ADJUSTMENT OF HORIZONTAL CONTROL POINT NETWORKS BY PARTITIONING

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Adjustment of horizontal control point networks is expediently made by the indirect measurement method. This choice is supported by two factors. First, because network adjustment by indirect measurements is rather suitable for computer processing, second, confidence indices necessary for precision testing the network are easier obtained by this method than by direct adjustment.

In the adjustment of greater — national or continental — networks, the solution of the normal equation systems presents the greatest problem. This can be helped by partitioning (cutting up) the coefficient matrices of the normal equation [1]. This method is also serviceable in adjusting smaller networks (for industrial plants or independent networks of motion tests). Its advantages are, partly, the accessibility of normal equations to smaller computers, and partly, that after repeated measurements of the nets or their fixing by further measurements, the computation needs not be entirely repeated.

Since the current use of telemetering instruments working on the physical principle, in general, networks are measured by combined (direction and distance) measurement.

In adjustment, first of all, network point coordinates are determined but in case of applying telemetering instruments, also determination of the instrument constant may be involved [6].

This study will be concerned with the adjustment of combined networks by partitioning. A possible method of partitioning will be discussed in detail, others may be deduced to the sense.

Adjustment of combined networks by partitioning

In adjusting indirect measurements, the first step is to determine the preliminary values of unknowns, such as the horizontal coordinates of the points and the orientation angles on all stands applied to direction measurement. The preliminary values of the coordinates can be calculated from the still inadjusted measurement results by various interpolation methods. As preliminary value of the orientation angle, in general, the average orientation angle obtained from the preliminary orientation on the stand is chosen.

Measurement results and unknowns are related by intermediate and correction equations [4].

Correction equations set up for every measured direction and measured length can be written together in matrix form as [2]:

$$\underset{(n,1)}{v} = \underset{(n,R)}{\mathbf{A}} \; \underset{(R,1)}{x} + \underset{(n,1)}{l} \tag{1}$$

where v vector of corrections;
x vector of the change of unknowns;
l vector of pure terms;
$\mathbf{A}$ coefficient matrix of the changes.

The number n of the rows of matrix $\mathbf{A}$ equals the number of all the measurements, i.e. the sum of direction and length measurements. The number R of columns of matrix $\mathbf{A}$ equals the number of unknowns, i.e. the sum of the numbers of orientation angles and coordinates.

The next step is to set up the normal equation as usual:

$$\underset{(R,R)}{(\mathbf{A}^* \; \mathbf{P} \; \mathbf{A})} \; \underset{(R,1)}{x} + \underset{(R,1)}{\mathbf{A}^* \; \mathbf{P} \; l} = \underset{(R,1)}{0} \tag{2}$$

where $\mathbf{P}$ is the weight matrix of measurement results.

Introducing symbols

$$\underset{(R,R)}{\mathbf{N}} = \underset{(R,R)}{\mathbf{A}^* \; \mathbf{P} \; \mathbf{A}} \tag{3}$$

and

$$\underset{(R,1)}{n} = \underset{(R,1)}{\mathbf{A}^* \; \mathbf{P} \; l} \tag{4}$$

the normal equation takes the form:

$$\underset{(R,R)}{\mathbf{N}} \; \underset{(R,1)}{x} + \underset{(R,1)}{n} = \underset{(R,1)}{0}\,. \tag{5}$$

If there is no known point in the network, matrix $\mathbf{N}$ is in principle a singular matrix, thus

$$\det \mathbf{N} = 0\,.$$

Such a network can be arbitrarily displaced and rotated.

The rate of the singularity is seen from the defect, difference between the dimension and the rank of the matrix [7]:

$$d(\mathbf{N}) = R(\mathbf{N}) - \varrho(\mathbf{N}) \tag{7}$$

where $d(\mathbf{N})$ is the defect of the matrix;
$R(\mathbf{N})$ — size of the matrix;
$\varrho(\mathbf{N})$ — rank of the matrix.

The defect depends on the character of the network and agrees with its degrees of freedom. Combined networks have a defect of three.

There are two methods to calculate the unknowns, either as a free network or with a fixed beginning point.

