

$$
\begin{aligned}
& K B C-K B D=\alpha=a(o-u)+b\left(o^{2}-u^{2}\right)+c\left(o^{3}-u^{3}\right)+\ldots \\
& K B E-K B D=\beta=a(h-u)+b\left(h^{2}-u^{2}\right)+c\left(h^{3}-u^{3}\right)+\ldots
\end{aligned}
$$

According to measurements by Stampfer, for identical instruments made in the workshop of the Technical University in Vienna, constants a little differ, although:
$a=630^{\prime \prime} \ldots 750^{\prime \prime}$ for different types of instruments;
$b$ has always a negative sign and is in the range $0.05^{\prime \prime} \ldots 0.1^{\prime \prime}$, a different value for each instrument;
$c=$ very little value, to be neglected for angles common in practice ( $8-10^{\circ}$ ).
In this way the formula of the vertical angle for Stampfer's instruments becomes, with a sufficient accuracy:

$$
\begin{align*}
& \alpha=a(o-u)-b\left(o^{2}-u^{2}\right) \\
& \beta=a(h-u)-b\left(h^{2}-u^{2}\right) \tag{2}
\end{align*}
$$

where the first term is the raw value of the angle, the second term corrects for the screw length, it being not an are but a chord.
This second term accounts for the effect of a thread imperfection (very small indeed for instruments made in the Viennese workshop).
3) To determine constants $a$ and $b$, two known angles have to be measured with the instrument which are fixed by two rods on the same distance, or by the same rod on two different distances. These constants are very accurately determined for the Viennese instruments and are recorded on the top of their cases. If the instruments are handled carefully, these values remain unchanged for years, but they are to be determined from time to time.

## § 346 Stampfer's Tables

Because angles $\alpha$ and $\beta$ are less than 8 to $10^{\circ}$, to ease calculations, Stampfer has expanded formulae for $H$ and $\bar{D}$ in series, as follows:

$$
\begin{align*}
& H=d\left(\frac{\beta}{\alpha}-\frac{2}{3} \frac{\beta^{3}}{\alpha}-\frac{1}{3} \beta \alpha+\beta^{2}\right) \\
& D=d\left(\frac{1}{\alpha}-\frac{\beta^{2}}{\alpha}-\frac{1}{3} \alpha+\beta\right) . \tag{3}
\end{align*}
$$

Terms in $H$ greater than order three, and in $D$ greater than order two have been neglected. Replacing angles $\alpha$ and $\beta$ by (2):

$$
\begin{aligned}
H & =d\left(\frac{h-u}{o-u}-\frac{2}{3} a^{2} \frac{(h-u)^{3}}{o-u}+\frac{b}{a}(h-u)-\frac{b}{a} \frac{(h-u)^{2}}{o-u}+a^{2}(h-u)^{2}\right) \\
D & =d\left(\frac{1}{a(o-u)}+\frac{b}{a^{2}} \frac{o+u}{o-u}-\frac{a(h-u)^{2}}{o-u}+a(h-u)\right)
\end{aligned}
$$

where $a$ and $b$ are in dimensions of arc.

If the instrument constants are given in arc-seconds, they have to be multiplied by $\sin 1^{\prime \prime}$ and put in these formulae. Stampfer made tables to simply read the terms of these formulae, where

$$
a=636.6^{\prime \prime} \quad \text { and } \quad b=0.07^{\prime \prime}
$$

These tables are very useful for those instruments, which have constants not very different from the tabulated values. For other instruments, the values are to be multiplied by other constant coefficients. The tables are found in "Anleitung zum Nivellieren" by Stampfer.

