MEMBRANE THEORY OF HYPERBOLIC COOLING TOWERS

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1. Scope

This study treats of the membrane theory of cooling towers with hyperbolic curvature, in the general case where axes of the directrix and of the shell of revolution do not coincide.

Investigations are limited to internal forces and deformations of the tower (shell) due exclusively to dead and wind loads.

2. Geometry

Coordinate axes r and z are assumed according to Fig. 1a.

l is the total shell height over supporting columns;

l_1 and l_2 are shell heights below and above throat, resp.;

h is the shell thickness (constant).

Equation of the directrix (distances of meridian curve points from the rotation axis):

$$r = c + a\sqrt{1 + \alpha \frac{z^2}{a^2}}, \tag{1}$$

where

$$\alpha = \frac{a^2}{b^2}.$$

a and b being the hyperbola axes (see Fig. 1). Introducing notations:

$$\xi = \frac{z}{a} \qquad \varrho = \sqrt{1 + \alpha\xi^2} \tag{2}$$

we have:

$$r = a\left(\varrho + \frac{c}{a}\right). \tag{3}$$

a, b and c are constants determined by the meridian curves.

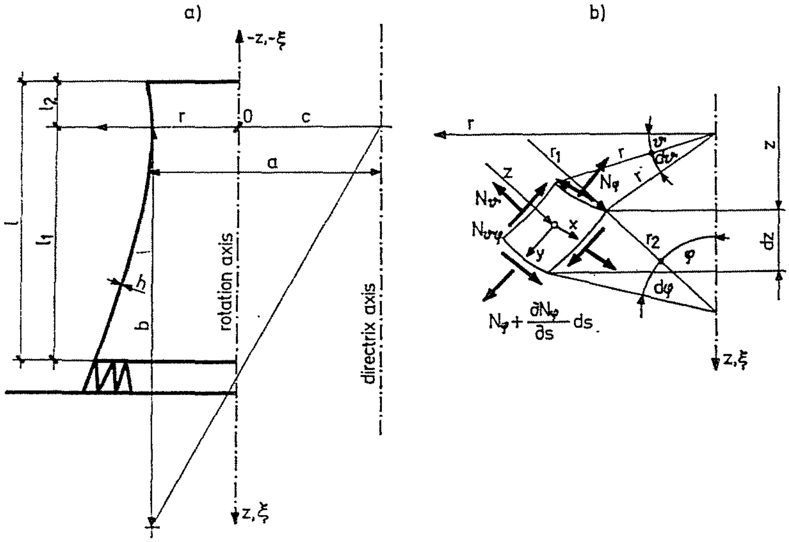

Fig. 1

Meridian curvature radius:

$$r_1 = -\frac{(r - c)^3}{\alpha a^2 \sin^3 \varphi} = -\frac{a \cdot \varrho^3}{\alpha \cdot \sin^3 \varphi}. \tag{4}$$

Curvature radius of the horizontal circle:

$$r_2 = \frac{c}{\sin \varphi} + a\sqrt{1 + \delta\alpha\xi^2} = \frac{r}{\sin \varphi}, \tag{5}$$

where:

$$\delta = 1 + \alpha. \tag{6}$$

Introducing notation

$$\bar{\varrho} = \sqrt{1 + \delta\alpha\xi^2} \tag{7}$$

yields

$$\sin \varphi = \frac{\varrho}{\bar{\varrho}}, \qquad \cos \varphi = \frac{\alpha\xi}{\bar{\varrho}}. \tag{8}$$

Further relationships being:

$$\frac{r}{r_1} = -\frac{\alpha\left(\varrho + \frac{c}{a}\right)\sin^3 \varphi}{\varrho^3}, \tag{9}$$

$$\frac{d\varrho}{d\xi} = \frac{\alpha\xi}{\varrho} = \frac{\cos \varphi}{\sin \varphi}, \qquad \frac{d\varphi}{d\xi} = -\frac{\alpha \sin^2 \varphi}{\varrho^3}. \tag{10}$$

3. Loads, notations

3.1 Shell surface load components

X — component tangential to the horizontal circle,
Y — component tangential to the meridian,
Z — normal component.

Components with a positive sign are seen in Fig. 1*b*.