In calculating as a free network, each unknown gets a change. In this case the problem can be solved by applying some so-called risk function [3], [8], [9].

To present adjustment of combined networks by partitioning, the computation with a fixed beginning point will be discussed where essentials of the procedure can be better followed.

In adjustment with a fixed beginning point as many unknowns as the defect have to be fixed, including the situation of the so-called beginning point and some direction. These three unknowns changing by zero, their column has to be cancelled from coefficient matrix $\mathbf{A}$ and so have these changes from vector $\mathbf{X}$, leading to correction equations:

$$\underset{(n,1)}{v} = \underset{(n,r)}{\mathbf{A}} \; \underset{(r,1)}{\mathbf{X}} + \underset{(n,1)}{l} \tag{8}$$

where $r = R - d$.

As there are as many rows in coefficient matrix $\mathbf{A}$ as the sum of direction and length measurements, and as many columns as the sum of the numbers of orientation angles and of not fixed coordinates, it is advisable to partition accordingly. Thus matrix $\mathbf{A}$ is quartered to:

$$\underset{(n,r)}{\mathbf{A}} = \begin{bmatrix} \underset{(n_L, r_1)}{\mathbf{A}_{\mathrm{I}}} & \underset{(n_L, r_2)}{\mathbf{A}_{\mathrm{II}}} \\ \\ \underset{(n_T, r_1)}{\mathbf{A}_{\mathrm{III}}} & \underset{(n_T, r_2)}{\mathbf{A}_{\mathrm{IV}}} \end{bmatrix} \tag{9}$$

where n_L — number of direction measurements;
n_T — number of length measurements;
r_1 — number of orientation angles;
r_2 — number of coordinates.

Because no orientation angles belong to length measurements, $\mathbf{A}_{\mathrm{III}} = \mathbf{0}$.

Cutting up matrix **A** determines also cutting up of vectors in the correction equation, to be written as:

$$\begin{bmatrix} \underset{(n_L,\,1)}{v_{\mathrm{I}}} \\ \\ \underset{(n_T,\,1)}{v_{\mathrm{II}}} \end{bmatrix} = \begin{bmatrix} \underset{(n_L,\,r_1)}{\mathbf{A}_{\mathrm{I}}} & \underset{(n_L,\,r_2)}{\mathbf{A}_{\mathrm{II}}} \\ \\ \underset{(n_T,\,r_1)}{\mathbf{0}} & \underset{(n_T,\,r_2)}{\mathbf{A}_{\mathrm{IV}}} \end{bmatrix} \begin{bmatrix} \underset{(r_1,\,1)}{x_{\mathrm{I}}} \\ \\ \underset{(r_2,\,1)}{x_{\mathrm{II}}} \end{bmatrix} + \begin{bmatrix} \underset{(n_L,\,1)}{l_{\mathrm{I}}} \\ \\ \underset{(n_T,\,1)}{l_{\mathrm{II}}} \end{bmatrix}. \tag{10}$$

The next step — necessary for setting up the normal equations — is to establish weight matrix **P**, characterizing the confidence of the measurements. Establishment of the weight relation consists of two parts. Relation between measurements partly of the same type, partly of different types — direction and length measurements — have to be determined.

Considering the measurements to be independent, the weight matrix will be a diagonal matrix.

This supposition is permissible for practical calculations.

In conformity with partitioning the correction equations, weight matrix **P** is divided into two parts:

$$\underset{(n,\,n)}{\mathbf{P}} = \langle \underset{(n_L,\,n_L)}{\mathbf{P}_{\mathrm{I}}}, \underset{(n_T,\,n_T)}{\mathbf{P}_{\mathrm{II}}} \rangle \tag{11}$$

where $\mathbf{P}_{\mathrm{I}}$ and $\mathbf{P}_{\mathrm{II}}$ contain the weights of the direction measurements and of the length measurements, respectively.

