ANALYSIS OF SKEW SUPPORTING COLUMNS OF COOLING TOWERS

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1. Introduction

The typical colonnade grid as supporting system for cooling tower shells provides for the entry surface of air flow, technically motivating its application. Also the height of the columnar zone depends on technology conditions. Statically, the colonnade is expected to transfer shell loads on the foundation, and to provide for outer, spatial stability. Columns, usually prefabricated, are monolithically connected to both the foundation and the shell edge ring. Consequently, shell edge motions raise additional flexural and torsional moments, usually neglected in calculations.

In analysing the columns' stability, correct assumption of the buckling form is of importance, requiring, in turn, to know the stiffness of the connection to the edge ring. A detailed analysis of forces and reactions in the column to shell edge connection will be presented, together with a calculation method for determining additional stresses and the buckling half-wavelength, illustrated on a numerical example.

2. Initial assumptions

The columns are usually arranged according to one of the schemes in Fig. 1.

The followings will rely on scheme a) but results hold also for scheme b). Scheme c) may be required for particularly high column zones, in general for cooling towers of dry operation.

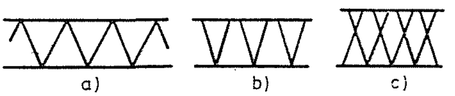

Fig. 1

Columns are assumed to be connected to closed, monolithic footing rings in the bottom, and to edge rings created by markedly thickening the shell edge.

The connection line between edge ring and the shell edge proper will be assumed to be where the shell part of constant or nearly constant thickness starts (Fig. 2).

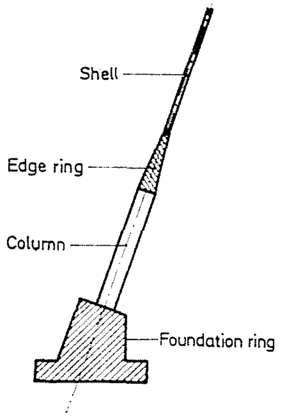

Fig. 2

The following assumptions are made for these principal structures:

— they are made of linear-elastic reinforced concrete;
— the cracks negligibly reduce the stiffness;
— column footings are rigidly clamped (foundation displacements are ignored);
— cross-sectional deformation of the edge ring is negligible compared to other displacements.

3. Geometry and stiffness conditions

3.1 Dimensions and cross-sectional characteristics

Computations will involve the following dimensions and cross-sectional characteristics (Fig. 3):

l — column length;
c — node spacing along the upper connection circle;
r — radius of the parallel ring belonging to the column to edge ring connection;
R — radius of the parallel circle belonging to the edge ring centroid;
z — vertical projection of the spacing of edge ring centroid;
α — slope of the meridian tangent to the lower shell edge (supposed to about equal the slope of the plane of columns O_1O_2);

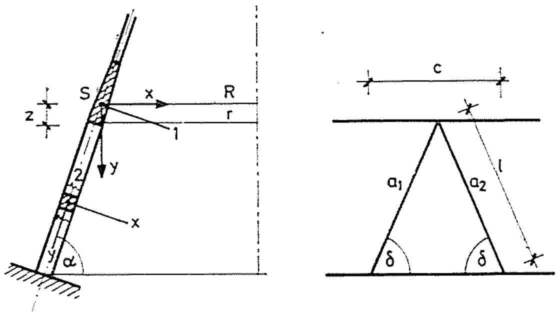

Fig. 3

δ	— slope of columns in the plane O_1O_2;
F_0, I_{x0}, I_{y0}	— cross-sectional area and moments of inertia of the column;
F, I_1, I_2	— cross-sectional area and moments of inertia of the ring in the principal directions;
I_x, I_y, I_{xy}	— ring cross section moments of inertia in directions x, y;
I_{p0}, I_p	— polar moments of inertia of column and ring cross section.