3.2 Dead load

The specific load:

$$g = h \cdot \gamma \tag{11}$$

$$X = 0, \quad Y = g \sin \varphi, \quad Z = g \cos \varphi. \tag{12}$$

3.3 Wind load

Value of the dynamic pressure: $q(z)$

$$\begin{aligned} X &= 0 \\ Y &= q(z) \cos \varphi \times f(\vartheta) \\ Z &= q(z) \sin \varphi \times f(\vartheta) \end{aligned} \tag{13}$$

where $f(\vartheta)$ is the function of the angular distribution of wind load (form factor).

Describing the angular distribution of wind load by a Fourier expression:

$$X = \sum_{n=0}^{n=i} X_n \sin n\vartheta \text{ (in the actual case } X_n = 0)$$

$$Y = \sum_{n=0}^{n=i} Y_n \cos n\vartheta, \tag{14}$$

$$Z = \sum_{n=0}^{n=i} Z_n \cos n\vartheta,$$

where

$$\begin{aligned} Y_n &= -q(z)\, j_n \cos \varphi, \\ Z_n &= q(z)\, j_n \sin \varphi. \end{aligned} \tag{15}$$

j_n being coefficients of the Fourier expression of form

$$\sum_{n=0}^{n=i} j_n \cos n\vartheta = j_0 + j_1 \cos \vartheta + j_2 \cos 2\vartheta + \ldots \tag{16}$$

For a dynamic pressure q acting on the vertical surface, to simplify calculations, variation with height of wind load is reckoned with as:

$$q(z) = q \sin \varphi. \tag{17}$$

3.4 Membrane forces

N_φ — normal force tangential to the meridian;
N_ϑ — normal force tangential to the horizontal circle;
$N_{\vartheta\varphi} = N_{\varphi\vartheta}$ — shear forces.

Applying the Fourier method:

$$\begin{aligned} N_\varphi &= \sum N_{\varphi n} \cos n\vartheta, \\ N_\vartheta &= \sum N_{\vartheta n} \cos n\vartheta, \\ N_{\vartheta\varphi} &= \sum N_{\vartheta\varphi n} \sin n\vartheta. \end{aligned} \tag{18}$$

4. Determination of membrane forces

4.1 Differential equation of the problem

Equilibrium equations [2]:

$$r_1 \frac{\partial N_\vartheta}{\partial \vartheta} + \frac{\partial (r N_{\varphi\vartheta})}{\partial \varphi} + N_{\vartheta\varphi} r_1 \cos \varphi = -r r_1 X, \tag{19}$$

$$r_1 \frac{\partial N_{\vartheta\varphi}}{\partial \vartheta} - r_1 N_\vartheta \cos \varphi + \frac{\partial (r N_\varphi)}{\partial \varphi} = -r r_1 Y, \tag{20}$$

$$\frac{N_\vartheta}{r_2} + \frac{N_\varphi}{r_1} = -Z. \tag{21}$$

Let us replace variable φ by z. Taking Eq. (5) and

$$\frac{d}{d\varphi} = r_1 \sin \varphi \frac{d}{dz} \tag{22}$$

into consideration, we obtain:

$$\frac{\partial N_\vartheta}{\partial \vartheta} + \frac{\partial (r N_{\varphi\vartheta})}{\partial z} \sin \varphi + N_{\vartheta\varphi} \cos \varphi = -r X, \tag{23}$$

$$\frac{\partial N_{\vartheta\varphi}}{\partial \vartheta} - N_\vartheta \cos \varphi + \frac{\partial (r N_\varphi)}{\partial z} \sin \varphi = -r Y, \tag{24}$$

$$N_\vartheta \sin \varphi + \frac{r}{r_1} N_\varphi = -r Z. \tag{25}$$

From (25):

$$N_\vartheta = -\frac{r}{\sin\varphi} Z - \frac{r}{r_1 \sin\varphi} N_\varphi . \tag{26}$$