Setting up the normal equations means to form products $\mathbf{A^*PA}$ and $\mathbf{A^*Pl}$:

$$\underset{(r,\,r)}{\mathbf{A^*\,P\,A}} = \begin{bmatrix} \underset{(r_1,\,r_1)}{\mathbf{A}_{\mathrm{I}}^*\;\mathbf{P}_{\mathrm{I}}\;\mathbf{A}_{\mathrm{I}}} & \underset{(r_1,\,r_2)}{\mathbf{A}_{\mathrm{I}}^*\;\mathbf{P}_{\mathrm{I}}\;\mathbf{A}_{\mathrm{II}}} \\ \\ \underset{(r_2,\,r_1)}{\mathbf{A}_{\mathrm{II}}^*\;\mathbf{P}_{\mathrm{I}}\;\mathbf{A}_{\mathrm{I}}} & \underset{(r_2,\,r_2)}{\mathbf{A}_{\mathrm{II}}^*\mathbf{P}_{\mathrm{I}}\mathbf{A}_{\mathrm{II}}} + \underset{(r_2,\,r_2)}{\mathbf{A}_{\mathrm{IV}}^*\mathbf{P}_{\mathrm{II}}\mathbf{A}_{\mathrm{IV}}} \end{bmatrix} \tag{12}$$

$$\underset{(r,\,l)}{\mathbf{A^*\,P}\,l} = \begin{bmatrix} \underset{(r_1,\,1)}{\mathbf{A}_{\mathrm{I}}^*\;\mathbf{P}_{\mathrm{I}}\;l_{\mathrm{I}}} \\ \\ \underset{(r_2,\,1)}{\mathbf{A}_{\mathrm{II}}^*\,\mathbf{P}_{\mathrm{I}}\,\mathbf{l}_{\mathrm{I}}} + \underset{(r_2,\,1)}{\mathbf{A}_{\mathrm{IV}}^*\,\mathbf{P}_{\mathrm{II}}\,l_{\mathrm{II}}} \end{bmatrix} = \begin{bmatrix} \underset{(r_1,\,1)}{n_{\mathrm{I}}} \\ \\ \underset{(r_2,\,1)}{n_{\mathrm{II}}} \end{bmatrix}. \tag{13}$$

For solving the normal equations the inverse of hypermatrix $\mathbf{N}$ (12) has to be produced. For the sake of simplicity, hypermatrix $\mathbf{N}$ is denoted by:

$$\underset{(r,\,r)}{\mathbf{N}} = \begin{bmatrix} \underset{(r_1,\,r_1)}{\mathbf{N}_{\mathrm{I}}} & \underset{(r_1,\,r_2)}{\mathbf{N}_{\mathrm{II}}} \\ \\ \underset{(r_2,\,r_1)}{\mathbf{N}_{\mathrm{III}}} & \underset{(r_2,\,r_2)}{\mathbf{N}_{\mathrm{IV}}} \end{bmatrix} \tag{14}$$

Inverse matrix $\mathbf{N}^{-1}$ is wanted in the same form:

$$\underset{(r,\,r)}{\mathbf{N}^{-1}} = \begin{bmatrix} \underset{(r_1,\,r_1)}{\mathbf{H}_{\mathrm{I}}} & \underset{(r_1,\,r_2)}{\mathbf{H}_{\mathrm{II}}} \\ \\ \underset{(r_2,\,r_1)}{\mathbf{H}_{\mathrm{III}}} & \underset{(r_2,\,r_2)}{\mathbf{H}_{\mathrm{IV}}} \end{bmatrix}. \tag{15}$$

There are two ways of computing values for $\mathbf{H}_{\mathrm{I}}$, $\mathbf{H}_{\mathrm{II}}$, $\mathbf{H}_{\mathrm{III}}$ and $\mathbf{H}_{\mathrm{IV}}$ ([5], [7]), namely either:

$$\begin{aligned}
\underset{(r_2,\,r_2)}{\mathbf{H}_{\mathrm{IV}}} &= (\underset{(r_2,\,r_2)}{\mathbf{N}_{\mathrm{IV}}} - \underset{(r_2,\,r_2)}{\mathbf{N}_{\mathrm{III}}\,\mathbf{N}_{\mathrm{I}}^{-1}\,\mathbf{N}_{\mathrm{II}}})^{-1} \\
\underset{(r_1,\,r_2)}{\mathbf{H}_{\mathrm{II}}} &= -\underset{(r_1,\,r_2)}{\mathbf{N}_{\mathrm{I}}^{-1}\,\mathbf{N}_{\mathrm{II}}\,\mathbf{H}_{\mathrm{IV}}} \\
\underset{(r_2,\,r_1)}{\mathbf{H}_{\mathrm{III}}} &= -\underset{(r_2,\,r_1)}{\mathbf{H}_{\mathrm{IV}}\,\mathbf{N}_{\mathrm{III}}\,\mathbf{N}_{\mathrm{I}}^{-1}} \\
\underset{(r_1,\,r_1)}{\mathbf{H}_{\mathrm{I}}} &= \underset{(r_1,\,r_1)}{\mathbf{N}_{\mathrm{I}}^{-1}} - \underset{(r_1,\,r_1)}{\mathbf{H}_{\mathrm{II}}\,\mathbf{N}_{\mathrm{III}}\,\mathbf{N}_{\mathrm{I}}^{-1}}
\end{aligned} \tag{16}$$

or

$$\begin{aligned}
\underset{(r_1,\,r_1)}{\mathbf{H}_{\mathrm{I}}} &= (\underset{(r_1,\,r_1)}{\mathbf{N}_{\mathrm{I}}} - \underset{(r_1,\,r_1)}{\mathbf{N}_{\mathrm{II}}\,\mathbf{N}_{\mathrm{IV}}^{-1}\,\mathbf{N}_{\mathrm{III}}})^{-1} \\
\underset{(r_2,\,r_1)}{\mathbf{H}_{\mathrm{III}}} &= -\underset{(r_2,\,r_1)}{\mathbf{N}_{\mathrm{IV}}^{-1}\,\mathbf{N}_{\mathrm{III}}\,\mathbf{H}_{\mathrm{I}}} \\
\underset{(r_1,\,r_2)}{\mathbf{H}_{\mathrm{II}}} &= -\underset{(r_1,\,r_2)}{\mathbf{H}_{\mathrm{I}}\,\mathbf{N}_{\mathrm{II}}\,\mathbf{N}_{\mathrm{IV}}^{-1}} \\
\underset{(r_2,\,r_2)}{\mathbf{H}_{\mathrm{IV}}} &= \underset{(r_2,\,r_2)}{\mathbf{N}_{\mathrm{IV}}^{-1}} - \underset{(r_2,\,r_2)}{\mathbf{H}_{\mathrm{III}}\,\mathbf{N}_{\mathrm{II}}\,\mathbf{N}_{\mathrm{IV}}^{-1}}\quad.
\end{aligned} \tag{17}$$

Eqs (16) and (17) are advisably used if the matrices $\mathbf{N}_{\mathrm{I}}$ and $\mathbf{N}_{\mathrm{IV}}$, resp., are easy to invert.

In adjusting combined networks by partitioning according to the above, the use of the first method is suitable because here matrix $\mathbf{N}_{\mathrm{I}}$ is a diagonal matrix.

After having produced the inverse matrix, the changes of the unknowns can be computed as solutions of the normal equations:

$$\begin{bmatrix} \underset{(r_1,1)}{x_{\mathrm{I}}} \\ \\ \underset{(r_2,1)}{x_{\mathrm{II}}} \end{bmatrix} - = \begin{bmatrix} \underset{(r_1,r_1)}{\mathbf{H}_{\mathrm{I}}} & \underset{(r_1,r_2)}{\mathbf{H}_{\mathrm{II}}} \\ \\ \underset{(r_2,r_1)}{\mathbf{H}_{\mathrm{III}}} & \underset{(r_2,r_2)}{\mathbf{H}_{\mathrm{IV}}} \end{bmatrix} \begin{bmatrix} \underset{(r_1,1)}{n_{\mathrm{I}}} \\ \\ \underset{(r_2,1)}{n_{\mathrm{II}}} \end{bmatrix} . \tag{18}$$

The adjusted values of the orientation angles and the coordinates are obtained by reducing the preliminary values and the changes.

The measurement corrections are computed from (10).

The adjustment is checked in the usual way by definite orientation of the points.

After adjustment, the computation of different confidence numbers may be necessary.