3.2 Stiffness parameters

3.21 The pair of columns O_1O_2

These two columns, rigidly clamped in the bottom and freely displaced in the top but with ends rigidly connected, are first exposed to moment $M^* = 1$ in the meridian plane, then to a radial force $Q^* = 1$. Let us determine angular rotation φ and radial displacement Δ for both cases. In the first case, moment M^* has to be decomposed into flexural and torsional components M_1, M_2, M_{1T} and M_{2T} to comply with conditions of equilibrium, symmetry, and compatibility with column end connections.

Because of the symmetry conditions:

$$\begin{aligned} |M_1| &= |M_2| = |M|, \\ |M_{1T}| &= |M_{2T}| = |M_T|. \end{aligned} \tag{1}$$

Symmetry permits to write one non-trivial equilibrium equation:

$$2M \sin\delta - 2M_T \cos\delta = M^* . \tag{2}$$

Compatibility equation expresses the mutual impossibility of relative rotation for the column ends i.e. the angular rotation vector of any end cross section to be normal to the t-axis (Fig. 4):

$$\frac{M}{EI_{y0}}\, l \cos\delta + \frac{M_T}{GI_{p0}}\, l \sin\delta = 0 . \tag{3}$$

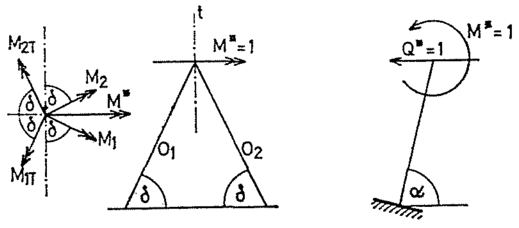

Fig. 4

Eqs (2) and (3) yield:

$$M = \frac{M^*}{2} \frac{\sin \delta}{\sin^2 \delta + a \cos^2 \delta}$$

$$M_T = -M \frac{a \cos \delta}{\sin \delta} = -\frac{M^*}{2} \frac{a \cos \delta}{\sin^2 \delta + a \cos^2 \delta} \tag{4}$$

where

$$a = \frac{GI_{p0}}{EI_{yi}}$$

permitting, in turn, to determine end face displacement φ_M, Δ_M:

$$\varphi_M = l \left(\frac{M}{EI_{y0}} \sin \delta - \frac{M_T}{GI_{p0}} \cos \delta \right) = \frac{l}{2EI_{y0}} \frac{1}{\sin^2 \delta + a \cos^2 \delta}$$

$$\Delta_M = \frac{l^2}{2} \sin \alpha \left(\frac{M}{EI_{y0}} \sin \delta - \frac{M_T}{GI_{p0}} \cos \delta \right) = \frac{l^2 \sin}{4EI_{y0}} \frac{1}{\sin^2 \delta + a \cos^2 \delta} . \tag{5}$$

Similar deductions yield displacements for the case $Q^* = 1$:

$$\varphi_Q = \frac{l^2 \sin \alpha}{4EI_{y0}} \frac{1}{\sin^2 \delta + a \cos^2 \delta}$$

$$\Delta_Q = \frac{l^3 \sin^2 \alpha}{6EI_{y0}} \frac{1}{\sin^2 \delta + a \cos^2 \delta} . \tag{6}$$

Let us now determine the displacement of a similar type, due to a horizontal, tangential force $T = 1$ acting at the common node, a problem analogous to that of moment decomposition (Fig. 5).

Due to antimetry:

$$| N_1 | = | N_2 | = N$$

$$| T_1 | = | T_2 | = T. \tag{7}$$

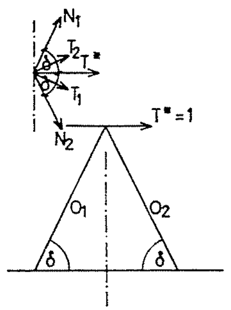

Fig. 5

The equilibrium equation:

$$2T \sin \delta + 2N \cos \delta = T^*. \tag{8}$$

The compatibility equation will rely on the condition that the vertical component of the displacement of the common node hence of any column end is zero.