The system of differential equations (23) to (25) can be reduced to

$$\frac{\partial(r^2 N_{\vartheta\varphi})}{\partial z} - \frac{r}{r_1 \sin^3\varphi} \frac{\partial(r N_\varphi \sin\varphi)}{\partial\vartheta} = -\frac{r^2}{\sin\varphi}\left(X - \frac{1}{\sin\varphi}\frac{\partial Z}{\partial\vartheta}\right), \tag{27}$$

$$\frac{\partial(r N_\varphi \sin\varphi)}{\partial z} + \frac{1}{r^2}\frac{\partial(r^2 N_{\vartheta\varphi})}{\partial\vartheta} = -\frac{r}{\sin\varphi}(Y\sin\varphi + Z\cos\varphi). \tag{28}$$

Taking surface and internal loads by Eqs (14) and (18), resp., into consideration (Fourier's method) yields:

$$\frac{d(r N_{\varphi n}\sin\varphi)}{dz} + \frac{n}{r^2}(r^2 N_{\vartheta\varphi n}) = -\frac{r}{\sin\varphi}(Y_n \sin\varphi + Z_n\cos\varphi) \tag{29}$$

$$\frac{d(r^2 N_{\vartheta\varphi n})}{dz} - \frac{\alpha\, a^2 n r}{(r-c)^3}(r N_{\varphi n}\sin\varphi) = -\frac{r^2}{\sin\varphi}\left(X_n + \frac{n}{\sin\varphi} Z_n\right). \tag{30}$$

Introducing new variable (see in (2))

$$\xi = \frac{z}{a} \tag{31}$$

yields:

$$\frac{d}{dz} = \frac{1}{a}\frac{d}{d\xi}. \tag{32}$$

Introducing reduced internal forces

$$U_n = U_n(\xi) = r N_{\varphi n}\sin\varphi, \tag{33}$$

$$V_n = V_n(\xi) = r^2 N_{\vartheta\varphi n} \tag{34}$$

and notations

$$A_n = -\frac{a^2\left(\varrho + \frac{c}{a}\right)}{\sin\varphi}(Y_n\sin\varphi + Z_n\cos\varphi), \tag{35}$$

$$B_n = \frac{a^3\left(\varrho + \frac{c}{a}\right)^2}{\sin\varphi}\left(X_n + \frac{n}{\sin\varphi} Z_n\right) \tag{36}$$

the derivative being, in short form:

$$\frac{\partial}{\partial\xi}(\ldots) = (\ldots)^{\cdot}$$

leads to the differential equations

$$U_n^{\cdot} + \frac{n}{a\left(\varrho + \frac{c}{a}\right)^2} V_n = A_n , \tag{37}$$

$$V_n^{\cdot} - \frac{n\alpha a\left(\varrho + \frac{c}{a}\right)}{\varrho^3} U_n = B_n. \tag{38}$$

Introducing reduced membrane force

$$H_n = \left(\varrho + \frac{c}{a}\right) U_n = N_{\varphi n} a\left(\varrho + \frac{c}{a}\right)^2 \sin\varphi \tag{39}$$

transforms (37) and (38) to:

$$\left(\frac{H_n}{\varrho + \frac{c}{a}}\right)^{\cdot} + \frac{n}{a\left(\varrho + \frac{c}{a}\right)^2} V_n = A_n , \tag{40}$$

$$V_n^{\cdot} - \frac{n\alpha a}{\varrho^3} H_n = B_n. \tag{41}$$

Let us express V_n and $V_n^{\cdot}$ from (40) and (41):

$$V_n = \frac{a}{n}\left(\varrho + \frac{c}{a}\right)^2 A_n - \frac{a}{n}\left[\left(\varrho + \frac{c}{a}\right) H_n^{\cdot} - \frac{\cos\varphi}{\sin\varphi} H_n\right], \tag{42}$$

$$V_n^{\cdot} = -\frac{a}{n}\left(\varrho + \frac{c}{a}\right) H_n^{\cdot\cdot} + \frac{\alpha a}{n\varrho^3} H_n + \frac{2a}{n}\left(\varrho + \frac{c}{a}\right)\frac{\cos\varphi}{\sin\varphi} A_n + \\ + \frac{a}{n}\left(\varrho + \frac{c}{a}\right)^2 A_n^{\cdot} . \tag{43}$$

