CONVERSIONS BETWEEN GEOGRAPHICAL AND TRANSVERSE MERCATOR (UTM, GAUSS-KRÜGER) GRID COORDINATES

J. VARGA

Institute of Geodesy, Technical University, Budapest, H-1521

(Received: March 2, 1981.)

Presented by Prof. Dr. F. Sárközy, Director

To now, mutual conversions between Universal Transverse Mercator (UTM) grid coordinates and geographical coordinates applied computers or tables. Programmable desk and pocket calculators revolutionized calculations not only in the office but also on the terrain. A single magnetic card may replace volumes of tables. A programmable calculator, fed with correct data input provides correct results as against the multitude of sources of error, inherent in the use of tables. Another significant advantage of programmed calculation is a lower qualified operator needed to obtain error-free results, at a reduced running time.

The mutual conversion between UTM and geographical coordinates by means of a minicomputer will be presented below.

The chosen computer methods require low data and program storage capacity fit to a smaller programmable pocket calculator. The prepared programs with the indicated constants provide for a mean error of ± 1 mm in the grid coordinate and $\pm 0.0002''$ in the geographical coordinate, all along the ellipsoid.

Notations

a = semi-major axis of the ellipsoid;

b = semi-minor axis of the ellipsoid;

$c = a^2/b$, polar radius of curvature;

$e^2 = \dfrac{a^2 - b^2}{a^2}$, square of the first eccentricity;

$e'^2 = \dfrac{a^2 - b^2}{b^2} = \dfrac{e^2}{1 - e^2}$, square of the second eccentricity;

Φ = latitude of point;

Φ_1 = latitude of foot of perpendicular to central meridian from point;

Λ = longitude of point;

Λ_0 = longitude of origin (the central meridian) of the projection;

λ = difference of longitude from the central meridian;
$\lambda = \Lambda - \Lambda_0$ in the Eastern Hemisphere;
$\lambda = \Lambda_0 - \Lambda$ in the Western Hemisphere;
$t = \operatorname{tg} \Phi$;
$\eta^2 = e'^2 \cos^2 \Phi$;
$V = \sqrt{1 + \eta^2} = \sqrt{1 + e'^2 \cos^2 \Phi}$;
$M = c/V^3$, radius of curvature in the meridian;
$M = c/V$, radius of curvature in the prime vertical;
S = true meridional distance on the ellipsoid from the equator;
k_0 = central scale factor;
X_0 = false northing;
Y_0 = false easting;
x = grid distance from the equator;
y = grid distance from the central meridian;
$X = x + X_0$ = grid northing;
$Y = y + Y_0$ = grid easting.

Relationships between grid and geographical coordinates in the transversal conformal Mercator projections of the ellipsoid are:

Geographical to projection:

$$x = (S + A_2 \lambda^2 + A_4 \lambda^4) k_0,$$
$$y = (A_1 \lambda + A_3 \lambda^3 + A_5 \lambda^5) k_0, \tag{1}$$
$$X = x + X_0, \qquad Y = y + Y_0.$$

Projection to geographical:

$$x = \frac{X - X_0}{k_0}, \quad y = \frac{Y - Y_0}{k_0},$$
$$\Phi = \Phi_1 + B_2 y^2 + B_4 y^4,$$
$$\lambda = B_1 y + B_3 y^3 + B_5 y^5, \tag{2}$$
$$\Lambda = \Lambda_0 \pm \lambda.$$

After substitutions and transformations, (1) and (2) take the following, efficiently programmable form: [2]

$$x = k_0 \left\{ S + \frac{N \lambda^2}{2} \sin \Phi \cos \Phi \left[1 + \frac{\lambda^2}{12} \cos^2 \Phi (5 - t^2 + 9\eta^2 + 4\eta^2) + \right. \right.$$
$$\left. \left. + \frac{\lambda^4}{360} \cos^4 \Phi (61 - 58 t^2 + t^4) \right] \right\}, \tag{3}$$

$$y = k_0 \lambda N \cos \Phi \left[1 + \frac{\lambda^2}{6} \cos^2 \Phi (1 - t^2 + \eta^2) + \right.$$

$$\left. + \frac{\lambda^4}{120} \cos^4 \Phi \,(5 - 18\, t^2 + t^4 + 14\, \eta^2 - 58\, t^2 \eta^2) \right], \qquad (4)$$

$$\Phi = \Phi_1 - \frac{y^2 t_1}{2\, M_1 N_1} \left[1 - \frac{y^2}{12\, N_1^2} (5 + 3 t_1^2 + \eta_1^2 - 9 t_1^2 \eta_1^2) + \right.$$

$$\left. + \frac{y^4}{360\, N_1^4} (61 + 90\, t_1^2 + 45\, t_1^4) \right], \qquad (5)$$

$$\lambda = \frac{y}{N_1 \cos \Phi_1} \left[1 - \frac{y^2}{6\, N_1^2} (1 + 2\, t_1^2 + \eta_1^2) + \frac{y^4}{120\, N_1^4} (5 + 28\, t_1^2 + 24\, t_1^4) \right]. \qquad (6)$$