## § 347 My tables

I have calculated other tables for any Stampfer-type instrument, published 1859 in the Bulletin of the Hungarian Academy of Sciences. The development is set out from Eqs (3) above, but leads to following results:

$$
\begin{gathered}
\log H=\log d+\log (h-u)-\log (o-u)-M \frac{b}{a}(h-o)- \\
-\frac{M \sin 1^{\prime \prime 2} a^{2}}{3}(h-o)(h-o+h-u)\left(1-2 \frac{b}{a}(h+o)\right) \\
\log D=\log d-\log \left(a \sin 1^{\prime \prime}\right)-\log (o-u)+ \\
+M \frac{b}{a}(o+u)-M \sin 1^{\prime \prime 2} a^{2}(h-o)(h-u)\left(1-2 \frac{b}{a}(h+o)\right)- \\
-\frac{M \sin 1^{\prime \prime 2} a^{2}}{3}(o-u)^{2}
\end{gathered}
$$

where
$M$ is the module of $B r i g g ' s$ logarithm.

Table I

| $h=0$ <br> $o+u$ | - | 0.00010 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :--- |

The terms of these formulae have been compiled in two tables* I and II.

1) Table I contains fourth terms of $\log H$ and $\log D$, namely $M \frac{b}{a}(h-o)$ and $M \frac{b}{a}(o+u)$. The sign of the former is always opposite to that of $(h-o)$, the second is always $+; \frac{b}{a}$ being different for every instrument, it has to be calculated forever.

Let e.g. for a certain instrument $a=641^{\prime \prime}, \quad b=0.085^{\prime \prime}$, hence $b / a=0.000133$. Rows marked by 0.00013 , and by 0.00003 are applied, the first one without any change, the second divided by ten (because the values are valid for 10 times greater arguments than the ones in question). The obtained values have to be added.
2) In Table II following terms have been compiled:

Table II

| Argam. |  |  |
| :---: | :---: | :---: |
| 8.500 | 0.00108 | $\begin{equation*} \frac{M \sin 1^{\prime \prime 2}}{3} a^{2}(h-o)(h-o+h-u)\left(1-2 \frac{b}{a}(h+o)\right) \tag{4} \end{equation*}$ |
| 505 | 109 |  |
| 510 | 110 |  |
| 515 | 111 |  |
| 520 | 113 |  |
| 8.525 | 0.00114 |  |
| 530 | 115 | $\begin{equation*} \frac{M \sin 1^{1 / 2}}{3} 3 a^{2}(h-0)(h-u)\left(1-2 \frac{b}{a}(h+0)\right) \tag{5} \end{equation*}$ |
| 535 | 117 |  |
| 540 | 118 |  |
| 545 | 119 |  |
| 8.550 | 0.00121 | $\begin{equation*} \frac{M \sin 1^{\prime / 2}}{3} a^{2}(o-u)^{=} \tag{6} \end{equation*}$ |
| 555 | 122 |  |
| 560 | 124 |  |
| 565 570 | 125 |  |
| 570 | 126 |  |

which are very similar in form and are therefore reduced in one table. As concerns signs of term (4) it is negative if
$h>o$, or $h<u$, or $o>h>u$ and $(h-o)+(h-u)<0$,
namely if the instrument horizon is above or below the rod or intersects the lower part of the rod; but
it is positive if $o>h>u$ and $(h-o)+(h-u)>0$, namely if the instrument horizon intersects the upper part of the rod. Term (5) is negative if $h>o$, or $h<u$, namely if the instrument horizon is above or below the rod;

[^0]it is positive if $o>h>u$, namely if the instrument horizon intersects the rod.
Term (6) is always negative.
Those cases are interesting in practice, where the instrument horizon is above or below the rod; corrections in other cases may be neglected.

The argument of this table is the following (in the first column from the left):
$M$ for (4) $m+\log (h-o)+\log (h-u+h-o)-2 M \frac{b}{a}(h+o)$,
for $(5) n+\log (h-o)+\log (h-u)-2 M \frac{b}{a}(h+o)$,
for (6) $m+2 \log (o-u)$,
where $m=2 \log a, n=\log 3+2 \log a$,
(constant for every instrument and calculated for once).
The arguments are between 8.5 and 9.5 , higher values don't occur. If nevertheless the argument would be between 6.5 and 7.5 or between 7.5 and 8.5, these must be enlarged by 2 or 1 to exceed 8.5 and the tabulated value must be divided by 100 or $10, A$ being of logarithmic nature. Quantities $h-o$, $h-u,(h-0)+(h-u)$ are always positive, even if any of them would be still negative. Values of $m$ and $n$ and the logarithms of the arguments are calculated up to 4 decimals. If $h-o, h-u,(h-0)+(h-u)$ are less than 20, 3 decimals are sufficient. Computing $m+2 \log (o-u), 2$ decimals are enough. The values of $M \frac{b}{a}(h-o)$ are taken from Table $I$, always of negative sign.
3) Examples