$V_n^{\cdot}$ is expressed by relationships (41) and (42), yielding differential equation

$$H_n^{\cdot\cdot} + \frac{\alpha(n^2 - 1)}{\varrho^3\left(\varrho + \frac{c}{a}\right)} H_n = \left(\varrho + \frac{c}{a}\right) F_n \tag{44}$$

for the reduced internal force H_n, where:

$$F_n = A_n^{\cdot} + \frac{2\cos\varphi}{\left(\varrho + \frac{c}{a}\right)\sin\varphi} A_n - \frac{n}{a\left(\varrho + \frac{c}{a}\right)^2} B_n . \tag{45}$$

Solution of the differential equation yields the value of H_n, delivering, in turn' internal forces:

$$N_{\varphi n} = \frac{1}{a\left(\varrho + \frac{c}{a}\right)^2 \sin\varphi} H_n , \tag{46}$$

$$N_{\vartheta\varphi n} = -\frac{1}{na\left(\varrho + \frac{c}{a}\right)^2}\left[\left(\varrho + \frac{c}{a}\right) H_n^{\cdot} - \frac{\cos\varphi}{\sin\varphi} H_n\right], \tag{47}$$

$$N_{\vartheta n} = -\frac{a\left(\varrho + \frac{c}{a}\right)}{\sin\varphi}\left(Z_n + \frac{N_{\varphi n}}{r_1}\right). \tag{48}$$

4.2 Solution of the differential equation

Differential equation (44) will be applied for determining internal forces arising from unilateral loads (wind load components varying with angular position around the shell ($n \geq 1$).

Differential equation (40) is easier to handle for axisymmetric loads ($n = 0$) (axisymmetric part of dead and wind load).

4.21 Axisymmetric loads

4.211 Dead load

Assuming constant thickness, specific load is $g = h\gamma$. Taking Eqs (12), (35) and (36), as well as $n = 0$, $N_{\vartheta\varphi} = V_0 = 0$ into consideration,

$$A_0 = -\frac{a^2\left(\varrho + \frac{c}{a}\right)}{\sin\varphi} g , \qquad B_0 = 0 . \tag{49}$$

Making use of differential equation (40):

$$\left(\frac{H_0}{\varrho + \frac{c}{a}}\right)^{\cdot} = -\frac{a^2\left(\varrho + \frac{c}{a}\right)}{\sin\varphi} g , \tag{50}$$

hence:

$$N_\varphi = -\frac{ag}{\left(\varrho + \frac{c}{a}\right)\sin\varphi}\left[\int \frac{\varrho + \frac{c}{a}}{\sin\varphi} d\xi + C_1\right]. \tag{51}$$

Value of the relationship in brackets may be obtained by numerical integration.

Assuming $\sin\varphi = 1.0$ within the integral:

$$N_\varphi = -\frac{ag}{\left(\varrho + \frac{c}{a}\right)\sin\varphi}\left\{\frac{1}{2}\left[\varrho\xi + \frac{1}{\sqrt{\alpha}}\ln\left(\xi + \frac{1}{\sqrt{\alpha}}\varrho\right)\right] + \frac{c}{a}\xi + C_1\right\} \tag{52}$$

C_1 being a constant according to the boundary condition.

In knowledge of N_φ, N_ϑ may be obtained from Eq. (48). For a varying shell thickness, assuming it to be section-wise constant, starting from the shell upper edge, internal forces can be determined at arbitrary points.

4.212 Axisymmetric part of the wind load

Wind load (16) described by a Fourier polynomial includes a load of constant angular intensity ($n = 0$) as constant term j_0:

$$X_0 = 0 \quad Y_0 = -q(\xi)j_0\cos\varphi, \quad Z_0 = q(\xi)j_0\sin\varphi. \tag{53}$$

$$A_0 = 0, \quad B_0 = 0, \quad V_0 = 0, \tag{54}$$

hence:

$$H_0 = \left(\varrho + \frac{c}{a}\right)C_1. \tag{55}$$

In knowledge of H_0:

$$N_{\varphi 0} = \frac{1}{a\left(\varrho + \frac{c}{a}\right)}C_1. \tag{56}$$

C_1 being a constant to be calculated from the boundary condition.

$$N_{\vartheta 0} = -\frac{a\left(\varrho + \frac{c}{a}\right)}{\sin\varphi}\left(Z_0 + \frac{N_{\varphi 0}}{r_1}\right). \tag{57}$$

4.22 Unilateral wind loads

Determination of membrane forces arising from unilateral wind loads will have recourse to differential equation (44).