The most frequently used confidence numbers are contained in the variance-covariance matrix $\mathbf{M}_{(x)}$ of adjustment unknowns, produced from the weight coefficient matrix $\mathbf{Q}_{(x)}$ of unknowns:

$$\underset{(r,r)}{\mathbf{M}(x)} = m_0^2 \underset{(r,r)}{\mathbf{Q}(x)} . \tag{19}$$

(Matrix sizes refer to adjustment with a fixed beginning point.)

m_0 in the formula is the mean error of the weight unit obtained, for the adjustment of an independent network, as:

$$m_0 = \sqrt{\frac{v^* \mathbf{P}\, v}{|n - (R - d)|}} . \tag{20}$$

In partitioning, the square sum of corrections is computed as:

$$v^* \mathbf{P}\, v = v_{\mathrm{I}}^* \mathbf{P}_{\mathrm{I}}\, v_{\mathrm{I}} + v_{\mathrm{II}}{}^* \mathbf{P}_{\mathrm{II}}\, v_{\mathrm{II}} . \tag{21}$$

Weight coefficient matrix of the unknowns is, using the known formula:

$$\underset{(r,r)}{\mathbf{Q}(x)} = \underset{(r,r)}{(\mathbf{A}^* \mathbf{P}\ \mathbf{A})^{-1}} = \underset{(r,r)}{\mathbf{N}^{-1}} . \tag{22}$$

As in general only the weight coefficient or the variance—covariance matrix of the coordinates is wanted, it is sufficient to examine $\mathbf{H}_{\mathrm{IV}}$ obtained from (16).

Adjustment by partitioning of combined networks results in many facilities, compared to adjustment as a whole, to be pointed out in presenting the example.

Adjustment by partitioning of a four-point motion test network

For the sake of motion test measurements, a network of four control points was made with side lengths averaging 500 m. Direction measurements were made by seconds theodolite (type Zeiss Theo 010/A), and length measurements by an electrooptical geodimeter AGA 10.

The coordinate system of the network was set out with its origin at point 1, axis y across point 2, and directed from point 1 to point 2. Preliminary coordinates of the points were calculated by different interpolation procedures using the measurement results.

In adjusting the network, the beginning point ($y_1 = 0$; $x_1 = 0$) as well as the direction of axis y ($x_2 = 0$) were fixed. On this basis, according to symbols in (11):

$$n_L = 4\times 3 = 12$$
$$n_T = 6$$
$$r_1 = 4$$
$$r_2 = 4\times 2 - 3 = 5\ .$$

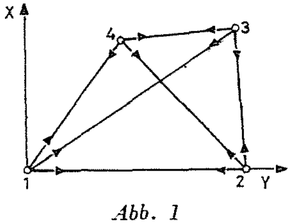

Abb. 1

When stating the weight relations the measurement results were considered as independent. Confidence of direction measurements was ± 3 sec, and that of length measurements ± 0.1 dm.

The normal equations were set up by computing the partitioned forms of form matrix **A**, weight matrix **P** and pure member vector l according to (12), (13). These matrix products are easy and fast to obtain with a pocket calculator.

It has to be pointed out that calculations further simplify upon adequately chosing the partitioning, further taking $\mathbf{P}_\mathrm{I}$ as a unit matrix, and the two minormatrices of $\mathbf{A^*PA}$ are the transposed of each other.

Table 1

		Z_1	Z_2	Z_3	Z_4	y_2	y_3	x_3	y_4	x_4	l	P
Direction measurements	1—2	—1	0	0	0	0	0	0	0	0	—0.3	1
	1—3	—1	0	0	0	0	+19.29	—26.88	0	0	+0.7	1
	1—4	—1	0	0	0	0	0	0	+41.64	—29.34	—0.3	1
	2—1	0	—1	0	0	0	0	0	0	0	+1.0	1
	2—3	0	—1	0	0	—56.65	+56.65	+2.41	0	0	—1.0	1
	2—4	0	—1	0	0	—35.39	0	0	+35.39	+30.87	0.0	1
	3—1	0	0	—1	0	0	+19.29	—26.88	0	0	—0.3	1
	3—2	0	0	—1	0	—56.65	+56.65	+2.41	0	0	—13.3	1
	3—4	0	0	—1	0	0	+8.84	—74.42	—8.84	+74.42	+13.7	1
	4—1	0	0	0	—1	0	0	0	+41.64	—29.34	—9.7	1
	4—2	0	0	0	—1	—35.39	0	0	+35.39	+30.87	—5.7	1
	4—3	0	0	0	—1	0	+8.84	—74.42	—8.84	+74.42	+15.3	1
Length measurements	1—2					+1.00	0	0	0	0	—0.06	900
	1—3					0	+0.81	+0.58	0	0	—0.23	900
	1—4					0	0	0	+0.58	+0.82	—0.03	900
	2—3					+0.04	—0.04	+1.00	0	0	—0.08	900
	2—4					+0.66	0	0	—0.66	+0.75	—0.09	900
	3—4					0	+0.99	+0.12	—0.99	—0.12	+0.03	900