The angle values have to be substituted in radians into formulae (3) to (6) to avoid calculation with powers of the analytical angle unit ϱ. S and Φ_1 can be determined as follows. Subscripts 1 in (5) and (6) indicate that for calculating the given quantity Φ_1 has to be substituted into the formula.

1. Determination of S (true meridional distance on the ellipsoid)

For determining S and Φ_1, VINCENTY suggested a useful direct method in [7]. For each ellipsoid 7 constant values have to be calculated and stored. Constants will be derived in a similar way, some of them are, however, simple to express from the others and therefore it is needless to store them separately.

LEE [3] calculates the plane coordinates by Ψ using the isometric latitude to obtain symmetrical terms for x and y, easy to program exempt from S. Disadvantage of this method is that series have to be taken into consideratian to a high order number likely to inhibit use of a minicomputer.

S is given by integrals:

$$S = \int_0^{\Phi} M \, d\, \Phi = a(1 - e^2) \int_0^{\Phi} \frac{d\Phi}{(1 - e^2 \sin^2 \Phi)^{3/2}}, \qquad (7)$$

or

$$S = \int_0^{\Phi} M \, d\, \Phi = c \int_0^{\Phi} \frac{d\Phi}{(1 + e'^2 \cos^2 \Phi)^{3/2}} \qquad (8)$$

showing the arc length S of the meridian to be proportional to the area between curves of functions

$$\omega = (1 - e^2 \sin^2 \Phi)^{-3/2},$$

or

$$\omega = (1 + e'^2 \cos^2\Phi)^{-3/2}$$

and the coordinate axis $\omega = 0$. It is also seen that for determining S, storage of two constants for each ellipsoid may be enough; in one case these are a and e^2, in the other c and e'^2. Functions $\omega = f(\Phi)$ are continuous, mildly ascending in the interval $0 \leq \Phi \leq \pi/2$ fit to numerical integration by dividing interval 0 to Φ into relatively few parts to yield the area below the curves at the desired accuracy.

Let us write *Simpson*'s parabola formula for S divided to even n parts according to (8):

$$S = c \int_0^{\Phi} \omega \, d\Phi \approx \frac{ch}{3} (\omega_0 + 4\omega_1 + 2\omega_2 + 4\omega_3 + \ldots + 2\omega_{n-2} + 4\omega_{n-1} + \omega_n)$$

where $h = \Phi/n$ and ω_i is the substitution value of the function at the i-th place. $n = f(\Phi)$ namely with increasing Φ, the interval has to be divided into ever more parts. This method of calculating S with a pocket calculator takes much time, therefore the use of a faster desk computer is recommended. For $n = 30$, generally S is obtained with mm accuracy even near the poles.

In most practical works, calculations refer to the area of a single continent or country. In this case it is expedient to calculate S_0 once accurately for the limit parallel circle Φ_0 outside the boundary of the area, to store it in the memory as a constant and to divide only the interval Φ_0 to Φ into n parts.
n can be determined empirically in each case.

This latter method takes not much time even with a pocket calculator because $n \leq 10$. The great advantage of this method is that only c and e'^2 have to be stored, these constants being also necessary for solving Eqs (3) to (6).

For determining S, a faster method but needing an increased storage capacity will be shown below.