(This condition follows from antimetry but it is also confirmed by the existence of a top edge ring providing rigid clamping and exemptness from vertical displacement.)

$$T \frac{l^3}{12EI_{x0}} \cos \delta - N \frac{l}{EF} \sin \delta = 0\,. \tag{9}$$

From (8) and (9):

$$T = \frac{T^*}{2} \frac{\sin \delta}{\sin^2 \delta + b \cos^2 \delta}$$

$$N = \frac{T^*}{2} \frac{b \cos \delta}{\sin^2 \delta + b \cos^2 \delta} \tag{10}$$

where

$$b = \frac{l^2 F_0}{12 I_{x0}}$$

leading to the node displacement along the force:

$$\Delta_T = \frac{Tl^3}{12EI_{x0}} \sin \delta + \frac{N}{EF_0} \cos \delta = \frac{T^* l^3}{24EI_{x0}} \frac{1}{\sin^2 \delta + b \cos^2 \delta}\,. \tag{11}$$

3.22 The edge ring

The forces and reactions of the column to ring connection will be examined by means of the force method, considering meridional bending moments and shears arising in the lower ring edge as unknown. An important width of thickened zone of the lower shell edge will be considered as edge ring, compared to its elongation and bending stiffnesses, those of the shell edge proper are considered as negligible. Thus, in the primary system to be assumed, out of the 3 + 3 force and moment components representing the rigid connection between column tops, bending moment M_z and meridional shear Q_x will be considered as unknown, bending and torsional moments M_x and M_y are omitted, and — since unknowns are assumed to constitute an equilibrium force system — normal force N becomes identically zero. The non-negligible shear force T will not be considered as unknown but the respective connection will be managed in assuming the primary structure. Thereby the ring part of the primary system is a so-called skew circular ring elastically bedded along the Z-axis, its principal cross section axes being not coincident with coordinate axes x, y, z in the axis line plane. As first step in determining the unit coefficient of unknown connection forces, let us produce displacements affecting the ring axis and generated by unit load with the function

$$q(\vartheta) = 1 \cos n\vartheta$$

$$m(\vartheta) = 1 \cos n\vartheta \qquad (n = 0, 2, 3, \ldots).$$

In the general case, for the displacement functions of an elastically bedded skew circular ring, a system of differential equations with four unknowns and constant coefficient can be written, deduced in [1] (Fig. 6).

The discussed circular ring has the peculiarity that among the three-way bedding coefficients, that along x is zero, that along y is very high, practically infinite, and a finite value emerges only along z. Furthermore, no external force acts along y and z, displacement v along y has a trivial solution, permitting to reduce by one the number of unknowns and equations. On the other hand, at a difference from assumptions made in [1], reckoning with a finite value for the ring elongation stiffness EF is both justified and possible. Making use of the cyclic character of load functions and making similar stipulations on

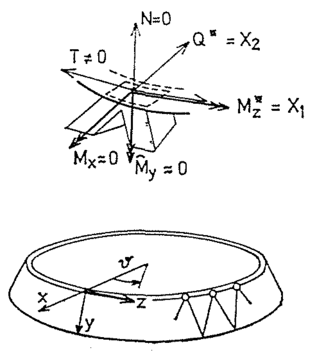

Fig. 6

the wanted solutions, particular solution of the inhomogeneous differential equation system corresponds at the same time to the general solution. Amplitudes u_0, φ_0, w_0 of functions u, φ, w, of the particular solution will be obtained after the usual reduction, by solving an algebraic equation system. In the following, function w will be useless and so will be its solution. This procedure will lead to the following solutions:

Case I: the ring is only exposed to unit load $q(\vartheta) = 1 \cos(n\vartheta)$

$$u_0^q = \frac{R^4}{E} \frac{(An^2 + K_z)(Tn^2 + I_x)}{D}$$
$$\varphi_0^q = \frac{R^3}{E} \frac{(An^2 + K_z) I_{xy}(n^2 - 1)}{D}. \tag{12}$$

Case II: the ring is only exposed to unit moment $m(\vartheta) = 1 \cdot \cos(n\vartheta)$

$$u_0^m = \frac{R^3}{E} \frac{(An^2 + K_z) I_{xy}(n^2 - 1)}{D}$$
$$\varphi_0^m = \frac{R^2}{E} \frac{[A(n^2 - 1) + K_z] I_y(n^2 - 1)n^2 + AK_z}{D} \tag{13}$$

with determinant

$$D = ATI_y(n^4 - n^2) + [A(n^2 - 1) + K_z](I_x I_y - I_{xy}^2)(n^2 - 1) n^2 + $$
$$+ K_z Tn^2[I_y(n^2 - 1)n^2 + A] + AK_z]_x$$

in the denominator.