In case of wind loads:

$$A_n = 0, \quad X_n = 0$$

hence

$$B_n = -\frac{na^3\left(\varrho + \frac{c}{a}\right)^2}{\sin^2\varphi}Z_n, \tag{58}$$

and so:

$$F_n = \frac{n^2a^2}{\sin^2 \varphi} Z_n \,. \tag{59}$$

Taking values of $q(z)$ and Z_n from (17) and (15), resp., into consideration yields the constant

$$F_n = n^2a^2 q j_n \,. \tag{60}$$

Solution of the inhomogeneous differential equation with a variable coefficient will be obtained by summing the solution H_{nH} of the homogeneous equation, and a particular solution H_{nP} of the inhomogeneous equation:

$$H_n = H_{nH} + H_{nP}. \tag{61}$$

In knowledge of particular solutions y_1 and y_2 of the homogeneous equation:

$$H_{nH} = C_1 y_2 + C_2 y_2 \,. \tag{62}$$

A particular solution of the inhomogeneous equation, according to the known method of varying the constants, can be written in the form:

$$H_{nP} = X_1 y_1 + X_2 y_2 \,, \tag{63}$$

where

$$X_1 = C_1(\xi), \quad X_2 = C_2(\xi). \tag{64}$$

$C_1(\xi)$ and $C_2(\xi)$ are known to be given by relationships:

$$C_1^{\cdot} = \frac{y_2}{\dot{y}_1 y_2 - y_1 \dot{y}_2} \left(\varrho + \frac{c}{a}\right) F_n \,, \tag{65}$$

$$C_2^{\cdot} = - \frac{y}{\dot{y}_1 y_2 - y_1 \dot{y}_2} \left(\varrho + \frac{c}{a}\right) F_n \,. \tag{66}$$

4.221 Load term $n = 1$

In this case, differential equation (44) becomes:

$$H_1^{\cdot\cdot} = \left(\varrho + \frac{c}{a}\right) F_1 \,, \tag{67}$$

where

$$F_1 = a^2 q j_1 \,. \tag{68}$$

Solution of the homogeneous equation:

$$H_{1H} = C_1 + C_2 \xi, \tag{69}$$

thus:

$$y_1 = 1 \quad \text{and} \quad y_2 = \xi. \tag{70}$$

A particular solution of the inhomogeneous equation:

$$H_{1P} = X_1 + X_2\xi. \tag{71}$$

$$X_1 = -F_1 \int \left(\varrho + \frac{c}{a}\right) \xi d\xi \,, \tag{72}$$

$$X_2 = F_1 \int \left(\varrho + \frac{c}{a}\right) d\xi \,. \tag{73}$$

The differential equation is solved as:

$$H_1 = C_1 + C_2\xi + X_1 + X_2\xi, \tag{74}$$

C_1 and C_2 being constants to be determined from boundary conditions.

In knowledge of H_1, membrane forces are delivered by Eqs (46) to (48).

4.222 Load terms $n > 1$

Wind load terms $n > 1$ prevent solution of the differential equation in closed form.

There exist several approximate procedures. For instance, expanding the variable coefficients into an infinite power series, two particular solutions of the homogeneous equation can be written by the method of indeterminate coefficients.

In their possession, the solution is obtained in the familiar way.

5. Membrane deformations

5.1 Differential equation of deformations

Displacements:

u — tangential to the ring;
v — tangential to the meridian;
w — normal.

Signs: u and v are positive along increasing ϑ and φ, resp., w is positive if it points outwards. Relationship between displacements and strains:

$$\varepsilon_\varphi = \frac{1}{r_1}\left(\frac{\partial v}{\partial \varphi} + w\right) \tag{75}$$

$$\varepsilon_\vartheta = \frac{1}{r}\left(v \cos\varphi + w \sin\varphi + \frac{\partial u}{\partial \vartheta}\right), \tag{76}$$

$$\gamma_{\vartheta\varphi} = -\frac{u}{r}\cos\varphi + \frac{\partial u}{r_1 \partial \varphi} + \frac{\partial v}{r \partial \vartheta}\,. \tag{77}$$