Table 1

Ellipsoid	c (m)	$e'^2 \cdot 10^{-3}$	α
Everest	6 398 547.993	6.682 201 989	6 366 680.291
Bessel	6 398 786.849	6.719 218 798	6 366 742.521
Clarke/66	6 399 902.551	6.814 784 832	6 367 399.690
Clarke/80	6 400 057.735	6.850 116 152	6 367 386.645
Hayford	6 399 936.608	6.768 170 197	6 367 654.500
Krassowsky	6 399 698.902	6.738 525 415	6 367 558.497
IUGG/1967	6 399 617.430	6.739 725 198	6 367 471.748

Series expanding (8) and integrating term by term:

$$S = A' c\,\Phi - \frac{B'}{2} c \sin 2\Phi + \frac{C'}{4} c \sin 4\Phi - \frac{D'}{6} c \sin 6\Phi + \dots \tag{9}$$

$$A' = 1 - \frac{3}{4} e'^2 + \frac{45}{64} e'^4 - \frac{175}{256} e'^6 + \frac{11025}{16384} e'^8 - \dots$$

$$B' = \quad + \frac{3}{4} e'^2 - \frac{15}{16} e'^4 + \frac{525}{512} e'^6 - \frac{2205}{2048} e'^8 + \dots$$

$$C' = \quad \frac{15}{64} e'^4 - \frac{105}{256} e'^6 + \frac{2205}{4096} e'^8 - \dots$$

$$D' = \quad \frac{35}{512} e'^6 - \frac{315}{2048} e'^8 + \dots \tag{10}$$

Reducing (9):

$$S = \alpha\bar{\Phi} - \beta \sin 2\,\bar{\Phi} + \gamma \sin 4\,\bar{\Phi} - \delta \sin 6\,\bar{\Phi} + \dots, \tag{11}$$

where

$$\alpha = A' c, \quad \beta = \frac{B'}{2} c, \quad \gamma = \frac{C'}{4} c, \quad \delta = \frac{D'}{6} c.$$

α, β, γ, δ are constants, to be determined separately for each ellipsoid. The values for the most frequently used ellipsoids are found in Table 1.

2. Determination of $\bar{\Phi}_1$

Fig. 1 shows that coordinate x of foot point P_1 on the central meridian at point P of latitude $\bar{\Phi}$ equals S_1, permitting (11) to be written as:

$$x = \alpha\bar{\Phi}_1 - \beta \sin 2\,\bar{\Phi}_1 + \gamma \sin 4\,\bar{\Phi}_1 - \delta \sin 6\,\bar{\Phi}_1 + \dots$$

β	γ	δ	$\beta/\alpha \cdot 10^{-3}$	$\gamma/\alpha \cdot 10^{-6}$	$\delta/\alpha \cdot 10^{-9}$
15 900.693 22	16.546 6	0.021	2.497 486	2.599	3.4
15 988.638 53	16.729 95	0.022	2.511 275	2.628	3.5
16 216.943 89	17.209 4	0.023	2.546 871	2.703	3.6
16 300.700 72	17.387 6	0.023	2.560 030	2.731	3.6
16 107.034 68	16.976 2	0.022	2.529 508	2.666	3.5
16 036.480 28	16.828 1	0.022	2.518 466	2.643	3.5
16 039.107 44	16.833 8	0.022	2.518 913	2.644	3.5

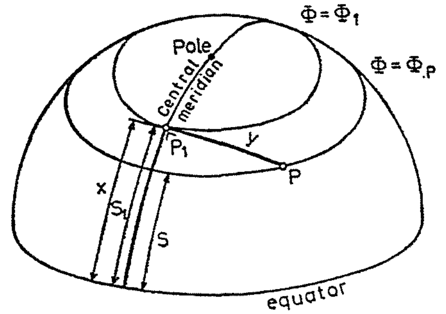


Fig. 1

Expressing Φ_1 at a good approximation:

$$\Phi_1 = \frac{x}{\alpha} + \frac{\beta}{\alpha} \sin 2\,\Phi_1 - \frac{\gamma}{\alpha} \sin 4\,\Phi_1 + \frac{\delta}{\alpha} \sin 6\,\Phi_1 + \ldots \tag{12}$$

As Φ_1 occurs on both sides of the sign of equality, it can be determined by iteration.

First a fair approximation value of $\bar{\Phi}_1$ is determined from $(\bar{\Phi}_1) = \dfrac{x}{\alpha}$. Substituting $\bar{\Phi}_1$ into the right-hand side of (12) yields a closer approximation for $\bar{\Phi}_1$. The number of re-substitutions depends on the needed accuracy but it is never more than three.

Disadvantage of the method is to need iteration, its advantage is, however, not to need storage of further constants that are easy to express from constants involved in calculating S.

In case of a computer with an increased storage capacity, also values of β/α, γ/α, δ/α can be stored, of them those referring to ellipsoids most frequently used are as well included in Table 1.

3. Programs

The programs were developed for a *Hewlett-Packard* 67/97 calculator.

For converting geographical to grid coordinates Eqs (3), (4) and (11); for converting grid to geographical coordinates Eqs (5), (6) and (12) were used. The programs can be accommodated on a magnetic card, and the constants belonging to an ellipsoid are stored on one side of a card.