Cross-sectional magnitudes not encountered under *3.1* are:

$A = R^2 \cdot F$ magnified value of the cross-sectional area;

$K = \dfrac{R^4 \cdot C_z}{E}$ magnified value of the bedding coefficient divided by E;

$T = \dfrac{J_p}{2(1 + \nu)}$ polar moment of inertia of the cross section, modified according to the Poisson's ratio.

Bedding coefficient C_z results from (11):

$$C_z = \frac{24EI_{x0}(\sin^2 \delta + b \cos^2 \delta)}{Cl^3}. \tag{14}$$

In Eqs (12) and (13), n is the load cycle number, possible with values $0, 2, 3, 4 \ldots$ etc. Case $n = 1$ has to be excluded since then the load produces no equilibrium force system. This case is, however, not critical in the analysis either of edge disturbances or of column buckling. Let us remark that in the case $n = 0$, hence in that of circular symmetric loading, displacement along z of any ring point is zero, suppressing the effect of elastic bedding. Now, displacements u_0, φ_0 equal the corresponding displacements of unbedded circular rings.

4. Force method for determining column stresses due to edge displacements

4.1 Fundamentals of the method

The precedings hint to the possibility of a method for computing column moments. Cutting the connection between capitals and edge ring suiting transfer of moment M_z and shear force Q results in the primary system (Fig. 7).

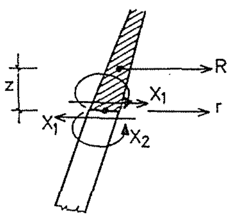

Fig. 7

Value of the moment corresponding to M_z will be unknown x_1, and that of radial force corresponding to force Q will be x_2. Unit coefficients will be determined from displacement values calculated under **3**. The corresponding load terms have to be indicated by previous calculation steps of shell and ring. These are primarily due to uneven warming of ring and foundation, and to ring expansion due to moist swelling. Among thermal movements, especially the effect of uniform peripheral heating is significant, that may be considered as of circular symmetric distribution. Also ring displacements due to shell membrane forces may be reckoned with, these seem, however, to develop negligible column stresses.

In knowledge of load terms and coefficients, the compatibility equation system can be solved and column moments calculated. For else than circular symmetric load terms, approximation by a trigonometric series of the form $y = \Sigma y_n \cdot \cos n\vartheta$ is imposed, solving the compatibility equation for every n. Respective coefficients can be determined as described under **3**.

4.2 *Unit coefficients*

Unit coefficients are easy to determine by using results under *3.2.* In applying the formulae, it has to be taken into consideration that magnitudes X_1, X_2 are specific values distributed along the circumference, and these connection forces act at the lower edge, rather than in the centroid axis of the ring, so that also the proper motions have to be found there. It is advisable to distinguish between parallel circle radii belonging to the centroidal axis and to the connection line. Accordingly, taking (5), (6), as well as (12) and (13) into consideration, unit coefficients become:

$$a_{11} = \frac{1}{E} \frac{Cl}{2I_{y0}} \frac{1}{\sin^2 \delta + a \cos^2 \delta} + \frac{r}{R} \varphi_0^m \tag{15}$$

$$\begin{aligned} a_{21} &= \frac{1}{E} \frac{Cl^2 \sin \alpha}{4I_{y0}} \frac{1}{\sin^2 \delta + a \cos^2 \delta} + \frac{r}{R} U_0^m - Z \frac{r}{R} \varphi_0^m \\ a_{12} &= \frac{1}{E} \frac{Cl^2 \sin \delta}{4I_{y0}} \frac{1}{\sin^2 \delta + a \cos^2 \delta} + \frac{r}{R} \varphi_0^q - Z \frac{r}{R} \varphi_0^m \end{aligned} \tag{16}$$

$$a_{22} = \frac{1}{E} \frac{Cl^3 \sin^2 \alpha}{6I_{y0}} \frac{1}{\sin^2 \delta + a \cos^2 \delta} + \frac{r}{R} U_0^q + Z^2 \frac{r}{R} \varphi_0^m . \tag{17}$$