Units ["] and [dm]

With symbols in (12), numerical values of minormatrices are:

$$\underset{(4,4)}{\mathbf{A}_{\mathrm{I}}^{*}\,\mathbf{P}_{\mathrm{I}}\,\mathbf{A}_{\mathrm{I}}} = \langle 3;\ 3;\ 3;\ 3;\rangle$$

$$\underset{(5,4)}{\mathbf{A}_{\mathrm{II}}^{*}\mathbf{P}_{\mathrm{I}}\mathbf{A}_{\mathrm{I}}} = \underset{(5,4)}{(\mathbf{A}_{\mathrm{I}}^{*}\mathbf{P}_{\mathrm{I}}\mathbf{A}_{\mathrm{II}})^{*}} = \begin{bmatrix} 0 & +92.04 & +56.65 & +35.39 \\ -19.29 & -56.65 & -84.78 & -8.84 \\ +26.88 & -2.41 & +98.89 & +74.42 \\ -41.64 & -35.39 & +8.84 & -68.19 \\ +29.34 & -30.87 & -74.42 & -75.95 \end{bmatrix}$$

$$\underset{(5,5)}{\mathbf{A}_{\mathrm{II}}^{*}\mathbf{P}_{\mathrm{I}}\mathbf{A}_{\mathrm{II}}} + \underset{(5,5)}{\mathbf{A}_{\mathrm{IV}}^{*}\mathbf{P}_{\mathrm{II}}\mathbf{A}_{\mathrm{IV}}} =$$

$$= \begin{bmatrix} +10\,213.25 & -6\,419.98 & -234.63 & -2\,893.27 & -1\,739.11 \\ & +8\,802.06 & -1\,586.86 & -1\,043.87 & +1\,210.82 \\ & & +13\,749.77 & +1\,210.82 & -11\,088.67 \\ & & & +7\,704.11 & -1\,491.99 \\ & & & & +15\,829.44 \end{bmatrix}.$$

(Of the symmetrical matrices only the upper triangles are presented.)

According to (13), $\mathbf{A}^{*}\,\mathbf{P}\,l$ is easy to compute since $n_{\mathrm{I}} = 0$ — in calculating the pure terms, the sum of deviations from the arithmetic mean being zero — the second part of the vector is:

$$\underset{(1,5)}{n_{\mathrm{II}}^{*}} = [+1007.98;\ -684.22;\ -2392.75;\ -970.22;\ +2311.37].$$

Based on (16) the inverse matrix $\mathbf{N}^{-1}$ may be calculated as a simple reciprocal. First the value of $\mathbf{H}_{\mathrm{IV}}$ has to be determined, involving two simple matrix multiplications, a subtraction inversion of a 5×5 matrix. In the entire computation this is the greatest inversion but even so, it is easily done by means of a higher capacity pocket calculator or a simpler desk computer.

$$\underset{(5,5)}{\mathbf{H}_{\mathrm{IV}}} = \begin{bmatrix} +0.00037 & +0.00029 & +0.00008 & +0.00020 & +0.00009 \\ & +0.00052 & -0.00003 & +0.00034 & +0.00010 \\ & & +0.00031 & -0.00009 & +0.00016 \\ & & & +0.00047 & +0.00009 \\ & & & & +0.00021 \end{bmatrix}.$$