The prepared programs are shown by Program Listings 1 and 2, their use by Users' Instructions 1 and 2. The memory content is identical in both programs; this is illustrated in Table "Registers". δ/α was not separately stored but built in into the program as a constant. From Table 1 it is seen that its value may be taken for all the ellipsoids uniformly as $3.5 \cdot 10^{-9}$. Similarly also δ might have been built into the program with a value of 0.022.

The elaborated examples refer to the *Universal Transverse Mercator* grid system, with the following properties to be reckoned with in our calculations.

1. Ellipsoid: Hayford. The necessary constants are found in Table 1.
2. False northing, $X_0 = 0$ m (10 000 000 for Southern Hemisphere).
3. False easting, $Y_0 = 500\,000$ m.
4. Scale factor at the central meridian, $k_0 = 0.9996$.

As in the former case, the stored matter of the memories is recorded in the Data Sheet, supposing work on the Northern Hemisphere.

The programs can of course be used for any ellipsoid, even for tangential or intersecting position of the image plane. Remind that in tangential position $k_0 = 1$.

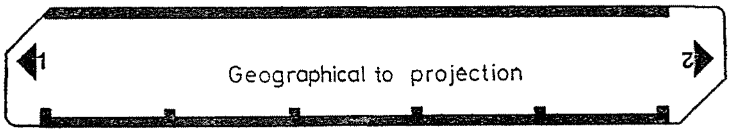
Data Sheet

6.367654500	+	06	0
1.610703468	+	04	1
1.697620000	+	01	2
2.200000000	–	02	3
2.529508000	–	03	4
2.666000000	–	06	5
9.996000000	–	01	6
0.000000000	+	00	7
0.000000000	+	00	8
0.000000000	+	00	9
5.000000000	+	05	A
0.000000000	+	00	B
0.000000000	+	00	C
6.399936608	+	06	D
6.768170197	–	03	E
0.000000000	+	00	I

Constants of ellipsoids missing from Table 1 are determined from Eqs (10) and (11).

The two programs confined suit conversion between belts, using the following method. Grid coordinates for one belt are converted to geographical coordinates, then, with the geographical longitude of the central meridian of the adjacent belt as input, grid coordinates of the point on the adjacent projection belt are calculated with the other program.

User Instructions



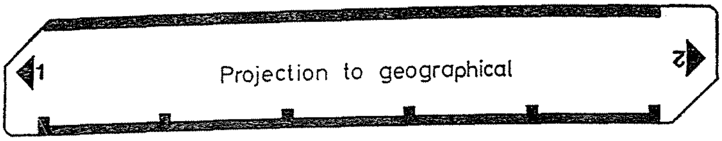
STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load sides 1 and 2 of			
	program card			
2	Load side 1 of data card			
	(Choose an ellipsoid), at first only			
-				
3	Longitude of the central meridian.	Λ_0	E	
	degree, if needed: deg., min., sec.			
4	Latitude, deg., min., sec.	Φ	ENT↑	
5	Longitude —″—	Λ		
6	Northern Hemisphere, East from Greenwich		A	
	West —″—		B	
	Southern Hemisphere, East —″—		C	
	West —″—		D	
-				
7	UTM grid coordinates			X (m)
8	—″—		R/S	Y (m)
	For a new case go to step 2,			
	or 3, or 4.			

EXAMPLE

STEP	INSTRUCTION	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load program			
2	Load data			
	(Hayford)			
3	Longitude of	9°	E	
	the cent. meridian			
4	Latitude	47° 15′ 38,4257″	ENT↑	
5	Longitude	6° 27′ 49,7791″		
6	N. Hemisphere, East.		A	
7	UTM, X.			5 237 353,489 m
8	UTM, Y.		R/S	308 121, 657 m

User Instructions 1

User Instructions



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Load sides 1 and 2 of				
	program card				
2	Load side 1 data card				
	(Choose an ellipsoid), at first only				
3	Longitude of the central meridian	Λ_0	E		
	degree, if needed: deg., min., sec.				
4	UTM grid coordinates	X (m)	ENT.↑		
5	— " —	Y (m)			
6	Northern Hemisphere, East from Greenwich		A		
	West — " —		B		
	Southern Hemisphere, East — " —		C		
	West — " —		D		
7	Latitude, deg., min., sec.				Φ
8	Longitude, — " —		R/S		Λ
	For a new case go to step 2,				
	or 3, or 4,				