For $a=737.4^{\prime \prime}, b=0.0436^{\prime \prime}, d=1^{\circ *}$, the corresponding constants are:
$\log d=0$
$\log \left(a \sin 1^{\prime \prime}\right)=0.55327-3$
$m=5.7354$
$n=6.2125$
$b / a=0.000059$.
The measurements gave following figures:

$$
\begin{aligned}
& h=39.895 \quad \text { hence } \quad h-u=39.020 \\
& o=2.054 \\
& u=0.875 \\
& o-u=1.179 \\
& h-o=37.84 \\
& h-o+h-u=76.86 \\
& h+o=42 \\
& o+u=2.9
\end{aligned}
$$

furthermore:


The corresponding $H$ value is $32.617^{\circ} \%$ identical with the result of the precise formula.

Calculation of the distance:

| $-\log \left(a \sin 1^{\prime \prime}\right)$ | $=-0.55327+3$ |  | $n=6.2125$ |
| :--- | :--- | :--- | :--- |
| $-\log 1.179$ | $=$ | -0.07151 | +8 |
| Table I | $=$ | -815 | $\log 37.84$ |
| Table II $(9.3796)$ | $=$ | 1.5780 |  |
| $\log D$ | $=$ | 2.36715 |  |

The corresponding value $\quad D=232.89^{\circ} *$
from the precise formula $\quad D=232.87^{\circ}$.
4) Difference between real and given horizontal distances in Table I is:
$f=0.0000001295 D^{2}$ cords of Vienna.
Replacing $D$ by its approximate value from screw turns:

$$
D=\frac{d}{a \sin \mathrm{I}^{\prime \prime}(o-u)}
$$

then

$$
f=\frac{5510 d^{2}}{a^{2}(o-u)^{2}}
$$

These values have been calculated in Table III, their arguments being a and ( $o-u$ ).
5) If the elevation angle exceeds $4^{\circ}$, it can't be measured in the horizontal position of the limb, because only the half screw length up and down is at our disposal. According to Stampfer, then the limb has to be set into a position about parallel to the sloping terrain, with its maximal slope in the vertical plane of the line of collimation. In such a position the whole length of the screw is at our disposal; but the screw setting corresponding to the horizontal sight

[^1]Table III

| --u | ${ }^{\text {a }}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 580 | 600 | 630 | 640 | 660 | 680 | 700 | 720 | 740 | 760 |
| 0.60 | 0.046 | 43 | 40 | 37 | 35 | 33 | 31 | 30 | 28 | 27 |
| 61 | 44 | 41 | 39 | 36 | 34 | 32 | 30 | 29 | 27 | 26 |
| 62 | 43 | 40 | 37 | 35 | 33 | 31 | 29 | 28 | 26 | 25 |
| 63 | 41 | 39 | 36 | 34 | 32 | 30 | 28 | 27 | 25 | 24 |
| 64 | 40 | 37 | 35 | 33 | 31 | 29 | 27 | 26 | 25 | 23 |
| 0.65 | 0.039 | 36 | 34 | 32 | 30 | 28 | 27 | 25 | 24 | 23 |
| 66 | 38 | 35 | 33 | 31 | 29 | 27 | 26 | 24 | 23 | 22 |
| 67 | 37 | 34 | 32 | 30 | 28 | 27 | 25 | 24 | 22 | 21 |
| 68 | 35 | 33 | 31 | 29 | 27 | 26 | 24 | 23 | 22 | 21 |
| 69 | 34 | 32 | 30 | 28 | 27 | 25 | 24. | 22 | 21 | 20 |

doesn't fall in its middle part any more but lower or higher, and the horizontal line of the telescope will thereby sink or rise. Be $g$ this change of the instrument height, expressed with sufficient accuracy by

$$
g=r \gamma
$$

where $r=$ distance of axes $B$ and $A$

$$
\gamma=\text { slope of the } \operatorname{limb}\left(\max .4^{\circ}\right)
$$