4.3 *Load terms*

Let us determine, as an example, the displacement due to temperature difference between foundation and ring, and to the moist swelling of the ring. According to [2], these two effects can be reckoned with combined as a temperature difference $\Delta t = 30$ °C. Thus, neglecting the ring rotation:

$$a_{10} = 0$$

$$a_{20} = \alpha \cdot \Delta t \cdot R.$$

4.4 *Compatibility equation and stresses*

Compatibility equation of the form

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix}$$

can be solved without difficulty. Column moments will be determined from forces X_i according to (4):

At the column top:

$$M^f = -\frac{X_1 C}{2} \frac{\sin \delta}{\sin^2 \delta + a \cos^2 \delta} \tag{18}$$

$$M^f_T = \frac{X_1 C}{2} \frac{a \cos}{\sin^2 \delta + a \cos^2 \delta} . \tag{19}$$

At the column bottom:

$$M^a = (X_2\, l \sin \delta \sin \alpha + X_1) \frac{C}{2} \frac{\sin \delta}{\sin^2 \delta + a \cos^2 \delta} \tag{20}$$

$$M^a_T = (X_2\, l \sin \delta \sin \alpha + X_1) \frac{C}{2} \frac{a \cos \delta}{\sin^2 \delta + a \cos^2 \delta} . \tag{21}$$

5. Determination of the half-wavelength of buckling

5.1 Fundamentals of the method

In checking the column load capacity, knowledge of the half-wavelength of buckling is needed, irrespective of the centricity or excentricity of compression. In examining the buckling length, cases of buckling in, and normal to, the plane of the pair of columns have to be distinguished. In the following, this latter case will be considered where elasticity of the connection to the edge ring asserts itself. From the precedings it is clear that, because of the ring elasticity in elongation and rotation, the column top end is partially clamped and supported. Spring constants of rotational and displacement stiffnesses may be determined according to **3**. Let us consider what n values have to be assigned to the respective ring stiffnesses. As seen from the numerical example, the ring on tangentially elastic bedding is the stiffest to radial displacement for $n = 0$ hence circular symmetric deformation, then its stiffness tends to decrease to a minimum about $n = 4, 5$. (Remark that without elastic bedding the minimum would be at $n = 2$.)

Variation of the torsional stiffness is less significant, and also its increase or decrease depends on the actual cross section. It seems thus advisable to consider the variation of the nodal rigidity and so, of the buckling length, as a function of the ring wave number n. It is a question whether a deformation type $n > 0$ may be concomitant to column buckling. This possibility cannot be excluded even for a circular symmetric solution of column normal forces, namely here the disturbances causing loss of indifferent equilibrium are distributed, more likely to be concomitant to minimum ring stiffnesses. This problem would better fit that of the shell stability. In this respect let us refer to [3] indicating a critical annular wave number $n = 4$ to 7 for the general buckling-type stability loss of shells. Similar data are found in [4]. Accordingly, the assumption $n = 4$ to 6 as the most adverse ring stiffness is realistic. At the same time the column footing can be considered to be safely clamped against rotation and displacement. In knowledge of spring constants of the elastic clamping of the column top, the buckling half-wavelength can be obtained by solving the known eigenvalue problem. In the analytic way it leads to transcendental equations. Computational difficulties may be eliminated by the graphic method in [5], leading to a fast solution of adequate accuracy. This method will be applied in the numerical example.