EXAMPLE

STEP	INSTRUCTION	INPUT DATA / UNITS	KEYS		OUTPUT DATA / UNITS
1	Load program				
2	Load data				
	(Hayford)				
3	Longitude of the	9°	E		
	cent. meridian.				
4	UTM, X	5 237 353,489 m	ENT.↑		
5	UTM, Y	308 121,657 m			
6	N. Hemisphere, East.		A		
7	Latitude				47° 15' 38,4257"
8	Longitude		R/S		6° 27' 49,7791"

User Instructions 2

Program Listing 1

```
*LBLB
SF0
GTOA
*LBLD
SF0
*LBLC
SF1
*LBLA
RAD
P⇄S
HMS→
RCLC
—
F0?
CHS
D→R
STO0
X⇄Y
HMS→
D→R
STO2
SIN
STO3
LSTX
COS
STO4
÷
X²
STO6
RCL4
X²
RCLE
×
STO5
1
+
√X
RCLD
X⇄Y
÷
RCL4
×
RCL0
×
STO7
1
RCL6
—
RCL5
+
RCL4
X²
RCL0
X²
×
STO8
6
÷
STO9
×
1
+
5
RCL6
1
8
×
—
RCL6
X²
+
RCL5
1
4
×
+
RCL5
RCL6
×
5
8
×
—
RCL8
X²
1
2
0
÷
STO8
×
+
RCL7
×
STOI
5
RCL6
—
RCL5
9
×
+
RCL5
X²
4
×
+
RCL9
2
÷
×
1
+
6
1
RCL6
5
8
×
—
RCL6
X²
+
RCL8
3
÷
×
+
RCL7
RCL0
×
RCL3
×
2
÷
×
RCL2
P⇄S
STO7
RCL0
×
+
RCL7
2
×
SIN
RCL1
×
—
RCL7
4
×
SIN
RCL2
×
+
RCL7
6
×
SIN
RCL3
×
—
RCL6
×
F1?
CHS
RCLB
+
CF0
CF1
DEG
R/S
RCLI
RCL6
×
RCLA
+
RTN
*LBLE
HMS→
STOC
R↓
RTN
R/S
```

Program Listing 2

```
*LBLB
SF0
GTOA
*LBLD
SF0
*LBLC
SF1
*LBLA
RAD
RCLA
—
RCL6
÷
F0?
CHS
STO7
X⇄Y
RCLB
—
RCL6
÷
ABS
RCL0
÷
STO8
STO9
3
STOI
*LBL0
RCL8
RCL9
2
×
SIN
RCL4
×
+
RCL9
4
×
SIN
RCL5
×
—
RCL9
6
×
SIN
3
.
5
EEX
9
CHS
×
+
STO9
DSZI
GTO0
STOI
RCL7
P⇄S
STO0
RCLD
RCLI
COS
X²
RCLE
×
STO1
1
+
√X
STO2
÷
STO3
÷
X²
STO4
X²
STO6
3
6
0
÷
RCLI
TAN
X²
STO5
9
0
×
6
1
+
RCL5
X²
4
5
×
+
×
RCL5
3
×
5
+
RCL1
+
RCL1
RCL5
×
9
×
—
RCL4
×
1
2
÷
—
1
+
RCL5
√X
×
RCL0
X²
×
RCL2
X²
X²
×
RCLD
X²
2
×
÷
CHS
RCLI
+
R→D
→HMS
CF0
CF1
R/S
RCL5
2
8
×
5
+
RCL5
X²
2
4
×
+
RCL6
×
1
2
0
÷
RCL5
2
×
RCL1
+
1
+
RCL4
×
6
÷
—
1
+
RCL0
×
RCL3
÷
RCLI
COS
÷
R→D
RCLC
+
→HMS
DEG
P⇄S
RTN
*LBLE
HMS→
STOC
R↓
RTN
R/S
```

Registers

0 α	1 β	2 γ	3 δ	4 β/α	5 γ/α	6 k_0	7	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9

A Y_0	B X_0	C Λ_0	D c	E e^2	I

4. Transformation of the programs for lower accuracy requirements

The enclosed programs can also be used for the highest accuracy requirements on the entire ellipsoid; therefore they exceed that needed for lower accuracy requirements. If a mean error of ± 1 cm is made up with, steps 72 to 83 and 99 to 107 in program 1 and steps 108 to 115 in program 2 may be omitted.