Be $N$ the screw-setting corresponding to the horizontal, in the horizontal position of the limb, and $h$ in its sloping position, then, in conformity with (2):

$$
\begin{aligned}
& \gamma=\mathrm{a} \sin \mathrm{l}^{\prime \prime}(h-N) \\
& g=\mathrm{ra} \sin \mathrm{l}^{\prime \prime}(h-N)
\end{aligned}
$$

Let the length of revolution $k$ be $1 / 100$ cord of Vienna, easy to measure by a caliper, then it is approximately

$$
2 r k a \sin 1^{\prime \prime}=1 / 100 \mathrm{cord}
$$

because the distance of the screw from axis $B$ is nearly $2 r$, thus

$$
r a \sin 1^{\prime \prime}=\frac{1}{200 k}
$$

then

$$
g=\frac{h-N}{200 k} \text { cords of Vienna, }
$$

to be added to the obtained $H$ value.
Calculated values of this expression have been compiled in Table IV affected with a sign + or -, according to $h \gtrless N$. The arguments of this table are $k$ and $h-N$.

Table IV

|  | $k$ |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| $h-M$ | 20 | 25 | 30 | 35 | 40 |  |
|  |  |  |  |  |  |  |
|  | 0.001 | 1 | 1 | 1 | 1 |  |
| 10 | 2 | 2 | 2 | 1 | 1 |  |
| 15 | 4 | 3 | 3 | 2 | 2 |  |
| 20 | 5 | 4 | 4 | 3 | 2 |  |
| 25 | 6 | 5 | 3 | 4 | 3 |  |

Be in the example above $N=22.6$ and $k=30$, then
from Table III we find for $(740.115)=-0.008^{\circ}$
from Table IV we find for (30.15) $=+0.003$
and so the final value of $H=32.617-0.005=32.612$ cords of Vienna.

## § 348 Criticism to Stampfer's instrument

Stampfer rendered a valuable service to engineering practice by his method, and especially, by his instrument. Although the usefulness of the screws as means to determine both angle differences and absolute elevation angles was already known, because former was recommended by T. Mayer (the elder), the latter by Hogrewe, according to Stampfer: but Stampfer was the first to achieve a result, and the confidence in the method was based on his instruments. For all the recognizance rightously due to his instruments, nevertheless some deficiencies, beginning to be burdensome in recent railway constructions, have to be mentioned:
a) The instrument has no coarse elevational motion, therefore it is unfit to set out straight lines in mountainous regions;
b) for elevation angles over $4^{\circ}$, the half screw-length isn't sufficient for goniometry, and because of the lack of coarse motion the limb has to be slanted; handling of the instrument is therefore tedious, tiresome and cumbersome. Hence it isn't well supplied just for the case where its use begins to prevail over other heighting apparatuses;
c) elevation angles of $8-10^{\circ}$ accessible to the instrument are insufficient, in practice angles of $15-20^{\circ}$ are required.

Deficiency a) is eliminated in recent instruments by Starke; where the telescope and the alidade may be up and down tilted about a horizontal pin and locked in any position; the slow-motion screw is on the other end of this pin. To apply this modification on existing instruments of Stampfer would be indeed very costly.

Concerning point b), I have suggested a modification in the construction, published in Vol. 1 of the Bulletin of the Hungarian Society of Engineers. This modification isn't expensive and could be achieved on every existing instruments for some Forints.