5.2 *Determination of the stiffness parameters of elastic clamping*

Let us determine values of the bending moment needed to unit rotation φ; and of the shear force needed to unit displacement Δ of the column top (Fig. 8). Applying (13), (18) and (17):

$$M_{(\varphi=1)} = \frac{C}{2} \frac{\sin \delta}{\sin^2 \delta + a \cos^2 \delta} \frac{R}{r} \frac{1}{\varphi_0^m}$$
$$Q_{(\Delta=1)} = \frac{C}{2 \sin \alpha} \frac{R}{r} \cdot \frac{L}{u_0^q + Z^2 \varphi_0^m} . \tag{22}$$

These values can be considered as spring constants of column clamping, and are directly applicable in the graphical procedure by W. MUDRAK [5].

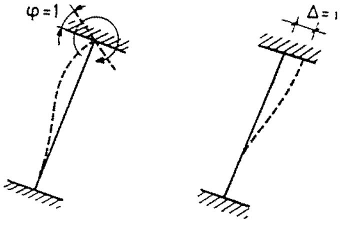

Fig. 8

6. Numerical example

Supporting system of a cooling tower has the characteristics seen in Fig. 9.

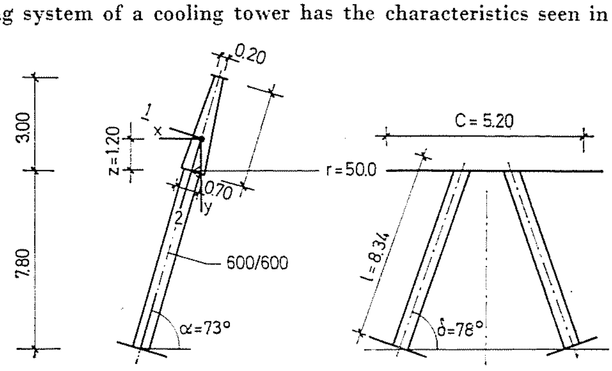

Fig. 9

Let us compute column moments due to a temperature difference $\Delta t = 30$ °C between ring and foundation, then determine the theoretical buckling length of the column.

6.1 Cross-sectional and dimensional data

Column cross section characteristics:

$$F_0 = 0.36 \text{ m}^2$$
$$J_{x0} = J_{y0} = 10.80 \cdot 10^{-3} \text{ m}^4$$
$$J_{p0} = 18.14 \cdot 10^{-3} \text{ m}^4.$$

Ring cross section characteristics:

$$F = 1.382 \text{ m}^2$$
$$Z = 1.250 \cdot \sin 73° = 1.200 \text{ m}$$
$$J_1 = 0.9734 \text{ m}^4$$
$$J_2 = 0.0563 \text{ m}^4$$
$$J_x = J_1 \cdot \cos^2 17° + J_2 \cdot \sin^2 17° = \ldots = 0.8950 \text{ m}^4$$
$$J_y = J_1 \cdot \sin^2 17° + J_2 \cdot \cos^2 17° = \ldots = 0.1347 \text{ m}^4$$
$$J_{xy} = (J_1 - J_2) \sin 17° \cdot \cos 17° = \ldots = 0.2564 \text{ m}^4$$
$$J_p = \frac{3.07 \cdot 0.45^3}{3} = 0.0933 \text{ m}^4$$
$$R = 50 - 1.25 \cdot \cos 73° = 49.64 \text{ m}$$
$$A = R^2 \cdot F = 49.64^2 \cdot 1.382 = 3405 \text{ m}^4$$
$$T = \frac{J_p}{2(1+\nu)} = \frac{0.0933}{2 \cdot 1.16} = 0.04022 \text{ m}^4.$$

6.2 Bedding parameters

Bedding coefficient C_z:

Substituted into (14):

$$C_z = E \, \frac{24 \cdot 10.80 \cdot 10^{-3} \sin^2 78° + 193.2 \cos^2 78°}{5.20 \cdot 8.34^3} = 0.800 \cdot 10^{-3} \cdot EN/\text{m}^2$$

$$b = \frac{8.34 \cdot 0.36}{12 \cdot 10.8 \cdot 10^{-3}} = 193.2.$$

Coefficient K_z:

$$K_z = 49.64^4 \cdot 0.8 \cdot 10^{-3} = 4858 \text{ m}^4.$$

6.3 Ring displacements due to unit forces

Let us compute the value of expressions (12), (13) for $n = 0, 2, 3, 4, 5, 6, 7$. Outcomes have been compiled in Table I.