Relationships between internal forces and displacements:

$$N_\varphi - \nu N_\vartheta = \frac{D}{r_1}\left(\frac{\partial v}{\partial \varphi} + w\right), \tag{78}$$

$$N_\vartheta - \nu N_\varphi = \frac{D}{r}\left(v \cos \varphi + w \sin \varphi + \frac{\partial u}{\partial \vartheta}\right), \tag{79}$$

$$N_{\vartheta\varphi} = \frac{D}{2(1+\nu)}\left(-\frac{u}{r}\cos \varphi + \frac{\partial u}{r_1 \partial \varphi} + \frac{\partial v}{r \partial \vartheta}\right), \tag{80}$$

$$D = Eh \tag{81}$$

ν being the contraction coefficient.

In possession of membrane forces in form (18), angular distribution of deformations may be assumed as:

$$u = \Sigma\, u_n \sin n\vartheta, \tag{82}$$

$$v = \Sigma\, v_n \cos n\vartheta, \tag{83}$$

$$w = \Sigma\, w_n \cos n\vartheta. \tag{84}$$

Now, replacing variable φ by z yields:

$$\frac{dv_n}{dz}\sin \varphi + \frac{w_n}{r_1} = \frac{1}{D}\left(N_{\varphi n} - \nu N_{\vartheta n}\right), \tag{85}$$

$$n u_n + v_n \cos \varphi + w_n \sin \varphi = \frac{r}{D}\left(N_{\vartheta n} - \nu N_{\varphi n}\right), \tag{86}$$

$$-\frac{u_n}{r}\cos \varphi + \frac{du_n}{dz}\sin \varphi - \frac{n}{r} v_n = \frac{2(1+\nu)}{D} N_{\vartheta\varphi n}. \tag{87}$$

From (86):

$$w_n \sin \varphi = -n u_n - v_n \cos \varphi + \frac{r}{D}\left(N_{\vartheta n} - \nu N_{\varphi n}\right). \tag{88}$$

Eliminating w_n reduces the system of differential equations (85) to (87) into:

$$\frac{d}{dz}\left(\frac{v_n}{\sin \varphi}\right) + \frac{n\alpha a^2}{(r-c)^3} u_n = \frac{1}{D r_1 \sin^3 \varphi}\left[N_{\varphi n}(r_1 \sin \varphi + \nu r) - N_{\vartheta n}(r + \nu r_1 \sin \varphi)\right], \tag{89}$$

$$\frac{d}{dz}\left(\frac{u_n}{r}\right) - \frac{n}{r^2}\frac{v_n}{\sin \varphi} = \frac{2(1+\nu)}{D r \sin \varphi} N_{\vartheta\varphi n}. \tag{90}$$

Considering variable ξ (see in (31) and (32)) and introducing reduced displacements

$$\mathcal{U}_n(\xi) = \frac{u_n}{r}, \tag{91}$$

$$\mathcal{V}_n(\xi) = -\frac{v_n}{\sin\varphi} \tag{92}$$

and notations

$$\mathcal{A}_n = \frac{2a(1+\nu)}{Dr\sin\varphi} N_{\vartheta\varphi n}, \tag{93}$$

$$\mathcal{B}_n = \frac{a}{Dr_1\sin^3\varphi}\left[N_{\varphi n}(r_1\sin\varphi + \nu r) - N_{\vartheta n}(r + \nu r_1 \sin\varphi)\right] \tag{94}$$

permits to write differential equations (89) and (90) as:

$$\dot{\mathcal{U}}_n + \frac{n}{a\left(\varrho + \frac{c}{a}\right)^2}\mathcal{V}_n = \mathcal{A}_n, \tag{95}$$

$$\dot{\mathcal{V}}_n = \frac{n\alpha a\left(\varrho + \frac{c}{a}\right)}{\varrho^3}\mathcal{U}_n = \mathcal{B}_n \tag{96}$$

where the point refers to derivation with respect to ξ. Introducing reduced displacement

$$\mathcal{H}_n = \left(\varrho + \frac{c}{a}\right)\mathcal{U}_n = \frac{u_n}{a} \tag{97}$$

transforms (95) and (96) to:

$$\left(\frac{\mathcal{H}_n}{\varrho + \frac{c}{a}}\right)^{\cdot} + \frac{n}{a\left(\varrho + \frac{c}{a}\right)^2}\mathcal{V}_n = \mathcal{A}_n, \tag{98}$$

$$\dot{\mathcal{V}}_n - \frac{n\alpha a}{\varrho^3}\mathcal{H}_n = \mathcal{B}_n. \tag{99}$$