Deficiency in point c) cannot be helped, because it is rooted in the construction principle of the instrument. Stampfer's screw measures the chord of the elevation angle or better its changes; up to 8-10 this chord differs from the arc by an infinitesimal amount, so that the elevation angle is rather accurately expressed by the two first terms of the series in Eq. (2). But for an angle of 12 to $20^{\circ}$, the subsequent higher-order neglected terms increase not to be neglected any more. Therefore, formulae and Tables for $H$ and $D$ get too extensive to be of practical use. To get rid of these deficiencies, I shall propose a different principle of goniometry for the measurement of angles; where the screw measures the arc rather than the chord, without impairing the accuracy of angle-measurement and the instrument stability compared to Stampfer's instrument.

## § 349 My correction on Stampfer's instrument

My correction concerning item b) is illustrated in Fig. 2. This construction differs from that by Stampfer only in that while there $K$ is the alidade lying directly on plate $J$, moving only in the plane of this plate, my modification consists in a third component $C$ between $K$ and $J$, acting as an alidade. On its two sides, there are bars $D$ ending in bearings, supporting pins $E$, extending bilaterally from $K$. A small bracket $a$ extends from $C$, supporting an excenter $F$ turning around an axis $b$ fastened to $K$. Spring $G$ presses $F$ to bracket $a$. On the flange of $F$, there are three planes ( $m, n, p$ ) filed at different distances from the axis. Upon turning axis $b$ by means of its crank these planes get one after another in contact with bracket $a$. In Fig. 2 the excenter $F$ rests


Fig. 2
on the middle plane $n$, in this position $K$ is about parallel with plate $J$. Now let us determine the screw position $M$, with the level parallel to the limb plane. Turning axis $b$ a quarter cycle forward or backward, planes $m$ or $p$ contact bracket $a$, and $K$ is tilted up or down by about $4^{\circ}$. The telescope has in this way three normal positions, that are achieved in an instant by turning axis $b$ with a quarter cycle. With Stampfer's instrument (according to his "Anleitung zum Nivellieren", §54), tilting of the limb is accomplished by setting the screw in position $M$; then the whole instrument is turned on its tripod-pivot; so that an imaginary diameter across a foot-screw aims at the rod; the level is set across this line, the diameter of the limb parallel to this is precisely levelled, then the limb may be set parallel to the terrain by means of one foot-screw.

Another advantage of the suggested modification is to lend a coarse vertical motion to the telescope, about eliminating deficiency a).
$\S 350$

1) Using instruments for angular measurement known so far fitted for elevation measurement, angular measurement has been done either by a screw (according to Stampfer's method determining the chord, else the tangent), or by means of a scaled arc. The first method (in conformity with statements in $\S 348$ ) is precise enough only up to 8 to $10^{\circ}$; the latter method is accurate within the reading accuracy of the vernier. This deficiency cannot be helped even as with the instrument of Breithatupt - by graduating the micrometer screw. Namely in measuring an angle by reading off this graduation the integer degrees by adjusting the pointer on the degree and the screw for the fractional degrees, the pointer adjustment error would be contained in the angle value, irrespective of the increased accuracy of screw measurement. The screw is absolutely preferred to the graduated circle but it is desirable that angles up to 20 to $25^{\circ}$ could be measured without complicated formulae, difficult handling, or excessive screw wear, such as for Stampfer's instrument.
2) With Stampfer's instrument, errors $\Delta \alpha$ and $\Delta \beta$ in angles $\alpha$ and $\beta$ produce different errors in elevation values $H$, modifying them by $\Delta H_{\alpha}$ and $\Delta H_{\beta}$, respectively, expressed according to higher mathematics, as

$$
\frac{\Delta H_{\alpha}}{H}=-\frac{\cos \beta}{\cos (\alpha-\beta)} \frac{\Delta \alpha}{\sin \alpha}, \quad \frac{\Delta H_{\beta}}{H}=\frac{\cos (\alpha-2 \beta)}{\cos (\alpha-\beta)} \frac{\Delta \beta}{\sin \beta}
$$