Table I

n	U_0^q [m²/N]	$\varphi_0^q = U_0^m$ [m/N]	φ_0^m [1/N]
0	$1.783 \cdot 10^3$	-10.291	$2.753 \cdot 10^3$
2	$6.777 \cdot 10^3$	99.45	$2.333 \cdot 10^3$
3	$12.865 \cdot 10^3$	423.0	$1.9408 \cdot 10^3$
4	$20.139 \cdot 10^3$	1014.0	$1.4949 \cdot 10^3$
5	$25.129 \cdot 10^3$	1639.5	$1.0100 \cdot 10^3$
6	$23.302 \cdot 10^3$	1798.0	$0.550 \cdot 10^3$
7	$17.154 \cdot 10^3$	1484.0	$0.255 \cdot 10^3$

6.4 Unit coefficients

The actual loading displacement being circular symmetric, unit coefficients will be calculated from ring displacement for $n = 0$, using (15), (16), (17).

Column twist parameter:

$$a = \frac{GJ_{p0}}{EJ_{y0}} = \frac{J_{p0}}{J_{y0}} \frac{1}{2(1+\nu)} = \frac{18.14 \cdot 10^{-3}}{10.80 \cdot 10^{-3}} \frac{1}{2 \cdot 1.16} = 0.724$$

$$\text{Term } \frac{1}{\sin^2 \delta + a \cos^2 \delta} = \frac{1}{\sin^2 78^\circ + 0.724 \cos^2 78^\circ} = 1.012$$

$$E_{\text{all}} = \frac{5.20 \cdot 8.34}{2 \cdot 10.80 \cdot 10^{-3}} \cdot 1.012 + \frac{50}{49.64} \cdot 2.753 \cdot 10^3 = 4.805 \cdot 10^{-3}\ \text{m}^{-2}.$$

$$Ea_{21} = \frac{5.20 \cdot 8.34^2 \cdot \sin 73^\circ}{4 \cdot 10.80 \cdot 10^{-3}} 1.012 - \frac{50}{49.64}(10.29 + 1.20 \cdot 2.753 \cdot 10^3) = 3.338 \cdot 10^3$$

$$Ea_{12} = Ea_{21} = 3.338 \cdot 10^3$$

$$Ea_{22} = \frac{5.20 \cdot 8.34^3 \cdot \sin^2 73^0}{6 \cdot 10.80 \cdot 10^{-3}} \cdot 1.012 + \frac{50}{49.64}(1.783 \cdot 10^3 + 1.20^2 \cdot 2.753 \cdot 10^3) =$$

$$= 48.871 \cdot 10^3.$$

6.5 Load term

$$Ea_{10} = 0$$

$$Ea_{20} = E\Delta t \cdot \alpha \cdot R = 2 \cdot 10^{10} \cdot 30 \cdot 10^{-5} \cdot 50 = 300 \cdot 10^6\ \text{N/m}$$

$$E = E_b \approx 2 \cdot 10^{10}\ \text{N/m}^2.$$

6.6 Solution of the equation system

$$\begin{pmatrix} 4.805 & 3.338 \\ 3.338 & 48.871 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -300 \cdot 10^3 \end{pmatrix}$$

$$X_1 = \frac{+300 \cdot 10^3 \cdot 3.338}{4.805 \cdot 48.871 \cdot -3.338^2} = +4.477\ \text{kNm/m}$$

$$X_2 = \frac{-300 \cdot 10^3 \cdot 4.805}{4.805 \cdot 48.871 - 3.338^2} = -6.444\ \text{kN/m}.$$