From (98):

$$\mathcal{V}_n = \frac{a}{n}\left(\varrho + \frac{c}{a}\right)^2\mathcal{A}_n - \frac{a}{n}\left[\left(\varrho + \frac{c}{a}\right)\dot{\mathcal{H}}_n - \frac{\cos\varphi}{\sin\varphi}\mathcal{H}_n\right], \tag{100}$$

$$\dot{\mathcal{V}}_n = -\frac{a}{n}\left(\varrho + \frac{c}{a}\right)\ddot{\mathcal{H}}_n + \frac{\alpha a}{n\varrho^3}\mathcal{H}_n + \frac{2a}{n}\left(\varrho + \frac{c}{a}\right)\frac{\cos\varphi}{\sin\varphi}\mathcal{A}_n + {} + \frac{a}{n}\left(\varrho + \frac{c}{a}\right)^2\dot{\mathcal{A}}_n. \tag{101}$$

Expressions (99) and (101) for $\mathcal{V}_n^{\cdot}$ yield differential equation of reduced deformation $\mathcal{H}_n$:

$$\mathcal{H}_n^{\cdot\cdot} + \frac{\alpha(n^2 - 1)}{\varrho^3 \left(\varrho + \frac{c}{a}\right)} \mathcal{H}_n = \left(\varrho + \frac{c}{a}\right) \mathcal{F}_n, \tag{102}$$

where:

$$\mathcal{F}_n = \mathcal{A}_n^{\cdot} + \frac{2 \cos \varphi}{\left(\varrho + \frac{c}{a}\right) \sin \varphi} \mathcal{A}_n - \frac{n}{a \left(\varrho + \frac{c}{a}\right)^2} \mathcal{B}_n . \tag{103}$$

5.2 *Solution of the differential equation*

Differential equation (102) will be applied for determining displacements due to wind loads. In case of axisymmetric loads, Eq. (96) will be applied.

5.21 *Axisymmetric loads*

5.211 Dead load

In knowledge of internal forces N_φ and N_ϑ, since $n = 0$ and $N_{\vartheta\varphi} = 0$,

$$\mathcal{A}_0 = 0, \tag{104}$$

$$\mathcal{B}_0 = - \frac{\alpha a}{D \sin^2 \varphi} \left[N_\varphi \left(\frac{1}{\alpha} - \nu \frac{\varrho + \frac{c}{a}}{\varrho^3} \sin^2 \varphi \right) + N_\vartheta \left(\frac{\varrho + \frac{c}{a}}{\varrho^3} \sin^2 \varphi - \frac{\nu}{\alpha} \right) \right] \tag{105}$$

In case of axisymmetric loads, $\mathcal{U}_n = 0$, thus, by solving differential equation (96)

$$\mathcal{V}_0^{\cdot} = \mathcal{B}_0 \tag{106}$$

we obtain:

$$v_0 = \alpha a \sin \varphi \int \frac{1}{D \sin^2 \varphi} \left[N_\varphi \left(\frac{1}{\alpha} - \nu \frac{\varrho + \frac{c}{a}}{\varrho^3} \sin^2 \varphi \right) + \right.$$
$$\left. + N_\vartheta \left(\frac{\varrho + \frac{c}{a}}{\varrho^3} \sin^2 \varphi - \frac{\nu}{\alpha} \right) \right] d\xi + C . \tag{107}$$

Value of integration constant C can be determined from the boundary conditions. Below, at the support:

$$\mathcal{U}_0 = \mathcal{V}_0 = 0. \tag{108}$$

In possession of $\mathcal{V}_0$, taking (88) into consideration:

$$w_0 = -v_0 \frac{\cos\varphi}{\sin\varphi} + \frac{r}{D\sin\varphi}\left(N_{\vartheta n} - N_{\varphi n}\right),$$

$$U_0 = 0. \tag{109}$$

For reinforced concrete cooling towers, ν may be considered as zero.