Since $\alpha$ and $\beta$ are small quantities, approximately:

$$
\frac{\Delta H_{\alpha}}{H}=-\frac{\Delta \alpha}{\alpha}, \quad \frac{\Delta H_{\beta}}{H}=\frac{\Delta \beta}{\beta}
$$

Hence:

$$
\Delta H_{z}: \Delta H_{\beta}=-\frac{\Delta \alpha}{\alpha}: \frac{\Delta \beta}{\beta}
$$

and since approximately $H=d \frac{\beta}{x}$, the last proportion becomes:

$$
\Delta H_{\alpha}: \Delta H_{\beta}=H \Delta \alpha: d \Delta \beta .
$$

This expression clearly indicates that measurement errors of angles $\alpha$ and $\beta$ affect the height accuracy differently, in different circumstances. Equal effect in both angles, i.e. $\Delta H_{\alpha}=\Delta H_{\beta}$, would result from the following relationship between these errors:

$$
H \Delta \alpha=d \Delta \beta \text { or } \Delta \alpha: \Delta \beta=d: H .
$$

The greater the height, the less error is allowed in $\alpha$.
This condition is met by none of the instruments of trigonometric levelling, because

1) for the measurement of both $\alpha$ and $\beta$, there is only one and the same measuring unit;
2) in every instrument known so far, angular measurement consists in observing the directions consecutively, - meanwhile the tripod must be considered as absolutely stable and motionless. Every change in the tripod position has the same effect on angles $\alpha$ and $\beta$, falsifying readings of both, an influence manifest by differences between consecutive readings.

In measuring angle $\alpha$, one action is preferred to two successive ones, it may become a factor in increasing the accuracy.
3) Developing a theoretically perfect trigonometric levelling instrument, one must keep following principles as standards:
a) The telescope must have both coarse and fine motion to enable the instrument to linear setting-out.
b) The elevation measurement has to be done by means of a screw, and range to $20-25^{\circ}$; this screw should measure the arc rather than some trigonometric function, and without excessive wear (like for the screw of Stampfer). An endless screw is useless, because the contact with only l-2 turns there concentrates.
c) Measurement of angles $\alpha$ and $\beta$ needs separate means, with relative accuracies of at least 3 to 1 .
d) Angle $x$ (affecting elevation by more than $\beta$ does) should be measured in a single action; except in very short ranges, where angle $\alpha$ is very great, unimpaired even by an error greater than usual.
e) The micrometer screw (as the most precious part of the instrument) should be used only for angular measurements rather than for other, secondary: purposes.