Working within a domain of 10° of the geographical latitude and having S_0 calculated up to a boundary parallel circle Φ_0, (3) to (5) may be replaced by simpler expressions:

$$x = k_0 \left[S_0 + M_m (\Phi_1 - \Phi_0) + \frac{M_m}{8} (\Phi_1 - \Phi_0)^3 \eta_m^2 (1 - \mathrm{tg}^2 \Phi_m) + \dots \right],$$

$$M_m = \frac{M_0 + M_1}{2}, \qquad \Phi_m = \frac{\Phi_0 + \Phi_1}{2},$$

$$y = k_0 \lambda N \cos \Phi \left[1 + \frac{\lambda^2}{6} \cos^2 \Phi (1 - t^2 + \eta^2) + \frac{\lambda^4}{120} \cos^4 \Phi (5 - 18 t^2 + t^4) \right],$$

$$\Phi = \Phi_1 - \left[\frac{y^2 V_1^2}{2 N_1^2} t_1 + \frac{y^4}{24 N_1^4} t_1 (5 + 3 t_1^2) + \dots \right],$$

where:

$$\Phi_1 = \Phi_0 + \frac{x_0}{N_0} V_0^2 - \frac{3}{2} \frac{x_0^2}{N_0^2} \eta_0^2 t_0 + \frac{x_0^3}{2 N_0^3} \eta_0^2 (t_0^2 - 1) + \dots,$$

or

$$\Phi_1 = \Phi + \frac{\lambda^2}{2} V^2 \sin \Phi \cos \Phi + \frac{\lambda^4}{24} \sin \Phi \cos^3 \Phi (5 - t^2) + \dots$$

These formulae can be composed to a program uniting the two programs above so that a single input on magnetic card before start suffices and thereafter also conversion between the projectional belts to and from can be carried out.

Similar simple formulae are those by FIELD [12]. Also nomograms are made for non-programmable pocket calculators facilitating conversions such as those by REITHOFER [10] for the domain $45° \leq \Phi \leq 57°$.

Summary

Most of difficulties in mutual computer conversions between Universal Transverse Mercator (UTM) grid and geographical coordinates arise in determining S (the meridian arc length from the equator) and Φ_1 (the ellipsoidal geographical latitude of the foot point which is on the central meridian).

Effective methods for such conversions are suggested, suiting pocket calculators with less storage capacities, just as are programs for calculators type HP 67/97 suitable for conversions on the entire surface of any ellipsoid, as indicated in the title.

References

1. HAZAY I.—TÁRCZY-H. A.: A Gauss—Krüger koordináták számítása. (Akadémiai Kiadó Budapest 1951.).
2. JORDAN—EGGERT—KNEISSL: Handbuch der Vermessungskunde, IV. (J. B. Metzlersche Verlagsbuchhandlung, Stuttgart, 1958.).
3. LEE, L. P.: The Transverse Mercator Projection of the Entire Spheroid. Empire Survey Review, XVI (123), 1962.
4. LEE, L. P.: The length of a meridian arc, and the Transverse Mercator Projection of the entire spheroid. Survey Review, XVII (133), 1964. pp. 343.
5. HAZAY I.: Vetülettan. (Tankönyvkiadó, Budapest, 1964.).
6. GDOWSKI, B.: The Transverse Mercator Projection of the entire spheroid. Survey Review, XVIII (141), 1966. pp. 339—341.
7. VINCENTY, T.: The meridional distance problem for desk computers. Survey Review, XXI (161), 1971. 136—140.
8. GERSTBACH, G.: Zur Lösung ellipsoidischer Aufgaben mit elektronischen Taschenrechnern. Zeitschrift für Vermessungswesen, 5. 1974. pp. 205.
9. HUBENY, K.: Isotherme Koordinatensysteme und konforme Abbildungen des Rotationsellipsoides. Mitteilungen der geodätischen Institute der Technischen Universität, 27. Graz. 1977.
10. REITHOFER, A.: Koeffiziententafeln und Rechenprogramme für die Gauss—Krüger-(UTM)-Koordinaten der Ellipsoide von Bessel, Hayford, Krassowsky und des Referenzellipsoids 1967. Mitteilungen der geodätischen Institute der Technischen Universität, 27, Graz. 1977.
11. JACKSON, J. E.: Transverse Mercator Projection. Survey Review, XXIV (188), 1978. pp. 278—285.
12. FIELD, N. J.: Conversions Between Geographical and Transverse Mercator Co-ordinates. Survey Review, XXV (195), 1980. pp. 228—230.

Dr. József VARGA, H-1521 Budapest