The value of $\mathbf{H}_{\mathrm{II}}$ is easy to obtain, namely the transposed of $\mathbf{N}_{\mathrm{I}}^{-1}\,\mathbf{N}_{\mathrm{II}}$ has already been determined in calculating $\mathbf{H}_{\mathrm{IV}}$, thus only the matrix multiplication has to be done:

$$\underset{(4,5)}{\mathbf{H}_{\mathrm{II}}} = -\begin{bmatrix} +0.0030 & +0.0074 & -0.0058 & +0.0086 & -0.0016 \\ -0.0026 & +0.0060 & -0.0023 & +0.0066 & +0.0023 \\ +0.0002 & +0.0119 & -0.0088 & +0.0094 & +0.0008 \\ +0.0013 & +0.0092 & -0.0069 & +0.0136 & +0.0025 \end{bmatrix}.$$

By computing $\mathbf{H}_{IV}$ and $\mathbf{H}_{II}$ the wanted minormatrices of the inverse matrix are obtained, namely to determine the changes, calculation of $\mathbf{H}_I$ and $\mathbf{H}_{III}$ in (18) is not necessary, as $n_I = 0$ and thus

$$\underset{(9,1)}{x} = -\begin{bmatrix} \underset{(4,4)}{\mathbf{H}_I} & \underset{(4,5)}{\mathbf{H}_{II}} \\ \underset{(5,4)}{\mathbf{H}_{III}} & \underset{(5,5)}{\mathbf{H}_{IV}} \end{bmatrix} \begin{bmatrix} \underset{(4,1)}{n_I} \\ \underset{(5,1)}{n_{II}} \end{bmatrix} = -\begin{bmatrix} \underset{(4,5)}{\mathbf{H}_{II}} & \underset{(5,1)}{n_{II}} \\ \underset{(5,5)}{\mathbf{H}_{IV}} & \underset{(5,1)}{n_{II}} \end{bmatrix} = \begin{bmatrix} \underset{(4,1)}{x_I} \\ \underset{(5,1)}{x_{II}} \end{bmatrix}$$

$$\underset{(1,4)}{x^*} = [+0.3;\ +2.3;\ -5.7;\ -4.2]$$

$$\underset{(1,5)}{x^*} = [-0.001;\ +0.088;\ +0.208;\ +0.080;\ -0.046]\,.$$

Corrections of measurement results may be computed from (10).

$$v_I^* = [-0.57;\ -3.46;\ +4.11;\ -1.32;\ +2.21;\ -0.88;\ +1.52;\ -2.06;\ +0.58;\ -0.79;\ -0.03;\ +0.70]$$

$$\underset{(1,6)}{v_{II}^*} = [-0.06;\ -0.04;\ -0.02;\ +0.12;\ +0.00;\ +0.07]\,.$$

The presented numerical example involved some partial results but in fact, the computation was not sharp to two decimals, just a rounded value for the sake of comprehensiveness. To balance calculation checkings, computation sharpness has to be pondered on the basis of the coefficients, the pure members and the weights.

According to (20), mean error of the unit weight:

$$m_0 = \sqrt{\frac{67{,}00}{18-(12-3)}} = \pm 2{,}73.$$

According to (19), the coordinate mean error values are:

$m_{y2} = \pm 5.3$ mm; $m_{y3} = \pm 6.2$ mm; $m_{x3} = \pm 4.8$ mm;

$m_{y4} = \pm 5.9$ mm; $m_{x4} = \pm 4.0$ mm.

Summary

The matrix partitioning presented has several practical advantages. It permits to solve extended normal equation systems or to find regularities in adjusting e.g. combined networks by the indirect measurement method. Use of partitioning — as shown in the numerical example — is recommended primarily for the rapid solution of direct practical problems. With the advent of high capacity computers, adjustment of a horizontal network consisting of

a few points would not justify partitioning of the coefficient matrix of normal equations. Remind, however, that the rapid solution of individual problems leaves no time to engage services of computing centres and in such cases adjustment can be carried out using pocket calculators or desk computers.

In both the theoretical discussion and presenting the numerical example, some correlations in repeated adjustment of combined networks, likely to result in important work savings, have been pointed out.

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