## § 351 The new trigonometric levelling instrument

1) I have constructed a new instrument relying on these principles, shown in Fig. 2. It resembles mostly Ertl's instrument, on a Reichenbachtype tripod with a limb $l$, alidade $D$, two motions (coarse, fine) and two rerniers reading minutes of arc. Two opposite columns $E$ emerge from $D$, topped by pivot $B$, down, in turn, by a trapezoidal cover contacting on three sides. On both ends of $B$, within the columns, there are two concentric pins turned bearing perforated sides $F$, their ends fastened together by U-shaped toggles $k$ forming thereby a basket, easily revolving without lateral motion on pivot $B$, of which it is detached. Pivot $B$ has also a U-bend in its middle to let the telescopeinto the vale of $k$ and to let its axis be in level with $B$. The telescope is removable from its support to be reversed: the level is set on top of the telescope and clamped down by the cover, not letting the telescope or the level falling from the instrument. A strong arm $b$ with a small screw $a$ is protruding from one side of basket $F$. The foot of column $E$ is pierced at $c$ with a hole. in which a mildly tapering pin - shown here only in cross section - is fitted. Pushing forward this pin in the hole, the end of screw a abuts on its protruding end, creating thereby a stable position between the basket holding the telescope or the level resting on it, and the alidade, a means to parallel the level and the limb. Pulling out this pin from the hole, this position is secured by a weak spring; arm $G$ can pass before pin $a$ without inhibiting the coarse motion of the telescope. There is a strong, concentric quadrant $M$ fastened on pivot $B$, inside arm $G$; this quadrant touches the face of arm $G$, releasably coupled to it 1 , means of clamping screw $b$ shown here only with its end. Quadrant $M$ is strongly connected with a horizontal micrometer screw $P$ on the alidade through a carriage $Q$, freely sliding forward and backward between fence-plates. This connection is due to two strong, flat spring-plates fixed on both sides of the quadrant at $e$ and $e^{\prime}$, crossing each other on its cylindrical surface, having their ends fixed on the other ends $f$ and $f$ ' of the carriage. This carriage has a longitudinal boring, the hole ends in a nut accommodating a spindle; a weak coiled spring between the nut and the screw-head produces a weak tension. Upon turning micrometer $P$, the carriage slides forward or backward causing the spring plates on the cylindrical surface of $M$ to coil up or down. Noting that plate ef in the middle of the cylinder is a bit wider than those $e^{\prime} f^{\prime}$ on its sides to prevent lateral pressures, and that plate ef can be tightened at will by means of a screw at $f$ we have a trigonometric levelling instrument, safe in all consider-
ations, which can measure arcs by a screw. This screw has a thread of 120 revolutions, 80 revolutions for $1^{\prime \prime *}$; the angular value of one revolution is $10-12^{\prime}$. The screw-head has 100 graduations. The screw is threaded very fine; reading of integer revolutions is eased by a counter inside the cross-clamp $R$ of the carriage side-fences, with two superposed openings; the upper one indicates the decades, the lower one the units, while fractions the index points to read off the screw-head $g$. The spindle is topped by an eccentric crank $V$ to accelerate turning, a lengthy operation. Apparently, upon loosening screw $b$, the telescope is able to wide motion up and down; by tightening it, the telescope may be tilted by the micrometer screw in fine motion, to measure elevation angles upwards or downwards, depending on the position of the quadrant.

Finally the telescope is constructed quite according to my distancemeasuring instrument, with a focus of $14-15^{\% *}$; the objective lens is cut into two halves, one is fixed in the telescope, the other is built in a movable tablet, raised or lowered by means of a micrometer screw $h$, serving for the measurement of angle $x$; one revolution of it equals about $4^{\prime}$. Complete revolutions are read off a little scale on the telescope side, fractions the screw head with 100 graduations.

To use this instrument for ordinary height measurement, the movable half objective has to be inactivated not to make disturbing double-images. This is accomplished by a semicircular fender inside the telescope, with a protruding handle ending in a knob $k$; by its help the fender is set across the telescope, fending off the rays incident on the movable half-lens not to form an image, thereby only images produced by the other half lens are seen.
2) I am going to describe this instrument and its theory in a separate booklet. What remains to me is to stress the possibility of applying Stampfer's formulae for height and distance measurements, provided the angular measurement is performed with micrometer screw $P$; especially in cases where the rod is close to the instrument, hence when angle $\boldsymbol{z}$ is big enough compared to the rod length. In such a case, $\alpha$ could only be determined by the telescope screw (maximum $2.5^{\circ}$ ), with targets very close to each other on the rod, i.e. taking $d$ smaller, impairing, in turn, the accuracy of the angular measurement.

[^2]
## Summary

This paper is a selection from the manual "Surveying" by István Kruspér, which was used also as a text-book.

Professor Stampfer at the Technical University in Vienna has constructed an instrument for the substitution of trigonometric levelling by stadia-measurement, suiting determination of both the height-difference and the distance of points. To use this apparatus, some tables were also necessary. Paragraphs $344-346$ describe this instrument and its applications. Knuspér has constructed more convenient tables for this instrument, scope of $\S 347$, still too extensive to be reproduced else but a few details. He gave the critics of Stampfer's device in $\S 348$ and his suggested improvement in $\S 349-350$. Subsequently he constructed a new instrument for height and distance determination, described in § 351 .


[^0]:    * Here only fragments of these tables will be reproduced.

[^1]:    * cords of Vienna

[^2]:    * Viennese inches.