6.7 Column forces

Bending moments:

$$M^f = \frac{4.477 \cdot 5.20}{2} \frac{\sin 78^\circ}{0.988} = 11.52\ \text{kNm}$$

$$M^a = (-6.44 \cdot 8.34 \cdot \sin 78^\circ \cdot \sin 73^\circ + 4.477)$$

$$\frac{5.20}{2} \cdot \frac{\sin 78^\circ}{0.988} = -117.88\ \text{kNm}.$$

Moments of torsion:

$$M_T^f = M^f \frac{a \cdot \cos}{\sin} = 11.52 \, \frac{0.724 \cdot \cos 78^\circ}{\sin 78^\circ} = 1.77 \text{ kNm}$$

$$M_T^a = -117.88 \, \frac{0.724 \cdot \cos 78^\circ}{\sin 78^\circ} = -18.14 \text{ kNm}.$$

Moments vary linearly between the two end points (Fig. 10).

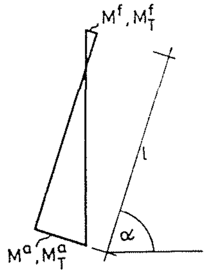

Fig. 10

6.8 Calculation of the buckling length

6.81 Spring constants of the column top:

Values of (22) will be calculated for $n = 0, 2, \ldots, 6$ (Table II).

Table II

n	$M(\varphi = 1)$ [Nm]	$Q(\Delta = 1)$ [N]
0	$0.942 \cdot 10^{-3}$	$0.476 \cdot 10^{-3}$
2	$1.111 \cdot 10^{-3}$	$0.270 \cdot 10^{-3}$
3	$1.336 \cdot 10^{-3}$	$0.175 \cdot 10^{-3}$
4	$1.734 \cdot 10^{-3}$	$0.123 \cdot 10^{-3}$
5	$2.567 \cdot 10^{-3}$	$0.103 \cdot 10^{-3}$
6	$10.166 \cdot 10^{-3}$	$0.114 \cdot 10^{-3}$

Parameters K^f and ψ will be formed from the quotient of spring constant by column stiffness coefficient:

$$K^f = M_{(\varphi=1)} \frac{l}{4EI_{y0}}$$

$$\psi = \frac{1}{Q_{(\Delta=1)}} \frac{EI_{y0}}{l^3} .$$

Column bottom:

$$K^a = \infty$$

Using these parameters, λ values are simply read off the nomogram in [5], yielding the buckling wavelength as:

$$l_0 = l \frac{\pi}{\lambda} .$$

Table III

n	K^j	φ	λ	l_0 [m]
0	0.182	0.039	5.60	4.68
2	0.215	0.069	3.85	6.81
3	0.258	0.106	3.40	7.71
4	0.335	0.151	3.40	7.71
5	0.496	0.180	3.15	8.32
6	1.963	0.163	3.20	8.19

Results of determinations for the assumed cases of n have been compiled in Table III. The column top connection is seen to be the softest for $n = 5$ where the half-wavelength of buckling is close to the real length. l_0 appears to be rather sensitive to the ring stiffness, so this result cannot be generalized.

Summary

Skew supporting columns of the cooling tower transmit shell membrane forces to the foundation. Because of the monolithic connection between capital and edge ring, shell edge displacements raise bending and torsional moments in the columns. The presented method lends itself to determine additional stresses and buckled forms of columns also exposed to other than circular symmetric effects.

References

1. Hegedűs, I.: Influence Function of Skew Circular Rings on Elastic Bedding. Per. Pol. C. E. Vol. 25 (1981) 1—2.
2. Orosz, Á.: Effects of Temperature on Reinforced Concrete Cooling Towers. Per. Pol. C. E. Vol. 25 (1981) 1—2.
3. Dulácska, E.—Farkas, Gy.—Nagy, J.: Stability of Hyperbolic Shells of Revolution.* Interim research report, Technical University, Budapest 1978.
4. Wittek, U.: Überblick und theoretische Einführung in das Stabilitätsverhalten von Kühlturmschalen. KIB Berichte, Essen 1977.
5. Mudrak, W.: Die Knickbedingung für den geraden Stab. Der Bauingenieur H. 5, 1941.

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