5.212 Axisymmetric part of the wind load

In knowledge of membrane forces $N_{\varphi 0}$ and $N_{\vartheta 0}$ (see under 4.212) displacements may be obtained from relationships similar to those for the dead load.

5.22 Unilateral wind loads

Disturbance term in the right-hand side of the differential equation may be obtained by assuming $\nu = 0$, and applying the neglect permitted in derivatives:

$$\mathcal{A}_n = \frac{2}{D\left(\varrho + \frac{c}{a}\right)\sin\varphi} N_{\vartheta\varphi n}, \tag{110}$$

$$\mathcal{A}_n^{\cdot} = -\frac{2\cos\varphi}{D\left(\varrho + \frac{c}{a}\right)^2\sin^2\varphi} N_{\vartheta\varphi n} + \frac{2}{D\left(\varrho + \frac{c}{a}\right)\sin\varphi} N_{\vartheta\varphi n}^{\cdot}, \tag{111}$$

$$\mathcal{B}_n = -\frac{a}{D\sin^2\varphi} N_{\varphi n} - \frac{\alpha a\left(\varrho + \frac{c}{a}\right)}{D\varrho^3} N_{\vartheta n}. \tag{112}$$

In compliance with the above:

$$\left(\varrho + \frac{c}{a}\right)\mathcal{F}_n = \frac{2\cos\varphi}{D\left(\varrho + \frac{c}{a}\right)\sin^2\varphi} N_{\vartheta\varphi n} + \frac{2}{D\sin^2\varphi} N_{\vartheta\varphi n}^{\cdot} +$$

$$+ \frac{n}{D\left(\varrho + \frac{c}{a}\right)\sin^2\varphi} N_{\varphi n} + \frac{\alpha n}{D\varrho^3} N_{\vartheta n}. \tag{113}$$

This relationship in the right-hand side of the differential equation can also be expressed in terms of function H_n determined in connection with membrane forces. Omitting deductions:

$$\left(\varrho + \frac{c}{a}\right) \mathcal{F}_n = \frac{2 \cos \varphi}{nDa \left(\varrho + \frac{c}{a}\right)^2 \sin^2 \varphi} H_n^{\cdot} +$$

$$+ \frac{n\left[\varrho^3 + 2\alpha \left(\varrho + \frac{c}{a}\right) \sin^2 \varphi\right]}{Da\varrho^3 \left(\varrho + \frac{c}{a}\right)^3 \sin^3 \varphi} H_n - \frac{2\varrho^3 + \alpha \left(\varrho + \frac{c}{a}\right) \sin^2 \varphi}{nDa\, \varrho^3 \sin \varphi} F_n , \qquad (114)$$

where:

$$F_n = n^2 a^2 q j_n .$$

Differential equation delivering reduced displacement $\mathcal{H}_n$ is similar to that of membrane forces, hence also its solution relies on similar principles.

Values of both integration constants in the solution can be determined from boundary conditions for the lower shell edge:

$$u_n = v_n = 0. \qquad (115)$$

Displacements u_n, v_n and w_n can be calculated in knowledge of reduced displacement $\mathcal{H}_n$ and reduced force H_n:

$$u_n = a\mathcal{H}_n \qquad (116)$$

$$\mathcal{V}_n = - \frac{2}{n^2 D \left(\varrho + \frac{c}{a}\right) \sin \varphi} \left[\left(\varrho + \frac{c}{a}\right) H_n^{\cdot} - \frac{\cos \varphi}{\sin \varphi} H_n\right] -$$

$$- \frac{a}{n}\left[\left(\varrho + \frac{c}{a}\right) \mathcal{H}_n^{\cdot} - \frac{\cos \varphi}{\sin \varphi} \mathcal{H}_n\right]. \qquad (117)$$

$$v_n = -\sin \varphi\, \mathcal{V}_n. \qquad (118)$$

Taking (88) into consideration:

$$w_n = - \frac{1}{\sin \varphi}\left[n u_n + v_n \cos \varphi + \frac{a^2 \left(\varrho + \frac{c}{a}\right)^2}{D \sin \varphi} Z_n - \frac{\alpha \sin \varphi}{D\varrho^3} H_n\right]. \qquad (119)$$

Summary

A membrane theory of hyperbolic cooling towers is presented for the general case where axes of the directrix and of the shell of revolution do not coincide.

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